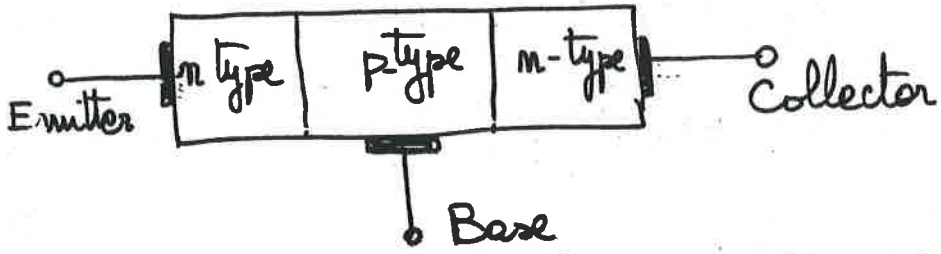
Si $Z=14$

$1s^2\,2s^2\,2p^6\,3s^2\,3p^2$

$\rightarrow sp^3$

Chapter 6: Bipolar Junction Transistors (BJTs)

npn transistor

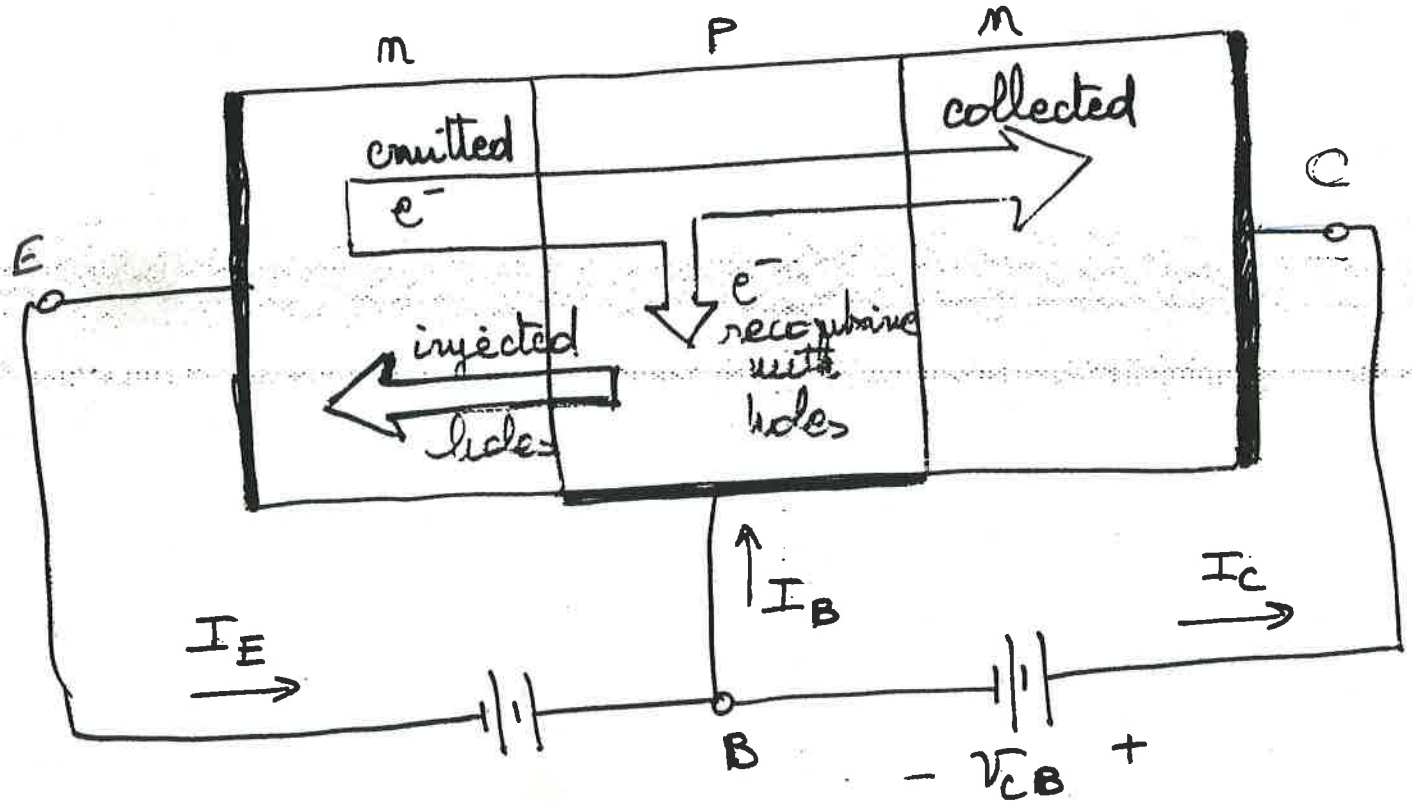


BJT Modes of operation

Mode	EB	CB
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

chapter 4

Active Mode



Collector Current

controlled by v_{BE}/V_T

current in third terminal

$$i_C = I_S e^{v_{BE}/V_T} \qquad V_T = \frac{kT}{q}$$

I_S = current scale factor.

