Thévenin's Theorem p. 141

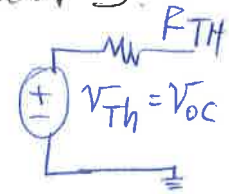
Statement

1. Given any linear circuit, rearrange it in the form of two network A & B, connected by two wires.

2. Disconnect network B. → replace it by infinite load. Define a voltage v_{oc} as the voltage appearing across the terminals of network A. → v_{Th}

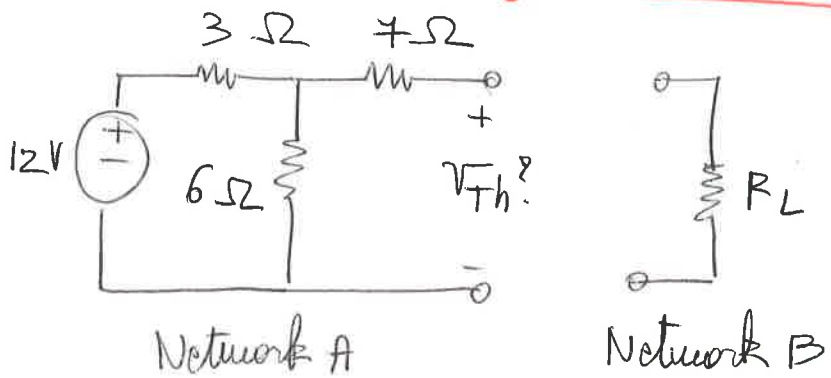
3. ZERO OUT every independent source in network A to form an inactive network (DO NOT TOUCH DEPENDENT SOURCES) → R_{TH}

4. Connect an independent voltage source v_{oc} ($= v_{TH}$) in series with the inactive network from part 3.



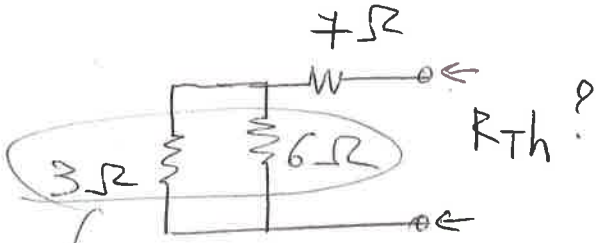
5. Connect network from step 4 back to network B. All currents and voltages in B will be the same as what would have been found without the Thévenin equivalent circuit for network A.

Back to Example 5.6 — Using Thevenin Theorem

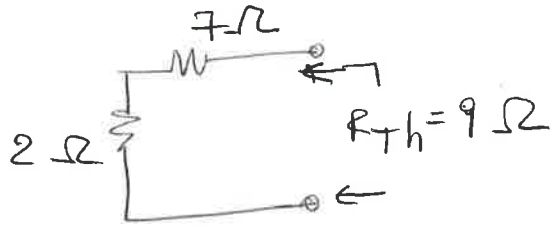


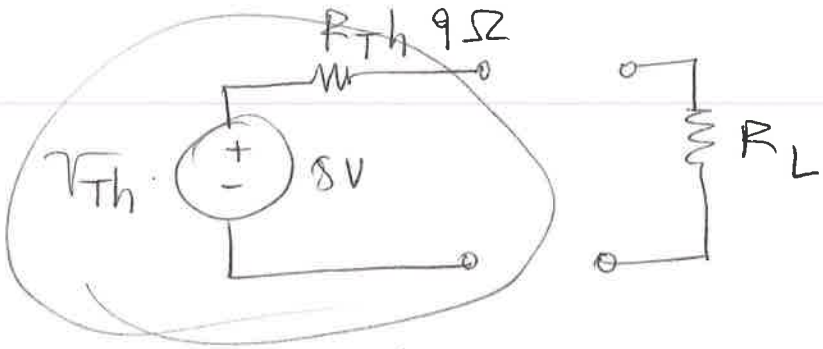
$$V_{Th} = \frac{6}{3+6} \cdot 12 = \frac{2}{3} \cdot 12 = 8\,V$$

R_{Th}



$$\frac{3 \cdot 6}{3+6} = \frac{18}{9} = 2\,\Omega$$





Norton Theorem p. 143

1. Given any linear circuit, rearrange it in the form of two networks (A & B). A is the network to be simplified.

   ⚠ If either network contains a dependent source, its controlling variable must be in the same network.

2. Disconnect network B, and short the terminals of A. Call i_{sc} the current flowing through the terminals of A.

3. "Zero out" every independent source in network A to form an inactive network. (Leave dependent sources unchanged).

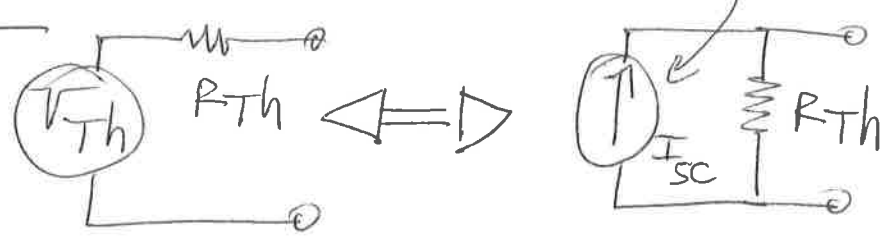
4. Connect an independent current source with value i_{sc} "in parallel" with the inactive network.

5. Connect network B to the two terminals of network B.

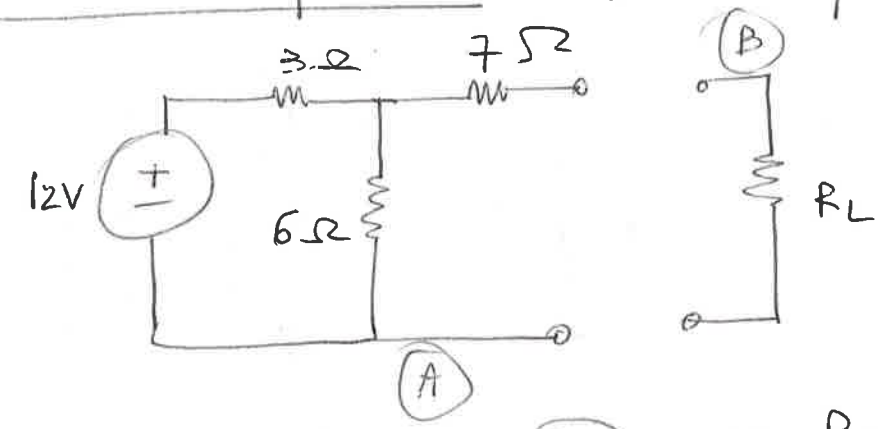
Connection: Thevenin & Norton

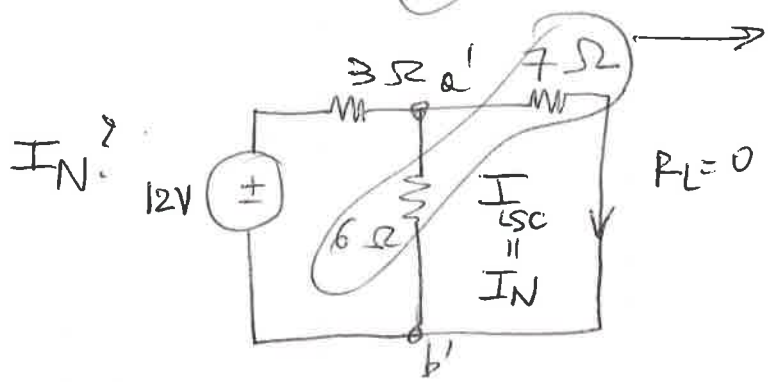
$$\frac{V_{Th}}{R_{Th}} = I_{sc} = \frac{V_{oc}}{R_{Th}}$$

$V_{oc} = V_{Th}$



Back to Example 5.6 (Norton Equivalent?)



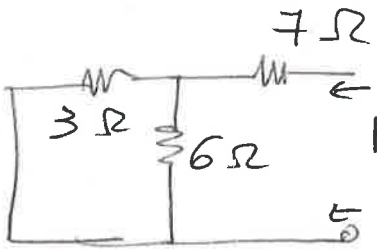


$$R_{eq} = \frac{7 \cdot 6}{13} = \frac{42}{13}\ \Omega$$

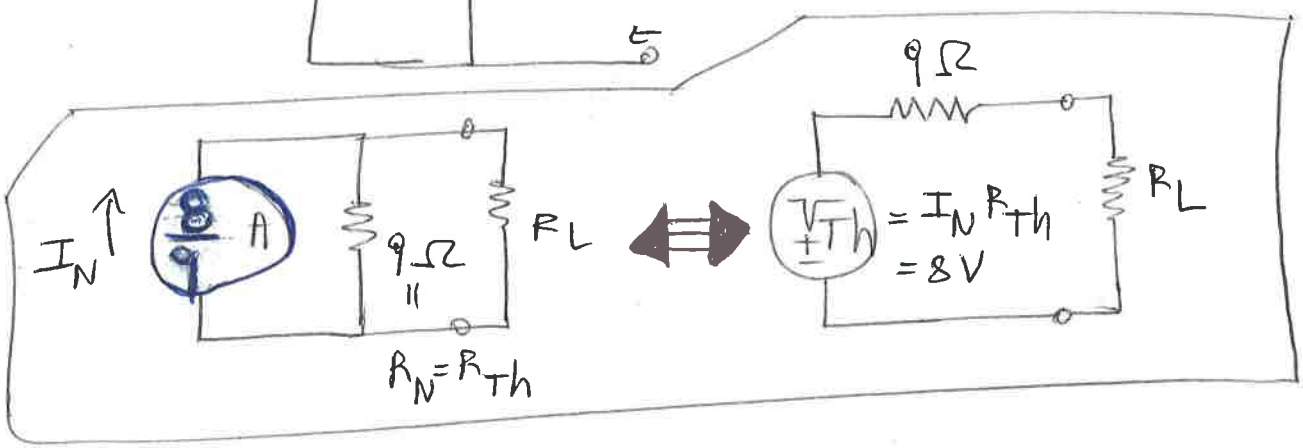
$$V_{a'b'} = \frac{\frac{42}{13}}{\frac{42}{13} + 3}\ 12 = \frac{42}{42+39} \cdot 12\ V$$

$$I_N = \frac{\cancel{42}^{6}}{81} \cdot \frac{12}{\cancel{7}} = \frac{72}{81}\ A = \frac{8}{9}\ A$$

R_N?