Base current

$$i_B = \frac{i_C}{\beta} \qquad v_{BE}/V_T$$

$$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$$

$\beta \simeq 100-200$ is called the common-emitter current gain

Emitter Current

$$i_E = i_C + i_B$$

$$i_E = \frac{\beta+1}{\beta} i_C = \frac{\beta+1}{\beta} I_S e^{v_{BE}/V_T}$$

$$i_C = \frac{\beta}{\beta+1} i_E = \alpha i_E$$

$$\rightarrow \boxed{\beta = \frac{\alpha}{1-\alpha} \quad \leftrightarrow \quad \alpha = \frac{\beta}{\beta+1}}$$

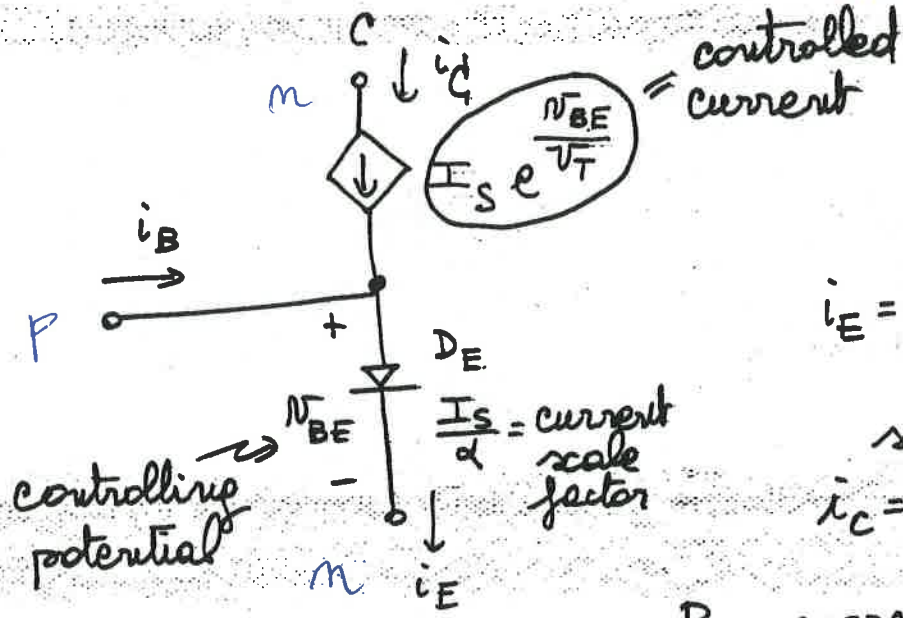
$\alpha \triangleq$ Common-base current gain

Typically $\alpha \simeq 1 \rightarrow \beta$ is large

is the goal to have a good transistor

$$i_E = \left(\frac{I_S}{\alpha}\right) e^{\frac{v_{BE}}{V_T}}$$

Equivalent Circuit Models (DC Spice simulations)

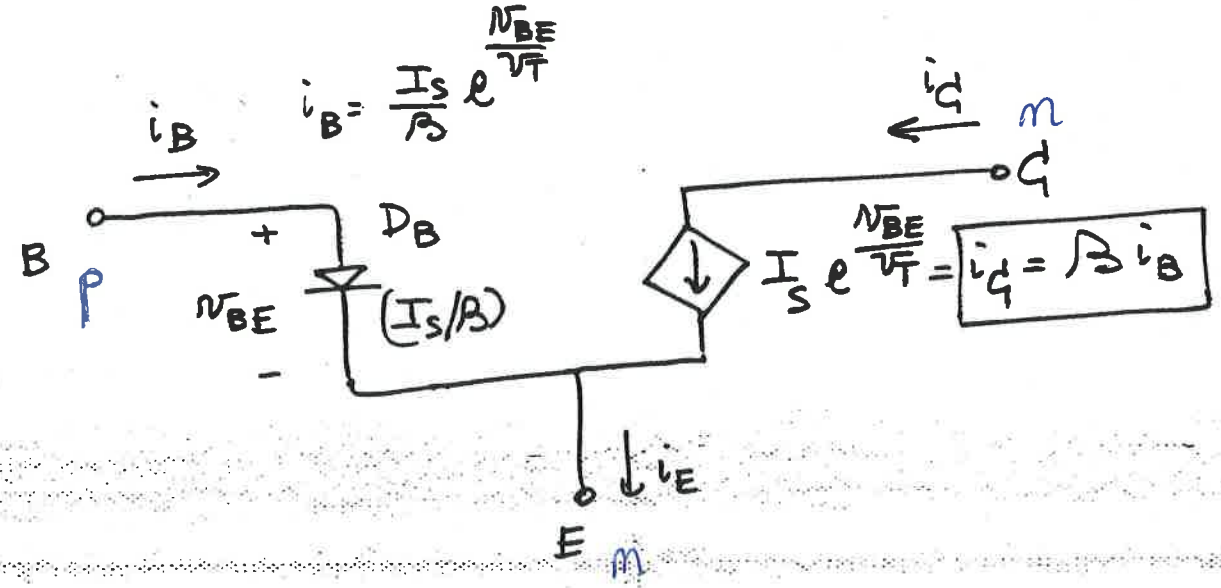


$$i_E = \underbrace{\left(\frac{I_S}{\alpha}\right)}_{\text{current scale factor}} e^{\frac{v_{BE}}{V_T}}$$

$$i_C = \alpha i_E$$

i_C slightly smaller than i_E

$\alpha \triangleq$ Common-Base current gain.



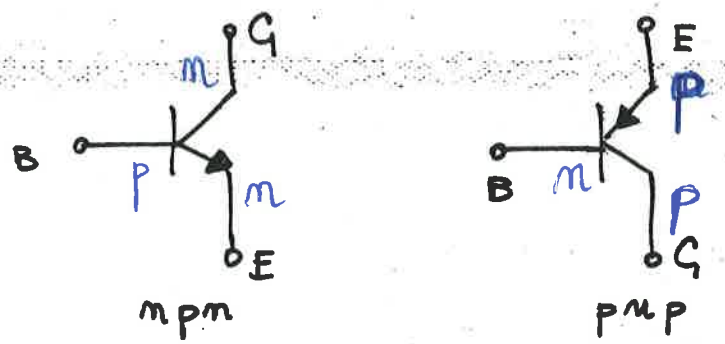
$\beta \triangleq$ Common-Emitter Current Gain.

§4.3 The pnp transistor - Active Mode

Base-Emitter junction is forward biased
Collector-base junction is reverse biased

$v_{BE} \rightarrow v_{EB}$

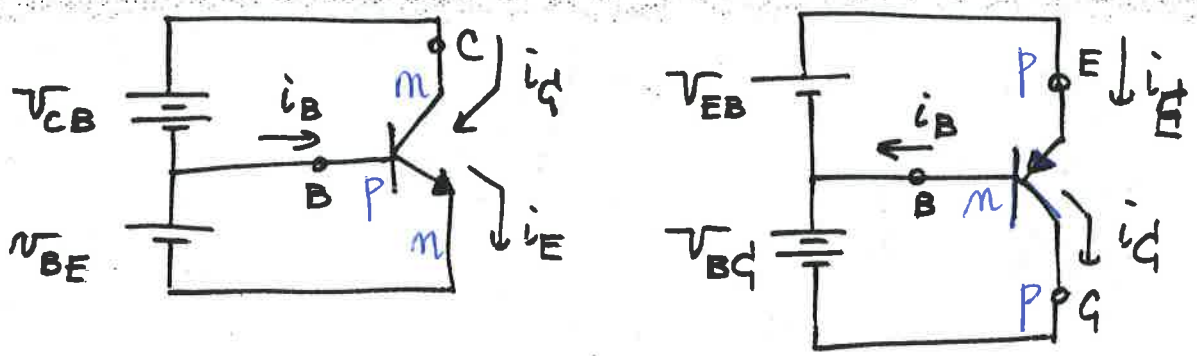
§4.4 Circuit Symbols and Convention



arrow is in the emitter leg.

the → points in the direction of normal current flow in the emitter.

Biasing the transistors in the active mode



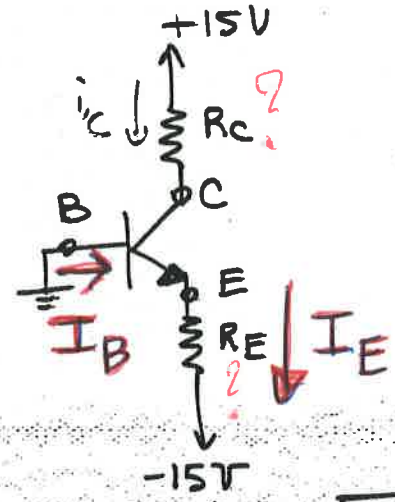
npn transistor with EB junction forward-biased will operate in the active mode as long as the collector is higher in potential than the base.

pnp transistor will operate in the active mode as long as the potential of the collector is lower than that of the base

Ex 4.1

$\beta = 100$

$v_{BE} = 0.7V$ at $i_C = 1mA$ Given



Design the circuit such that a current of 2mA flows through the collector and $V_C = +5V$