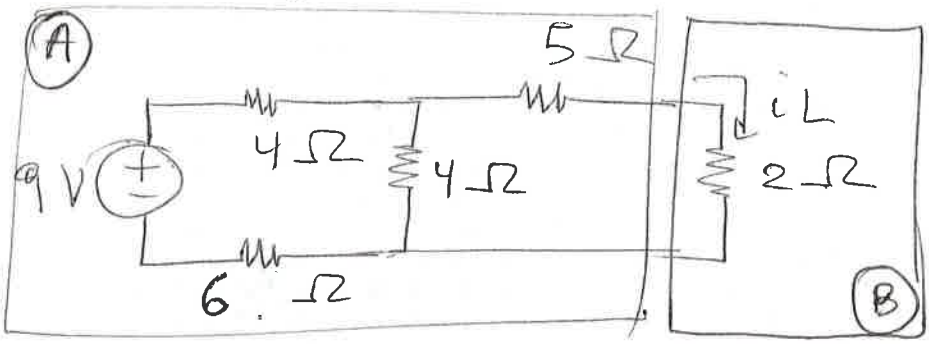


$$R_N = 7 + \frac{18}{9} = 9\ \Omega$$



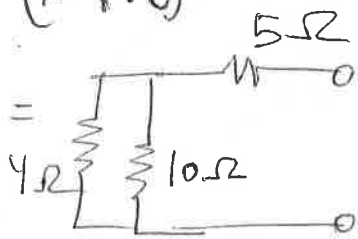
Practice problem 5.6 p. 142

Use Thévenin equivalent to find current through 2 Ω resistor in circuit below.



$$V_{Th} = \left(\frac{4}{4+4+6}\right) 9\,V = \frac{4}{14}\, 9\,V = 2.571\,V$$

$R_{Th} =$



$$5 + \frac{(4 \times 10)}{[4+10]} = 5 + \frac{40}{14} = 7.857\,\Omega$$

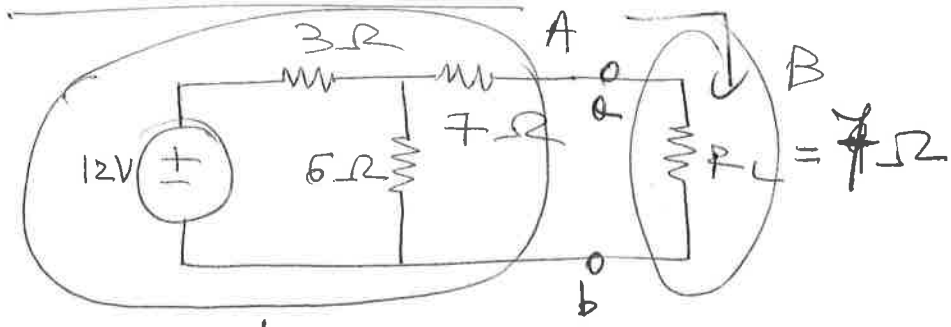
$\Rightarrow$ $i_L \overset{?}{=}$

$$i_L = \frac{V_{Th}}{R_{Th} + 2} = 260.8\,mA$$

Example 5.7 p.141

i_L?

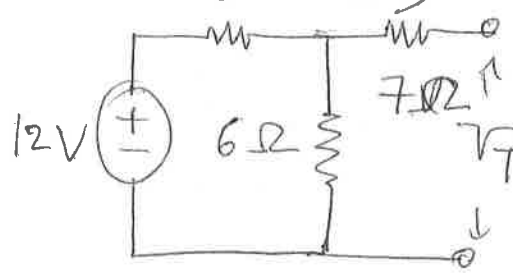
(1)



V_{Th}, R_{Th}?

V_{Th}?

(2)

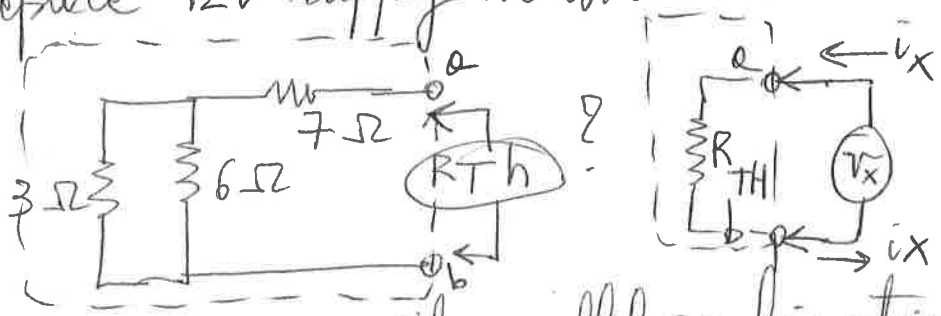


$R_L = \infty \rightarrow V_{ab} = V_{Th} = V_{oc}$

$$V_{Th} = V_{oc} = \frac{6}{[3+6]} \times 12 = 8V$$

R_{Th}? Replace 12V supply with a short circuit

(3)

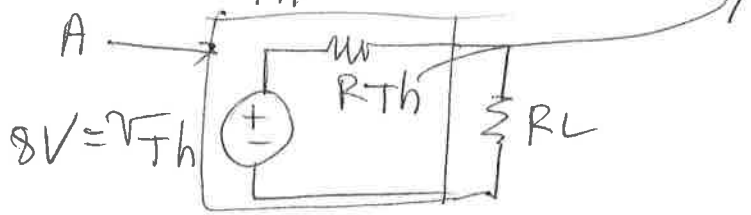


7Ω in series with parallel combination of 3Ω and 6Ω

$$R_{eq} = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$$

$\rightarrow R_{Th} = 7\Omega + 2\Omega = 9\Omega$

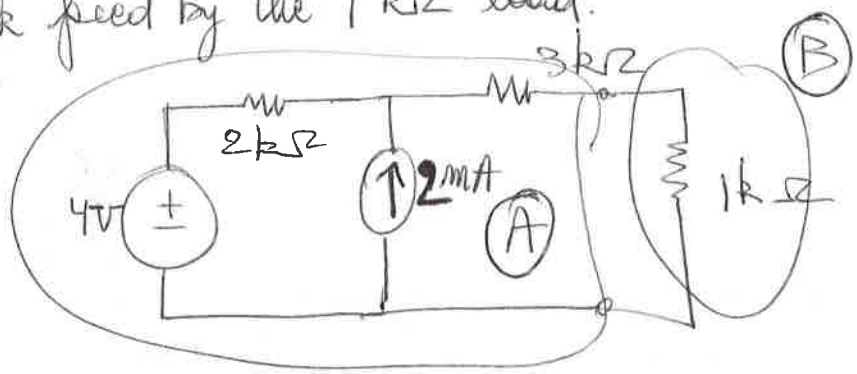
(4)



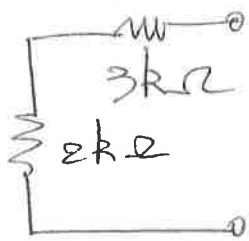
$$i_L = \frac{8V}{16\Omega} = 0.5A$$

Example 5.8 (p 144)

Find the Thévenin and Norton equivalent circuits for the network feed by the 1 kΩ load.