$V_C = +5V$
$I_C = 2mA$ $\rightarrow$ $R_C = \frac{10V}{2mA} = 5k\Omega$

$$i_C = I_S e^{\frac{v_{BE}}{V_T}}$$

$$v_{BE} = V_T \ln\left(\frac{i_C}{I_S}\right)$$

$$v_{BE}^2 = v_{BE}^1 + V_T \ln\left(\frac{i_C^2}{i_C^1}\right)$$

$i_C^1 = 1mA$ when $v_{BE}^1 = 0.7V$

$V_T = 25mV$

$\Rightarrow$ for $i_C^2 = 2mA \rightarrow v_{BE}^2 = 0.717V$

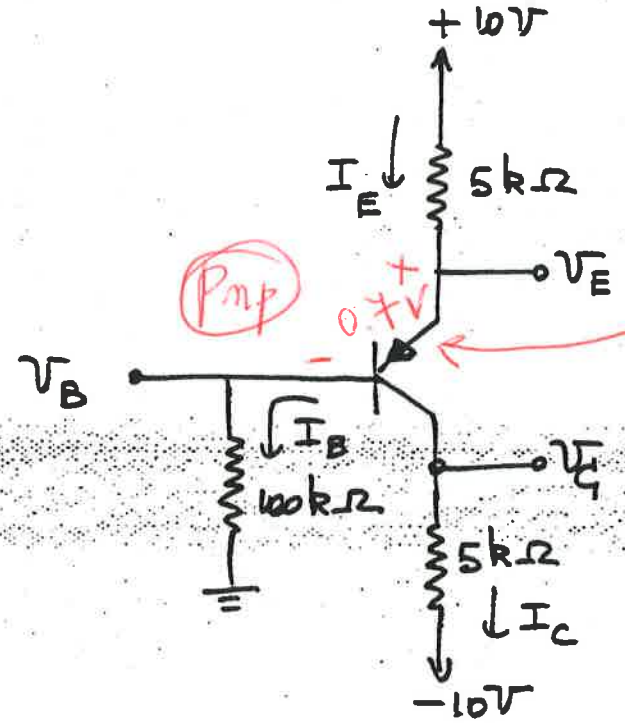
$V_B = 0V \rightarrow V_E = -0.717V$

$\beta = 100 \rightarrow \alpha = 0.99$

$\rightarrow I_E = \frac{I_C}{\alpha} = 2.02mA$ $(I_B = 0.02mA)$

$\Rightarrow$ $R_E = \frac{V_E - (-15)}{I_E} = 7.07k\Omega$

Exercise 4.9



$V_B = 1.$ volt
$V_E = 1.7$ V
Given

α? β?
V_C?
← FIND

$$I_E = \frac{10-1.7}{5} = 1.66 \text{ mA}$$

$$\frac{V_B}{100k\Omega} = I_B = \frac{1}{100} = 0.01 \text{ mA}$$

DON'T FORGET!

$$\Rightarrow \quad I_C = I_E - I_B = 1.66 - 0.01 = 1.65 \text{ mA}$$

$$\Rightarrow \quad \boxed{V_C = -10 + (1.65)\times 5 = -1.75 \text{ volt}}$$

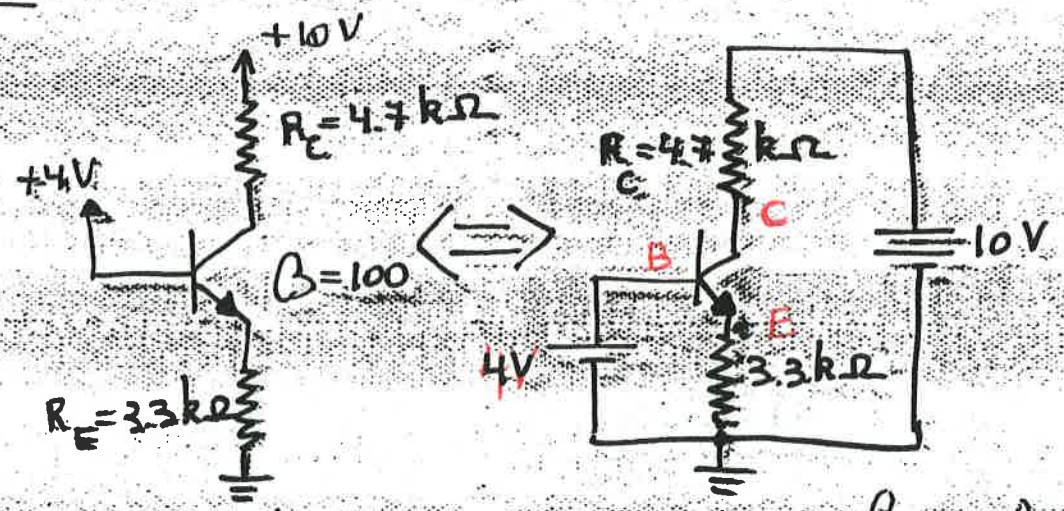
$$\Rightarrow \quad \alpha = \frac{I_C}{I_E} = \frac{1.65}{1.66} = \underline{0.994}$$

close to unity, as expected for transistor in forward active mode.

$$\beta = \frac{I_C}{I_B} = \frac{1.65}{0.01} = \underline{165}$$

large.