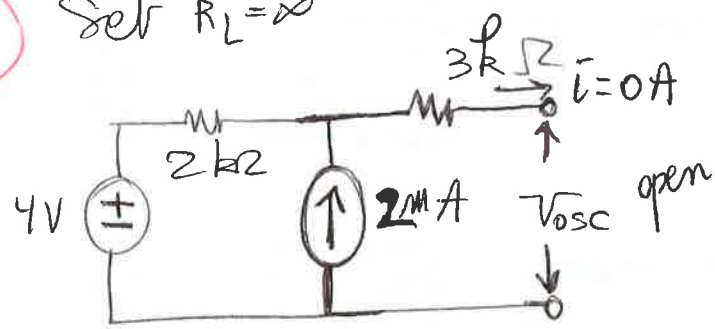
R_{Th}



$\Rightarrow R_{Th} = 5k\Omega$

ALL independent sources are simultaneously zeroed out

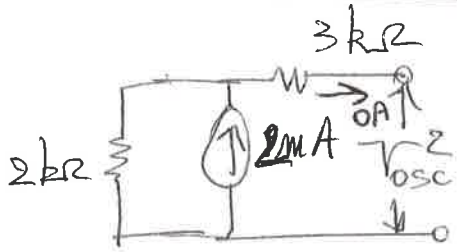
$V_{osc} = V_{Th}$ Set $R_L = \infty$



V_{osc} by principle of superposition

① set 2mA to zero → $\boxed{V_{osc}^1 = 4V}$

② Set 4V to zero

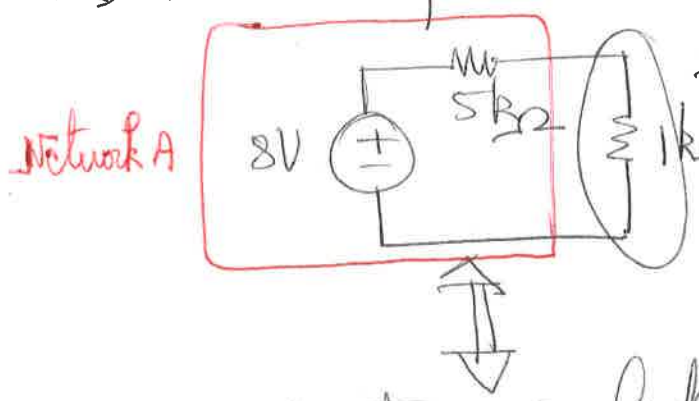


$$\boxed{V_{osc}^2 = 2k\Omega \times 2mA = 4V}$$

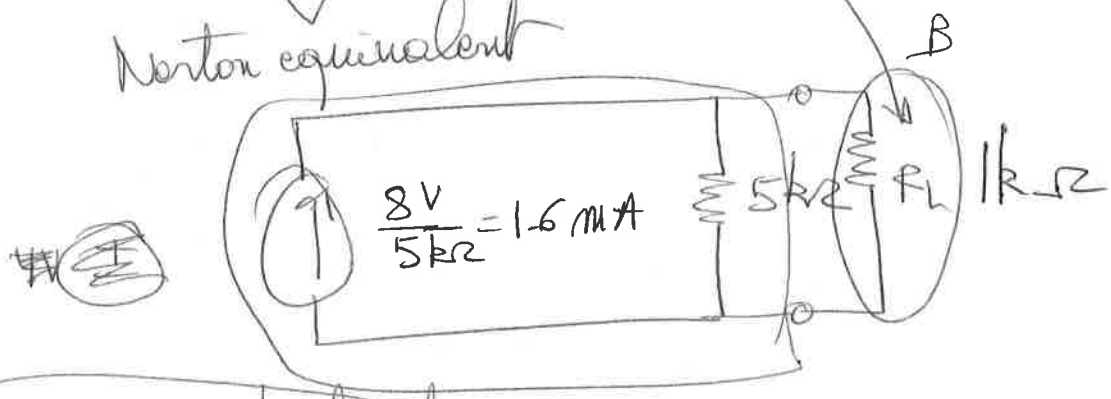
→ with both independent sources on

$$V_{osc} = V_{Th} = V_{osc}^1 + V_{osc}^2 = 8V$$

→ Thevenin equivalent



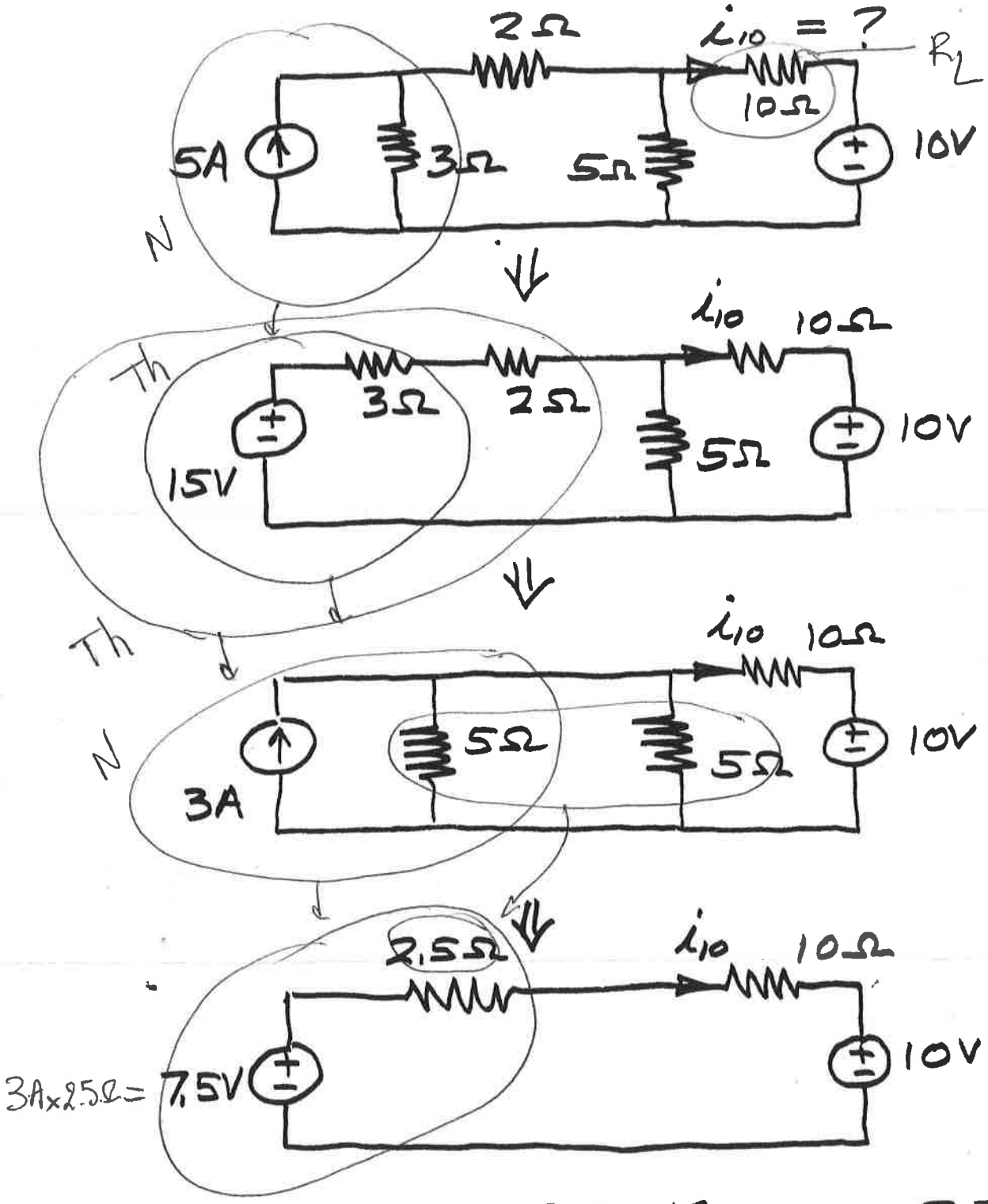
Norton equivalent



I_L can now be found easily using current divider rule

Do practice problem 5.7

~~When dependent sources are present~~

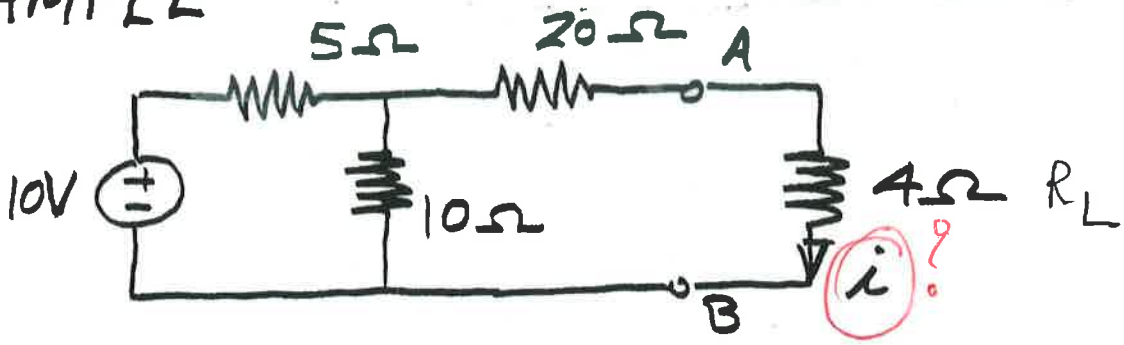


$$i_{10} = \frac{7.5 - 10}{2.5 + 10} = \frac{-2.5}{12.5}$$

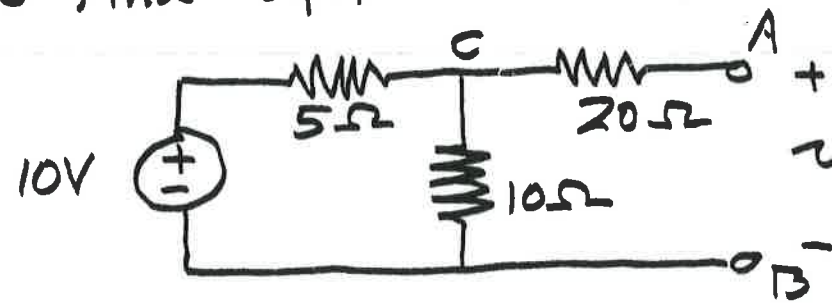
$$= -0.2\,A$$

SAME CIRCUIT using Thevenin and NORTON

EXAMPLE



To find v_T:



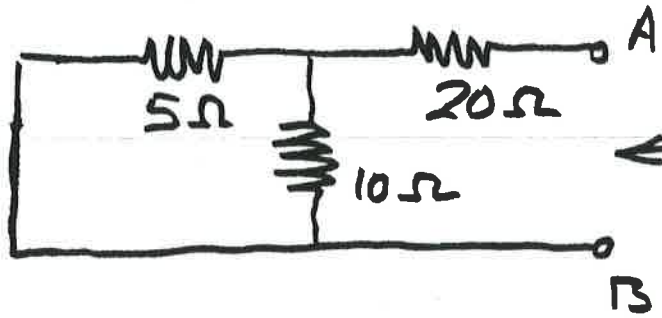
$$v_T = (v_{AB})_{oc}$$

$$= v_{CB}$$

$$= \left(\frac{10}{10+5}\right) 10$$

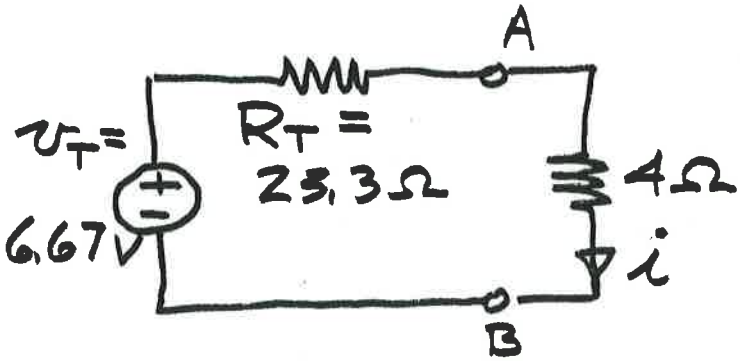
$$= 6.67\,V$$

To find R_T:



$$R_T = 20 + \frac{(5)(10)}{5+10}$$

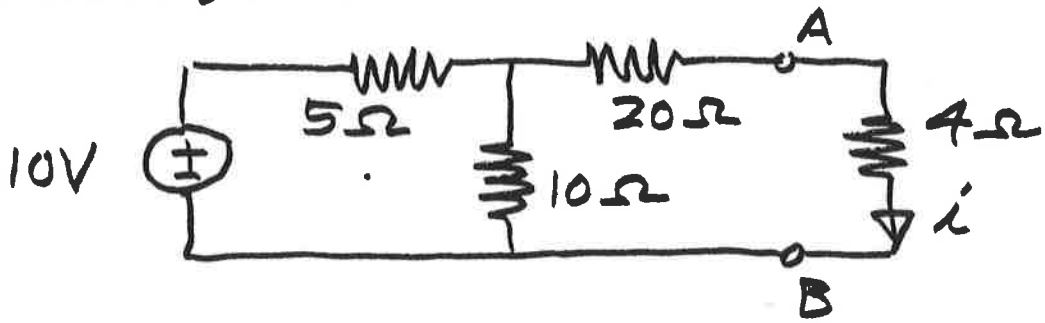
$$= 23.3\,\Omega$$



$$i = \frac{6.67}{4+23.3}$$

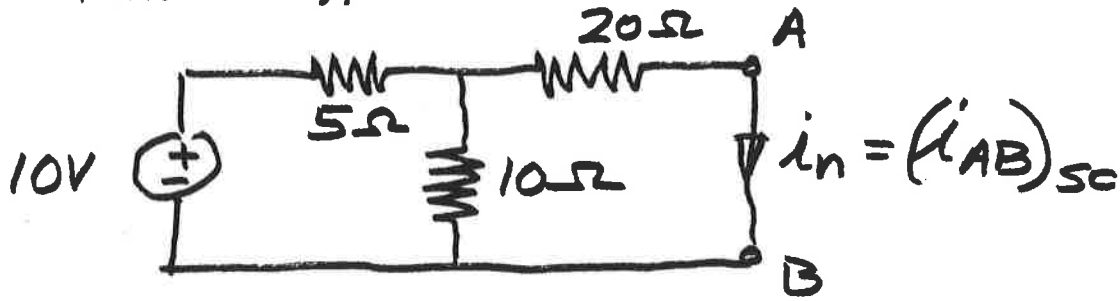
$$= 0.244\,A$$

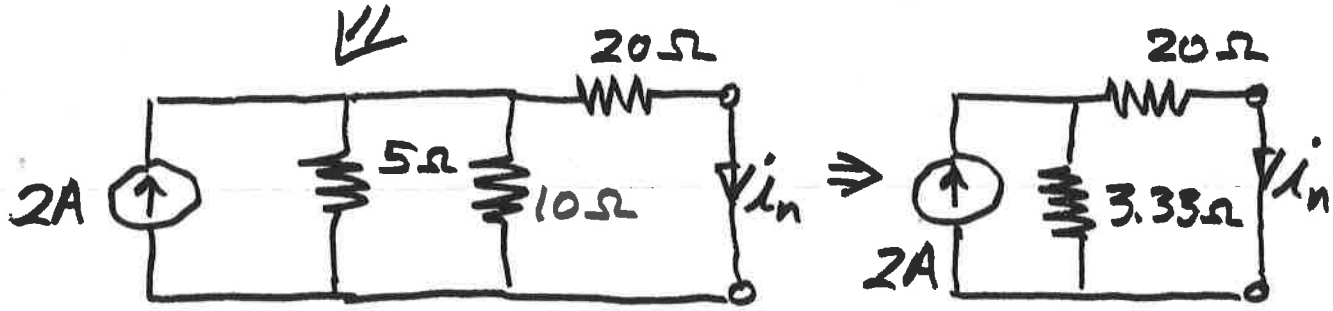
(Previous) EXAMPLE



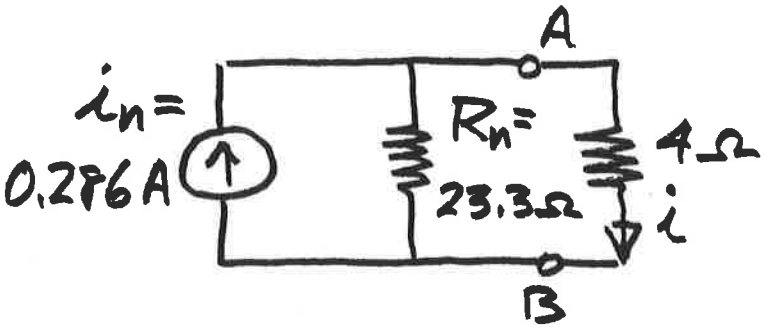
$R_n = R_T = 23.3\,\Omega$ as before.

To find i_n:





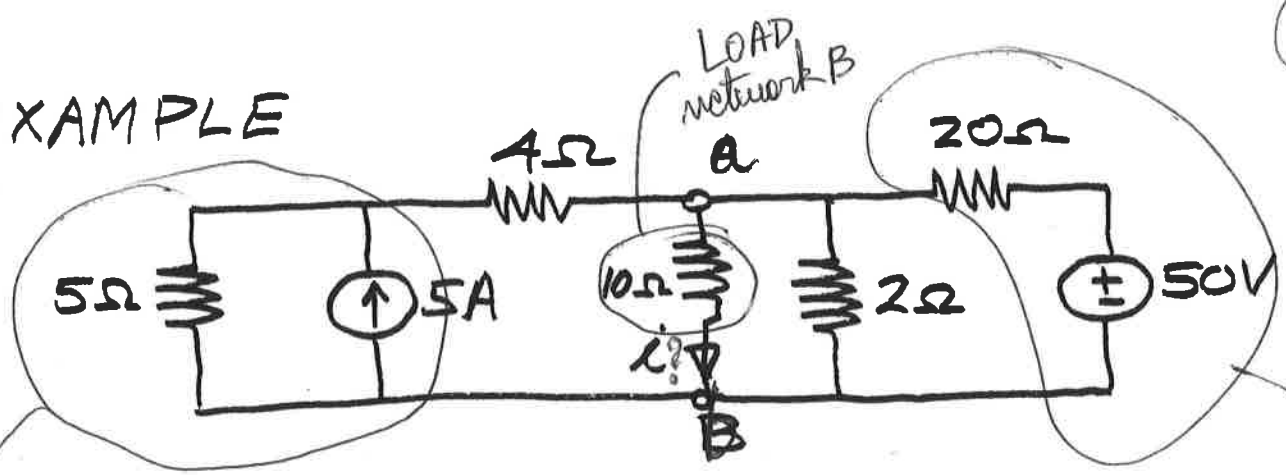
$$i_n = \left(\frac{3.33}{3.33+20}\right)2 = 0.286\,A$$



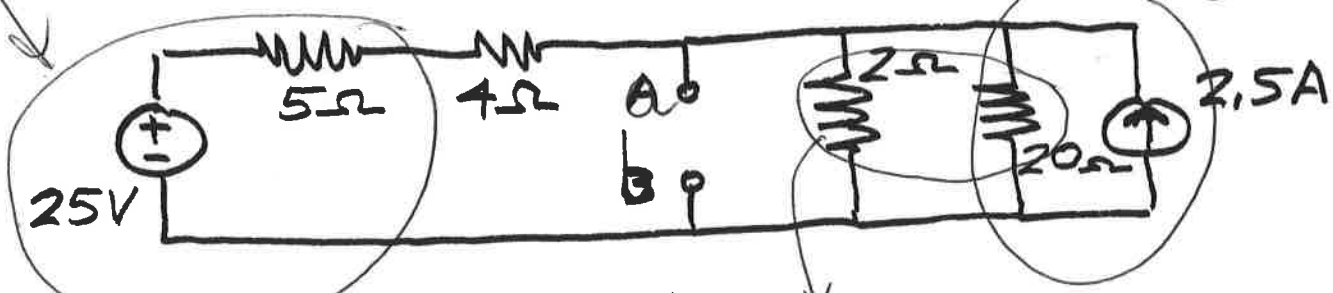
(And check)

$v_T = R_T i_n$

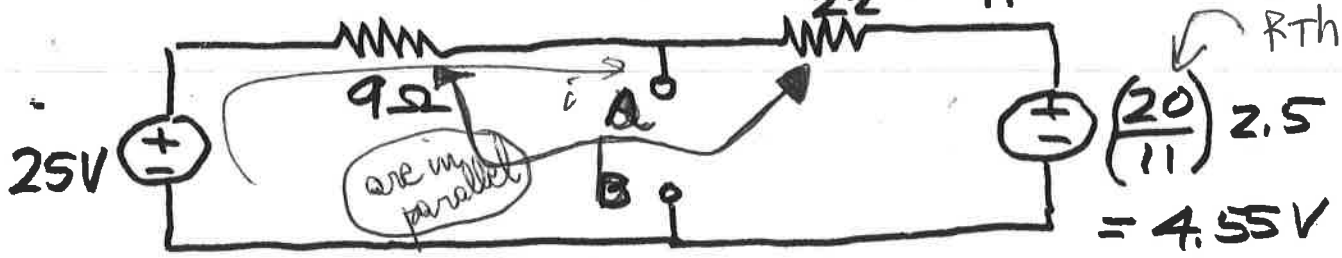
EXAMPLE



Use source transformations to replace network (all except 10Ω resistance) by Thevenin equivalent



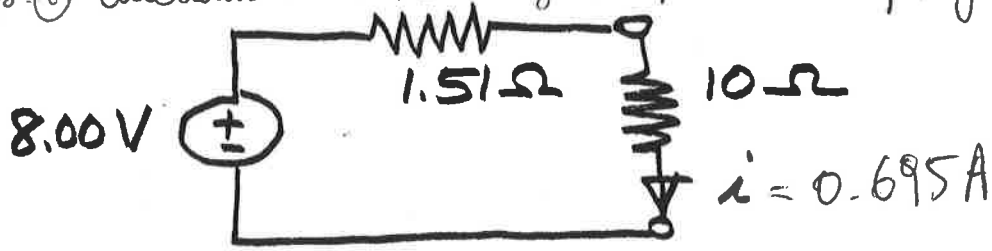
$$\frac{40}{22} = \frac{20}{11} = 1.82\,\Omega = R_{Th}$$



$$R_T = \frac{9(1.82)}{9+1.82} = 1.51\,\Omega$$

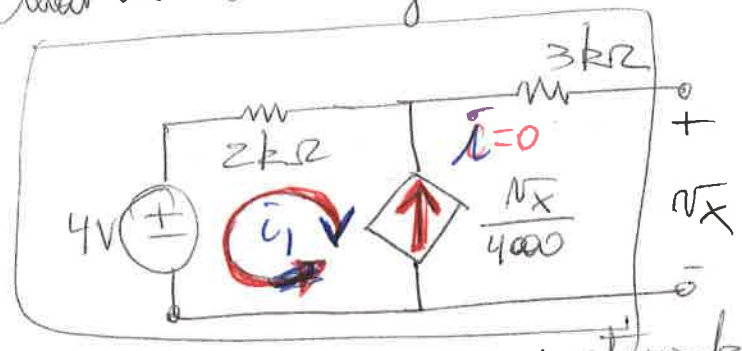
$$v_{ab} = v_T = 25 - 9\left(\frac{20.45}{10.82}\right) = 8.00\text{ V}$$

2 ways: (a) Calculate current through loop (or) Principle of superposition.



When dependent sources are present p.145

If network A contains a dependent source, we must ensure that the controlling variable CANNOT BE in network B.



Network A

What is V_{TH}, R_{TH} for circuit on the left?

$v_x = v_{oc} = V_{Th}$

$$-4 + 2\,10^3\left(\frac{-v_x}{4000}\right) + 3\times10^3(0) + v_x = 0$$

$i_1 = \frac{-v_x}{4000}$

$\longrightarrow v_x = V_{oc} = 8\,V$

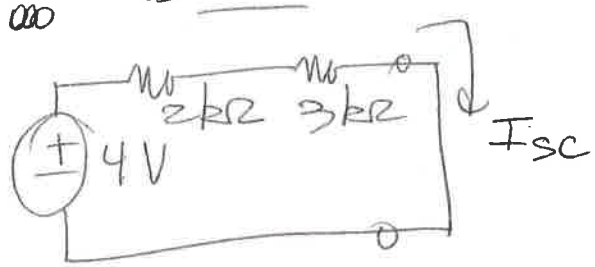
R_{TH}? $4V \to 0V$ but we do not know what $\frac{v_x}{4000}$ is

Let's go to Norton equivalent instead. Find I_{sc}

If we short-circuit terminals of network A

$\longrightarrow v_x = 0 \rightarrow$  $\frac{v_x}{4000}$ is zero

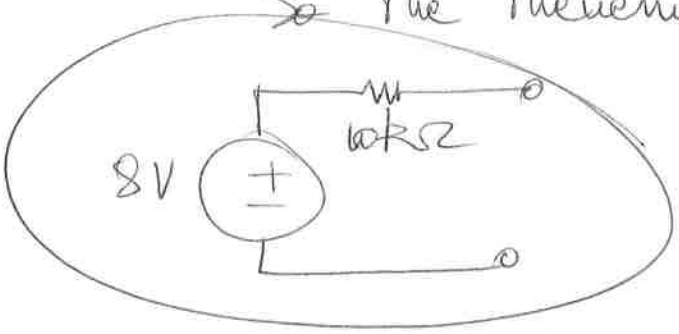
$\Rightarrow$ Network for I_{sc}



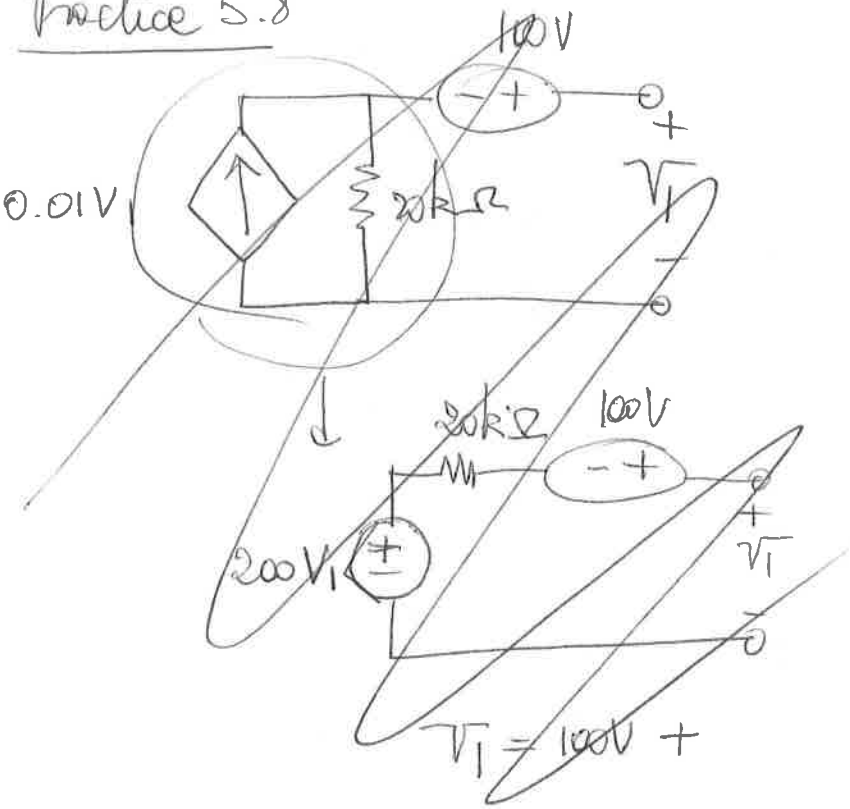
$$I_{sc} = \frac{4V}{5k\Omega} = 0.8\,mA$$

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{8}{0.8\,10^{-3}} = \underline{\underline{10\,k\Omega}}$$

So the Thevenin eq. is

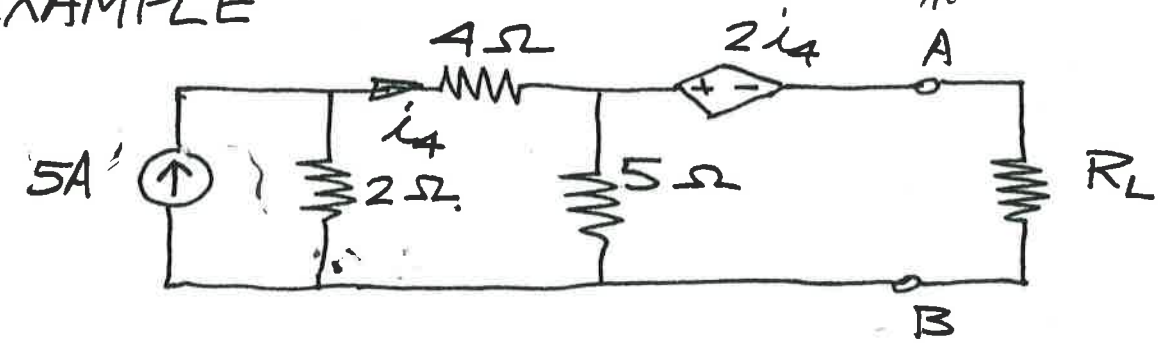


Practice 5.8

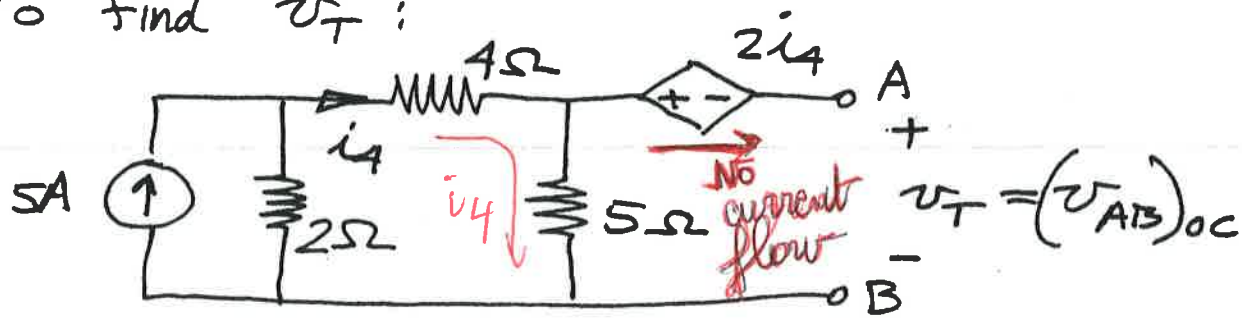


EXAMPLE

v_T & i_n → $R_{TH} = \frac{v_T}{i_n}$



To find v_T :