Ex 4.2

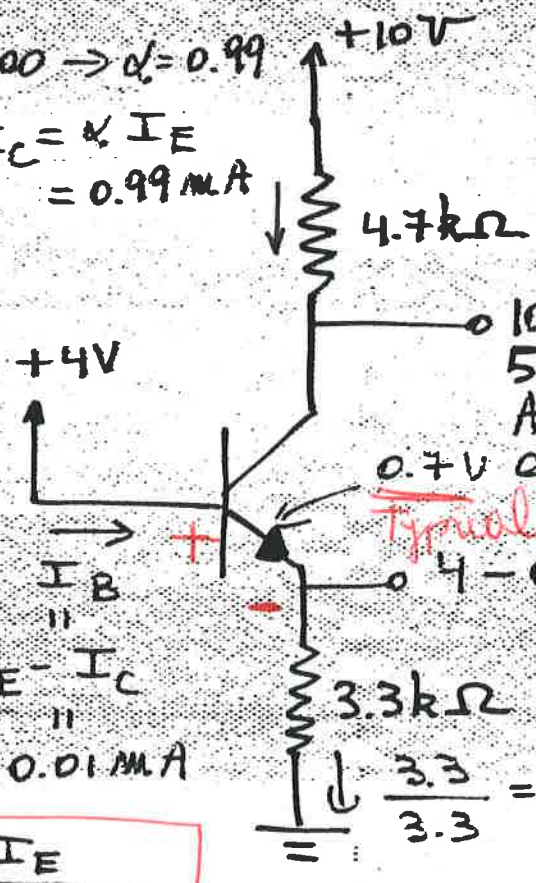


Assume Transistor in ACTIVE MODE → Carry Analysis
Determine all voltages and currents [$V_C, V_B, V_E, I_B, I_C, I_E$]

Typical Question

$\beta = 100 \rightarrow \alpha = 0.99$

(3) $I_C = \alpha I_E$
$= 0.99\,mA$



$10 - 0.99 \times 4.7 = 5.3V$ (4)
$5.3V > V_B \rightarrow V_{CB} > 0$ okay!
Assumption
0.7V across when forward biased
Typical
$4 - 0.7 = 3.3V$ (1)

$\frac{3.3}{3.3} = 1\,mA$ (2)
$= I_E$

$$I_B = \frac{I_E}{\beta + 1}$$

$$\left.\begin{aligned} I_C &= \alpha I_E \\ I_C &= \beta I_B \\ I_E &= I_B + I_C \end{aligned}\right\} \quad I_B = I_E - I_C = (1-\alpha) I_E$$

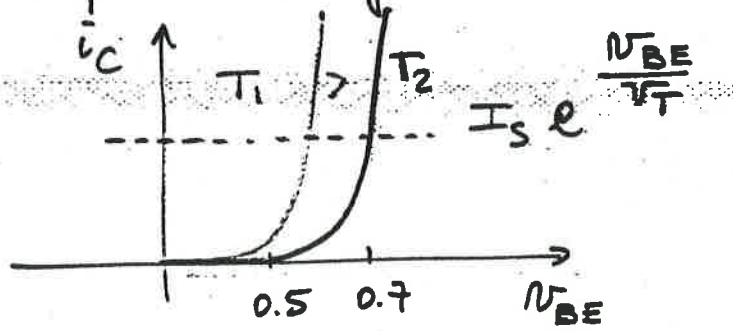
$$\alpha = \frac{\beta}{\beta+1}$$

$$\Rightarrow \boxed{I_B = \frac{I_E}{\beta+1}}$$

DC Analysis BJT in Forward Active Mode npn + pnp

4.5 Graphical Representation of Transistor Characteristics

(a)



Remember

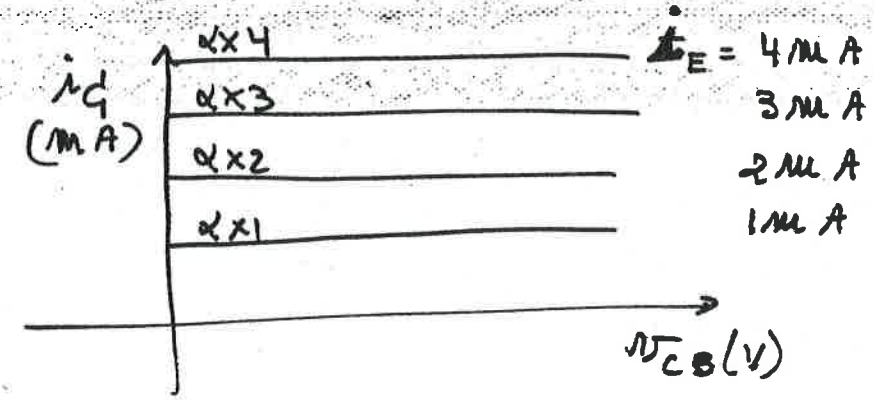
$$i_E = \frac{I_S}{\alpha} e^{v_{BE}/V_T}$$