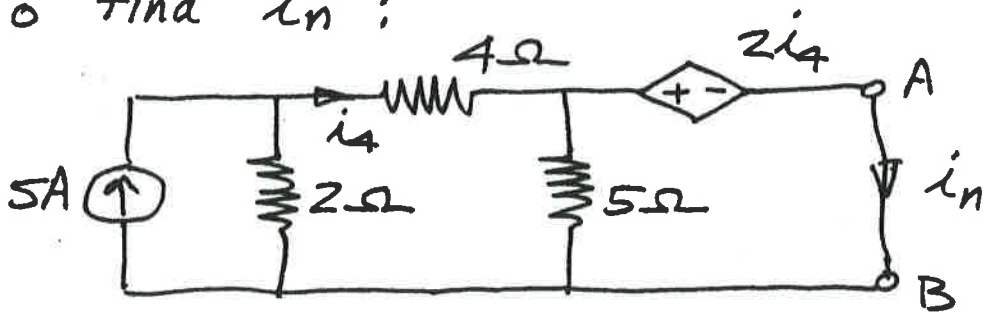


$$i_4 = \left(\frac{2}{2+9}\right) 5 = 0.909 \text{ A}$$

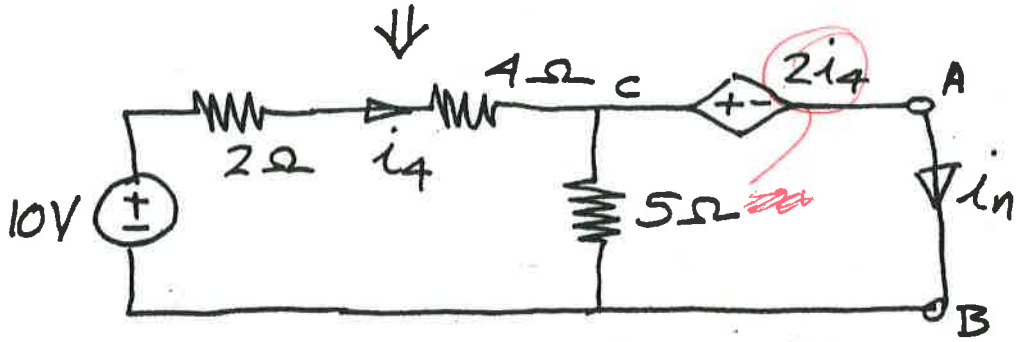
$$v_T = (v_{AB})_{oc} = -2i_4 + 5i_4 = 3i_4$$

$$= 2.73 \text{ V}$$

To find i_n :



⇓



$v_{CB} = 2i_4$

$$v_{CB} = 2i_4 = 10 - 6i_4$$

$$8i_4 = 10$$

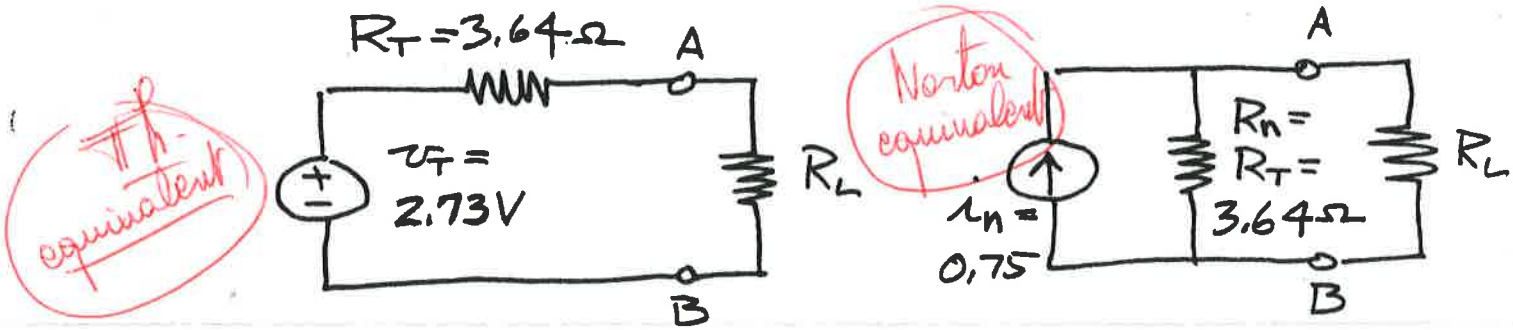
$$i_4 = \frac{10}{8} = 1.25$$

$$i_n = i_4 - \frac{v_{CB}}{5} = i_4 - \frac{2i_4}{5} = \frac{3}{5} i_4$$

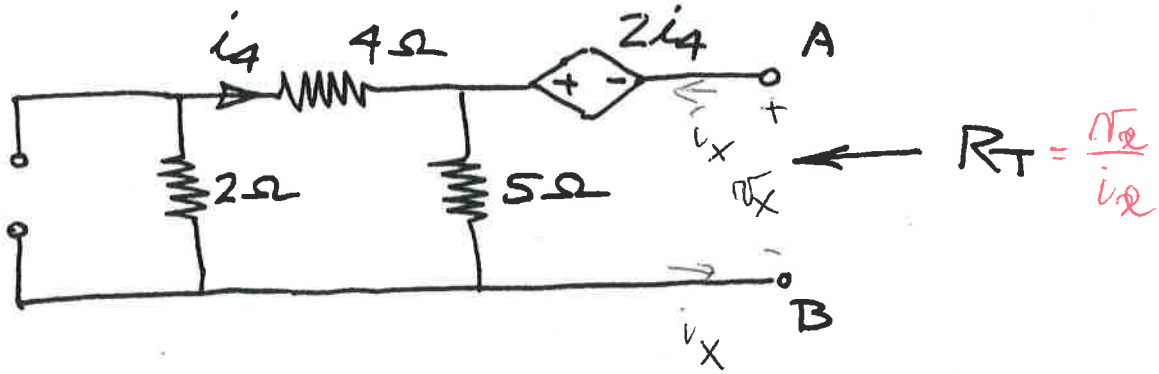
$$= 0.75 \text{ A}$$

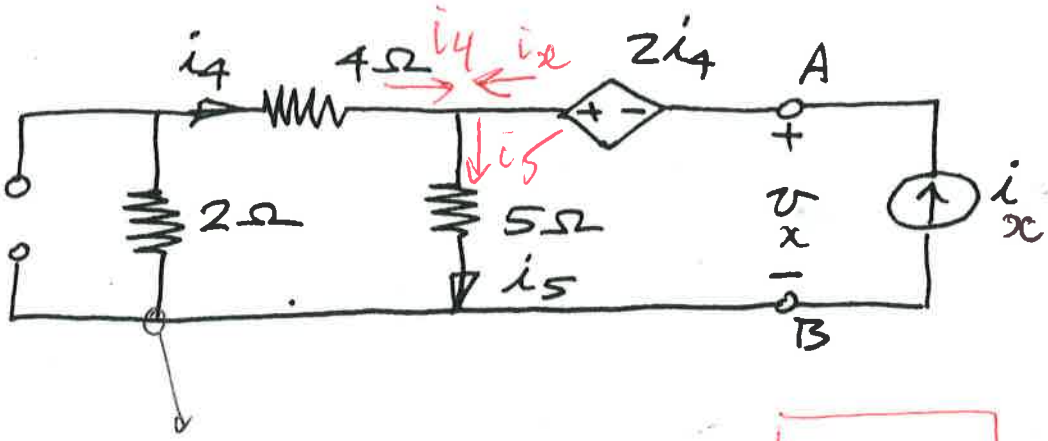
To find R_T:

$$R_T = \frac{v_T}{i_n} = \frac{2.73}{0.75} = 3.64\,\Omega$$



Or find R_T directly:





$$v_x = -2i_4 - 4i_4 - 2i_4 = \boxed{-8i_4}$$

$$= -2i_4 + 5i_5 = \boxed{-2i_4 + 5(i_x + i_4)}$$

$$-8i_4 = -2i_4 + 5(i_x + i_4)$$

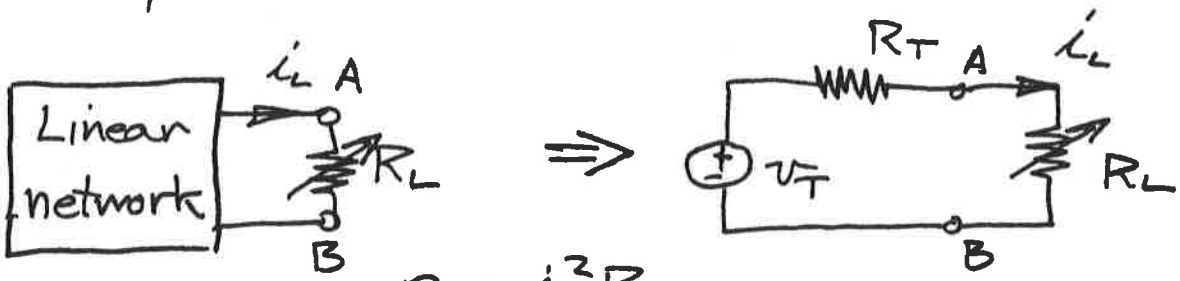
$$-11i_4 = 5i_x$$

$$i_4 = \frac{-5}{11} i_x$$

$$v_x = -8i_4 = \frac{40}{11} i_x$$

$$R_T = \frac{v_x}{i_x} = \frac{40}{11} = 3.64\ \Omega$$

Maximum power transfer theorem



$$p_L = i_L^2 R_L$$

For what value of R_L is p_L maximum?

$$i_L = \frac{v_T}{R_T + R_L}$$

$$p_L = i_L^2 R_L = \frac{v_T^2 R_L}{(R_T + R_L)^2}$$

$$\frac{dp_L}{dR_L} = 0 = v_T^2 \frac{(R_T+R_L)^2 - R_L(2)(R_T+R_L)}{(R_T+R_L)^4}$$

$$0 = R_T^2 + 2R_T R_L + R_L^2 - 2R_L R_T - 2R_L^2$$

$$= R_T^2 - R_L^2$$

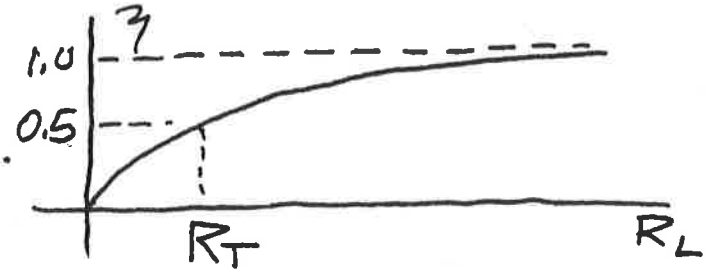
$$R_L^2 = R_T^2$$

$$\longrightarrow R_L = R_T$$

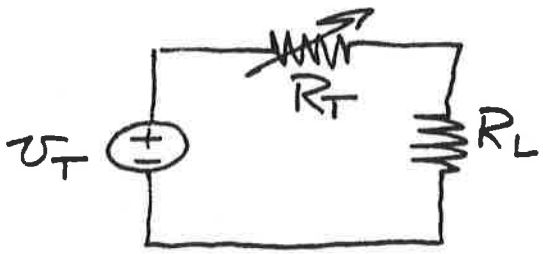
$$p_{L_{max}} = i_L^2 R_L = \frac{v_T^2}{(R_T+R_T)^2} R_T = \frac{v_T^2}{4R_T}$$

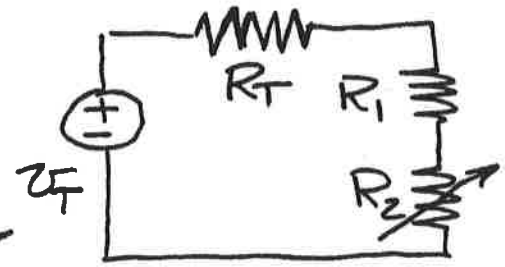
Note <u>efficiency</u> of power transfer to R_L:

$$\eta = \frac{P_o}{P_{in}} = \frac{i_L^2 R_L}{i_L^2 (R_L + R_T)} = \frac{R_L}{R_L + R_T}$$



Note other maximum power transfer questions





For max power to R_2:

$$R_2 = R_1 + R_T$$

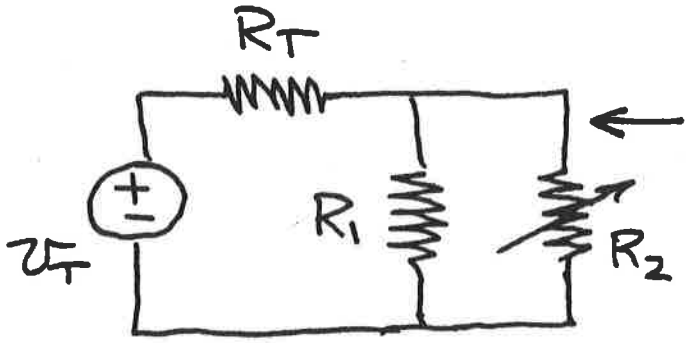
For max power to R_1:

$$R_2 = 0$$

For max power to $R_L = R_1 + R_2$:

$$R_1 + R_2 = R_T$$

$$R_2 = R_T - R_1$$

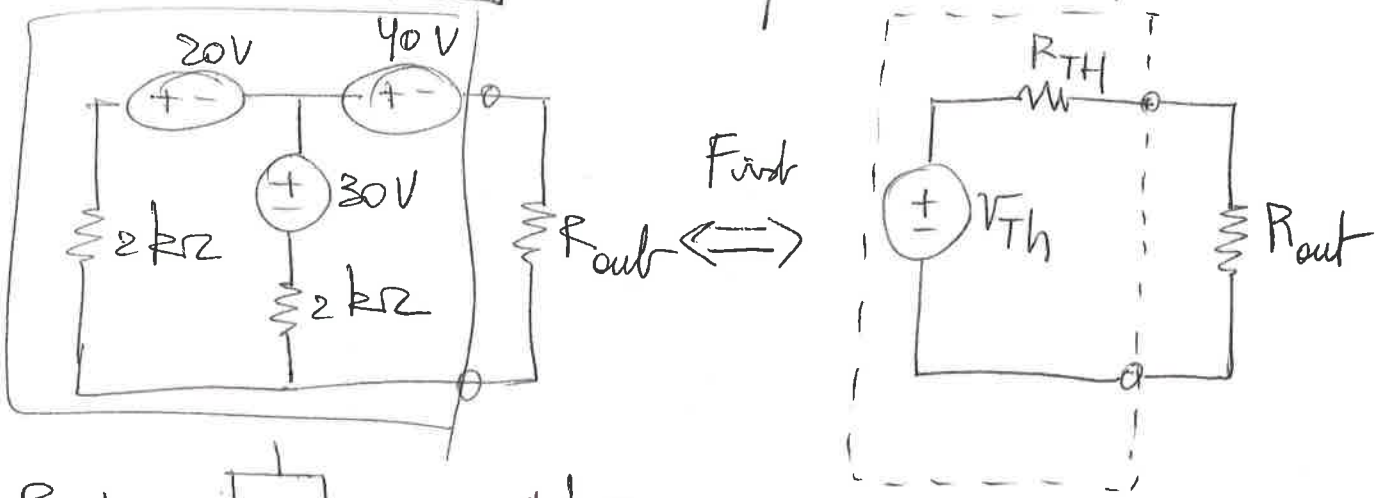


← Maximum power to R_2? R_1?

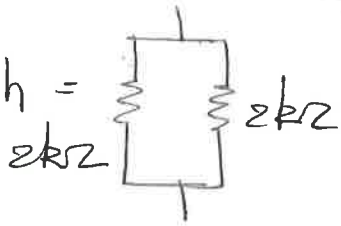
$$R_L = \frac{R_1 R_2}{R_1 + R_2} \ ?$$

Max. Power transfer?

Practice problem 5.10 p152

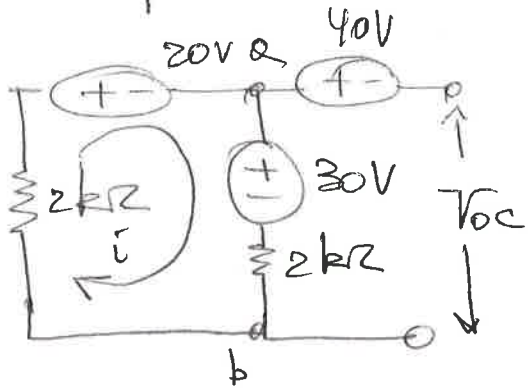


$R_{Th} =$



$= 1\,k\Omega$

V_{Th}



$V_{ab} = 40 + V_{oc} \Rightarrow V_{Th} = V_{ab} - 40V$

$2k\Omega\, i + 50 + 2k\Omega\, i = 0$

$4k\Omega\, i = -50$

$i = -\frac{50}{4}\,mA = -12.5\,mA$

$V_{ab} = 30 + 2k\Omega\, i = 30 - 25 = 5V$

$\rightarrow V_{Th} = -35V$

$R_{out} = R_{Th} = 1\,k\Omega$

$$P_L^{max} = \frac{V_{TH}^2}{4R_{TH}}$$

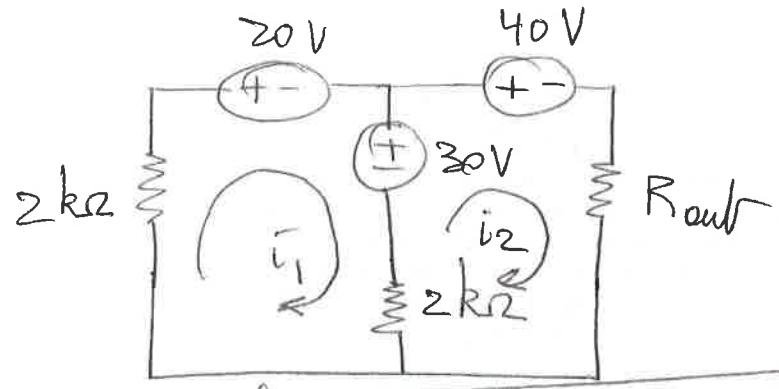
V_{Th}

-35

$P_L = \frac{1}{4}\left(\frac{35^2}{1\times10^3}\right) = \frac{35^2}{4}\,mW$

$= 306\,mW$

Practice 5.10 p152



$R_{out} = 3k\Omega$, find power delivered to it.

$$2k\Omega\, i_1 + 50 + 2k\Omega(i_1 - i_2) = 0$$

$$2k\Omega(i_2 - i_1) - 30 + 40 + R_{out}\, i_2 = 0$$

$$4k\Omega\, i_1 - 2k\Omega\, i_2 + 50 = 0$$

$$-2k\Omega\, i_1 + (R_{out} + 2k\Omega) i_2 + 10 = 0$$

$$(\times 2) \quad -2k\Omega\, i_1 + 5k\Omega\, i_2 + 10 = 0$$

$$-4k\Omega\, i_1 + 10k\Omega\, i_2 + 20 = 0$$

$$8k\Omega\, i_2 + 70 = 0 \quad (\text{sum})$$

$$i_2 = \frac{-70}{8k\Omega}\ A$$

$$R_{out}\, i_2^2 = P_{3k\Omega} = \frac{3k\Omega\ 70^2}{64(k\Omega)^2} = \frac{3 \times 70^2}{64}\ mW$$

$$\boxed{P_{3k\Omega} = \frac{210(70)}{64} = \cancel{306\,mW}\ \ 230\,mW}$$

$< P_L^{max} = 306\,mW$

Found on previous page

what 2 values of R_{out} will have exactly 20 mW delivered to them?