$$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T}$$

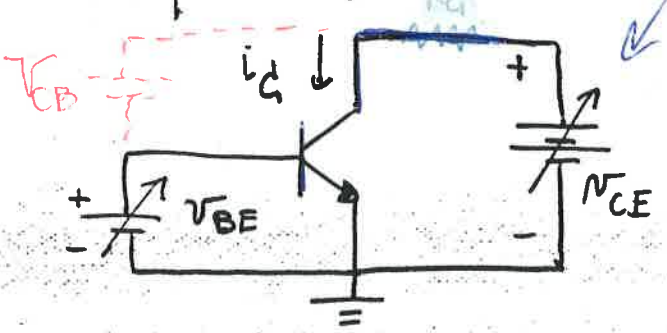
$$\boxed{\beta = \frac{\alpha}{1-\alpha}}$$

(b)

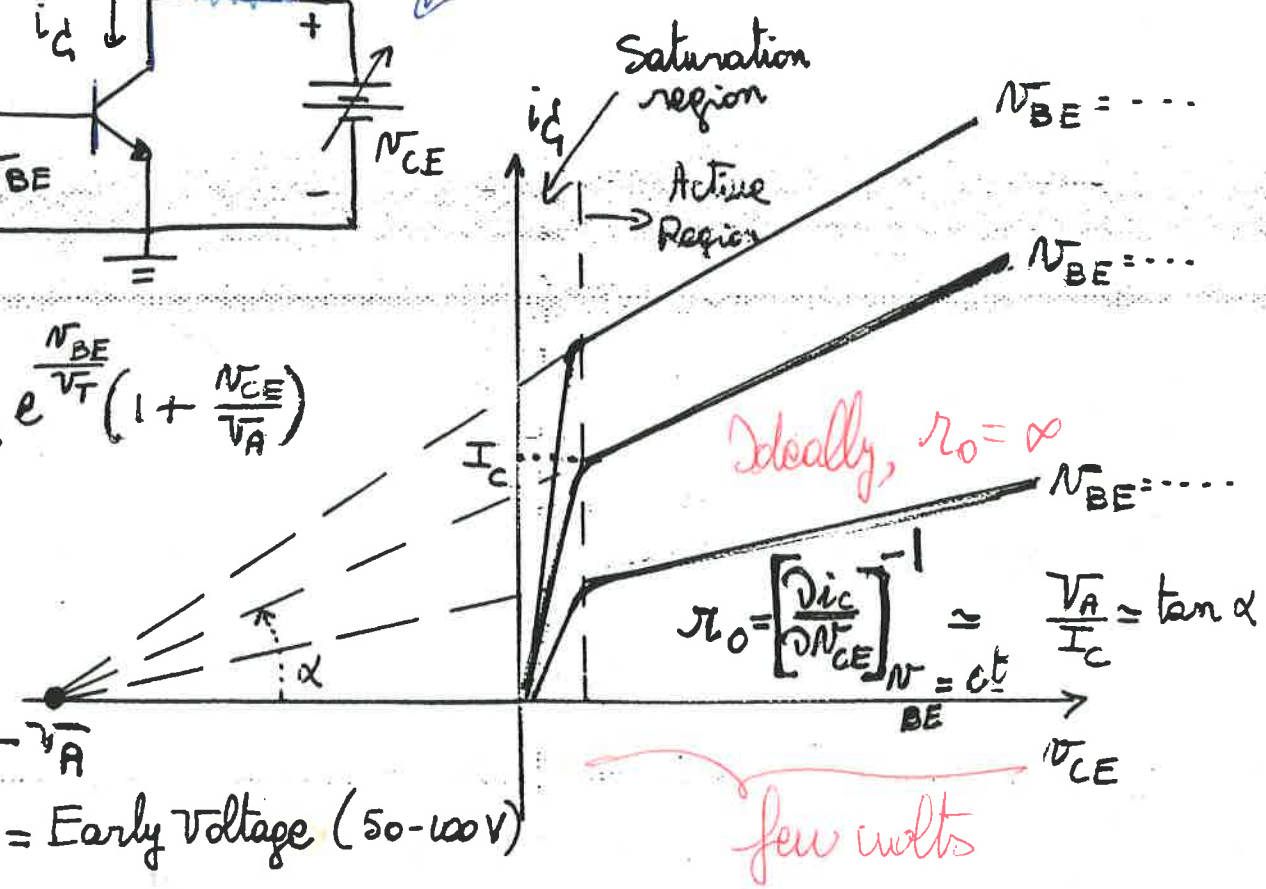
Ideal



In practice,



In practice



$$i_C = I_S e^{\frac{v_{BE}}{V_T}}\left(1 + \frac{v_{CE}}{V_A}\right)$$

Ideally, $r_o = \infty$

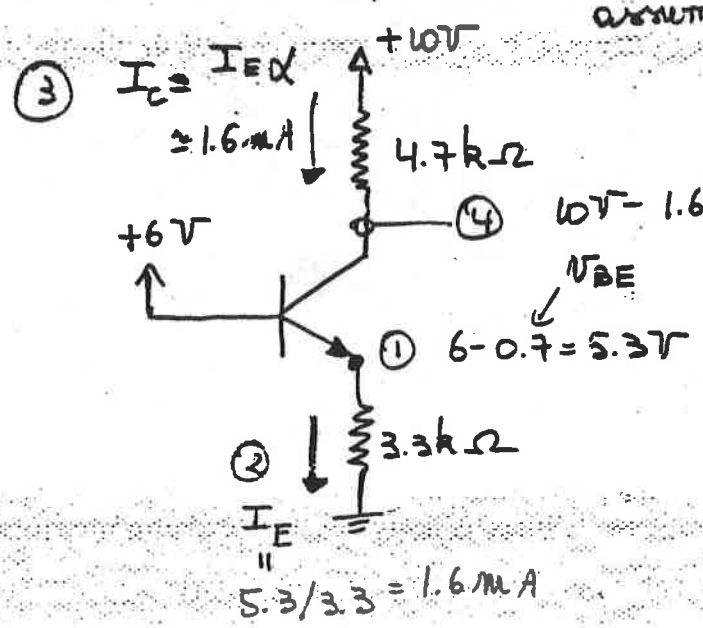
$$r_o = \left[\frac{\partial i_C}{\partial v_{CE}}\right]^{-1}_{v_{BE} = ct} \simeq \frac{V_A}{I_C} \simeq \tan\alpha$$

V_A = Early Voltage (50-100 V)

§4.6 Analysis of Transistor Circuits at DC

assume $\beta = 100$

$\rightarrow \alpha = \frac{\beta}{\beta+1} \simeq 0.99$

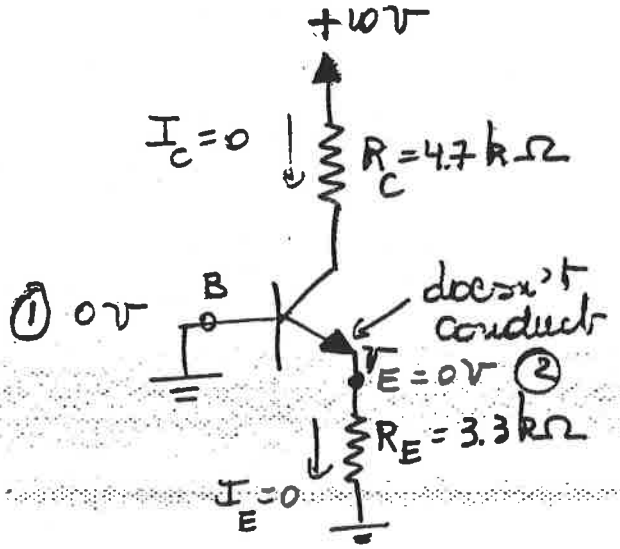


$10V - 1.6 \times 4.7 = 2.48V < 6V$ on base!

$\Rightarrow$ Conclusion

Transistor is NOT in Active Mode!

$\boxed{\text{Saturation Regime}}$



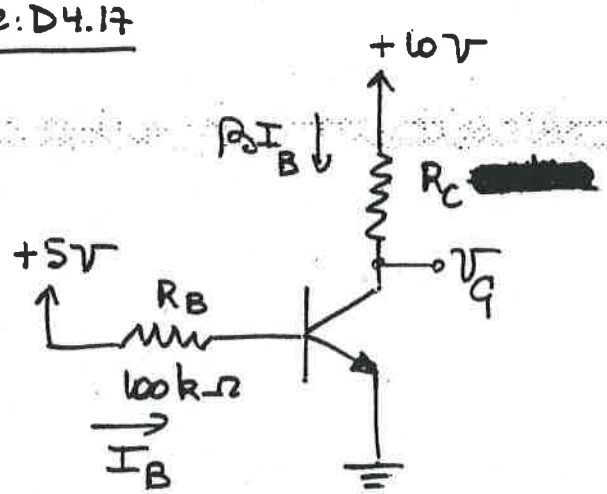
$I_C = I_B + I_E$

$\Rightarrow I_B = 0$

Both of the junctions are reverse-biased

$\Rightarrow \boxed{\text{cut-off regime.}}$

Exe: D4.17



Typical in practice

$\beta \in [50, 150]$

(a) R_C? so that all fabricated transistors will operate in the active mode.