$$\begin{cases} 4k\Omega\, i_1 - 2k\Omega\, i_2 + 50 = 0 \\ -2k\Omega\, i_1 + (R_{out} + 2k\Omega)\, i_2 + 10 = 0 \\ -4k\Omega\, i_1 + (2R_{out} + 4k\Omega)\, i_2 + 20 = 0 \end{cases}$$

ADD

$$2(R_{out} + 1k\Omega)\, i_2 = -70$$

$$i_2 = -\frac{35}{(R_{out} + 1k\Omega)}$$

$$P_{out} = \frac{R_{out}\, 35^2}{(R_{out} + 1k\Omega)^2} = 20\,mW$$

$$35^2 R_{out} = 20\, 10^{-3} (R_{out} + 1k\Omega)^2$$

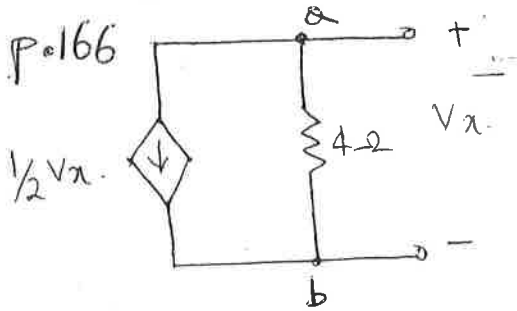
$$R_{out}^2 + 2k\Omega\, R_{out} + 10^6 = \frac{35^2}{20} 10^3 R_{out}$$

quadratic equation

→ 2 roots $\boxed{R_{out} = 59.2k\Omega \;\&\; 16.88\Omega}$

Determine Thevenin equivalent of circuit below

(53) p.166



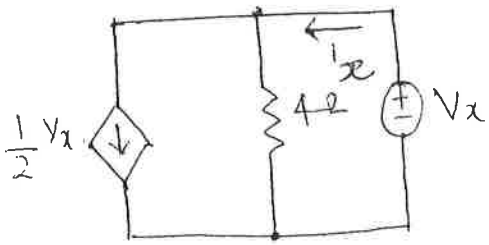
$V_{ab} = V_x$

$V_{ba} = \frac{1}{2}V_x\, 4 = 2V_x$

$\rightarrow V_x = -2V_x$

$\rightarrow \boxed{V_x = 0}$

$V_{Th} = 0$ (no independant sources)

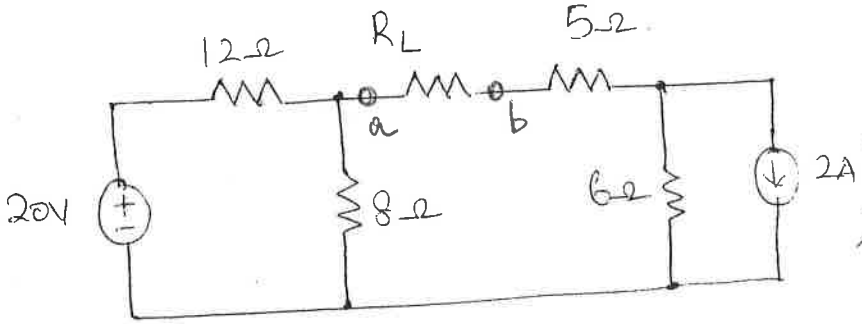


$i_x = \frac{V_x}{2} + \frac{V_x}{4} = \left(\frac{1}{2}+\frac{1}{4}\right)V_x$

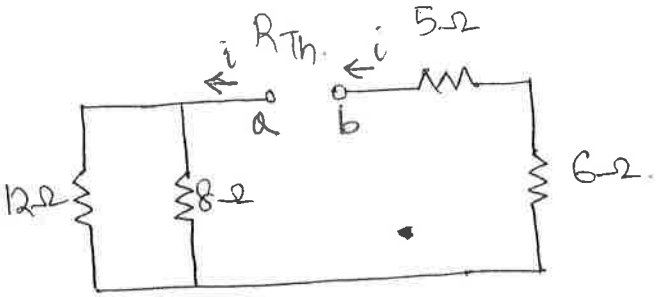
$R_{Th} \triangleq V_x/i_x = 1/(3/4) = 4/3 = 1.3\,\Omega$

∴ norton equivalent is 0A in parallel with 1.3Ω.

(61) p.167



If any value what-ever may be selected for R_L, what is the maximum power that could be dissipated in R_L?



$R_{Th} = \frac{V_{ab}}{i}$

8Ω is in parallel with 12Ω.

∴ $1/R = 1/8 + 1/12 = \frac{3+2}{24} = 5/24$

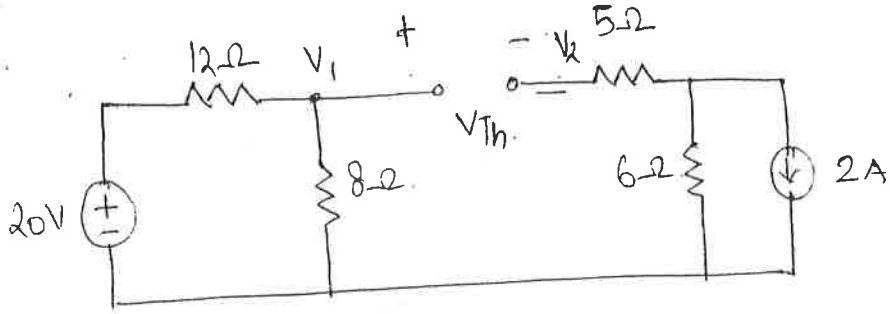
∴ $R = 24/5 = 4.8\,\Omega$

5Ω and 6Ω are series

∴ $R' = 5\Omega + 6\Omega$

∴ $R_{Th} = 11\Omega + 4.8\Omega$
$= 15.8\,\Omega$

∴ $R_{Th} \simeq 16\,\Omega$.



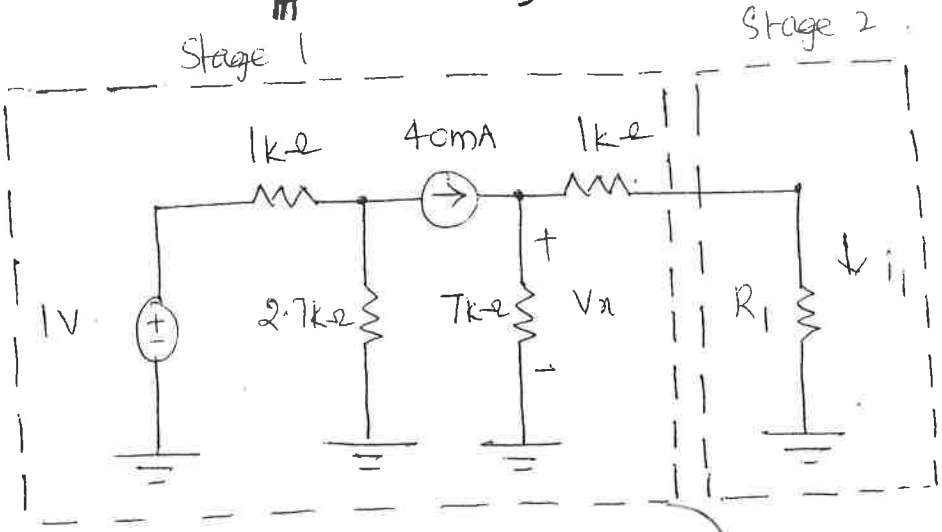
$$V_1 = 20\left(\frac{8}{8+12}\right) = 20 \cdot \frac{8}{20} = 8V$$

$$V_2 = -2 \times 6 = -12V$$

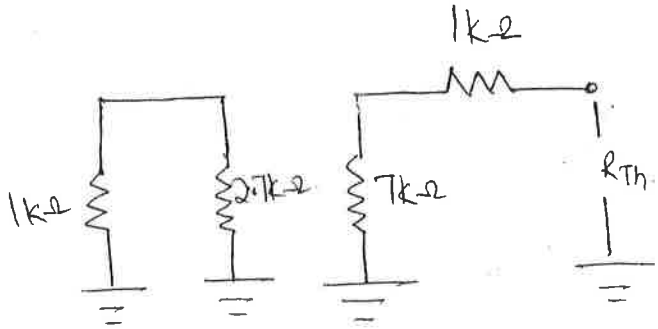
$$\therefore V_{Th} = V_1 - V_2 = 8V + 12V = 20V$$

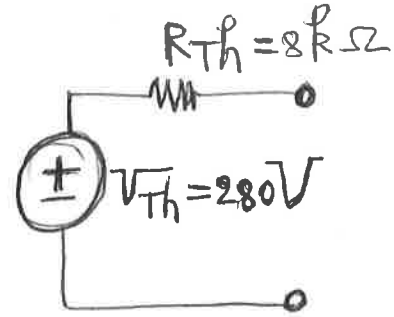
$$\therefore P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(20)^2}{4(16)} = \frac{\cancel{20}^{5} \times \cancel{20}^{5}}{\cancel{4} \times \cancel{16}\,4} = \frac{25}{4} = 6.25\,W$$

(E9) p169



Select R_1 so that the maximum power is transferred from stage 1 to stage 2.





$$R_{Th} = 7k\Omega + 1k\Omega = 8k\Omega$$

$$V_{Th} = (40 \times 10^{-3})(7 \times 10^{3}) = 280V$$

$$\therefore R_1 = R_{Th} = 8k\Omega$$