(b) What is the range of v_C that the circuit can exhibit and still be in the active mode?

$$I_B = \frac{5-0.7}{100} = 0.043\,mA$$

$$v_C = 10 - \beta I_B R_C = 10 - 0.043\,\beta R_C \quad (R_C \text{ in } k\Omega)$$

Design Transistor so that it is on the verge of saturation for $\beta = 150$, i.e, $v_C = 0.7\,V$

$$\Rightarrow \boxed{R_C = 1.44\,k\Omega}$$

$\forall\, \beta \le 150$, the BJT will operate in the active region.

Take $\beta = 50 \rightarrow \boxed{v_C = 6.9\,V}$

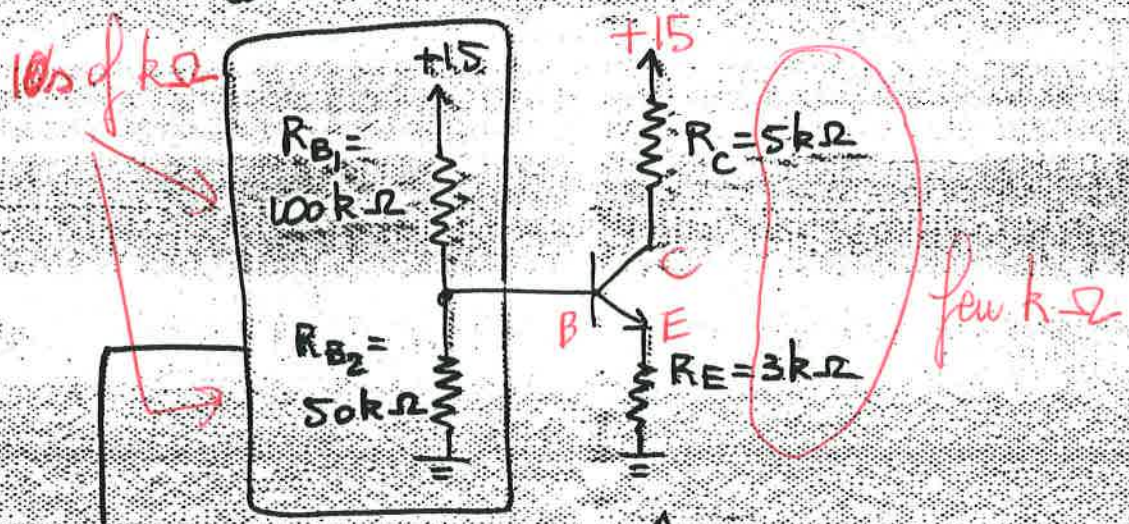
$\Rightarrow$ Collector voltage $v_C \in [0.7 \rightarrow 6.9]$ Volts for $\beta \in [150 \rightarrow 50]$

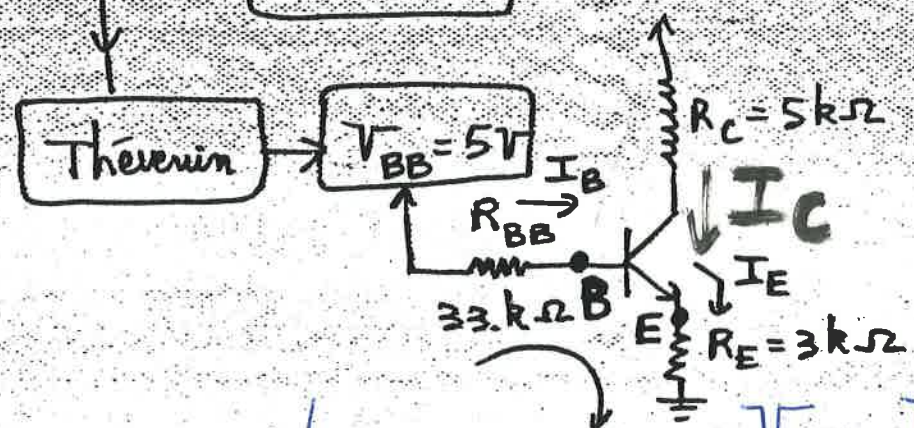
NOT ACCEPTABLE!

We will need to use feedback to control gain of amplifier

Example 4.7

$\boxed{\beta = 100}$ Determine voltages at all nodes & currents through all branches.





LOOP EQUATION – VERY IMPORTANT!

$$\left.\begin{aligned} V_{BB} &= I_B R_{BB} + V_{BE} + I_E R_E \\ I_B &= \frac{I_E}{\beta+1} \end{aligned}\right\} \Rightarrow I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_{BB}}{\beta+1}}$$

See p.117

$$\Rightarrow \quad I_E = 1.29\,mA$$

$$\Rightarrow \quad I_B = \frac{1.29}{101} = 0.0128\,mA \qquad \boxed{I_B = \frac{I_E}{\beta+1}}$$

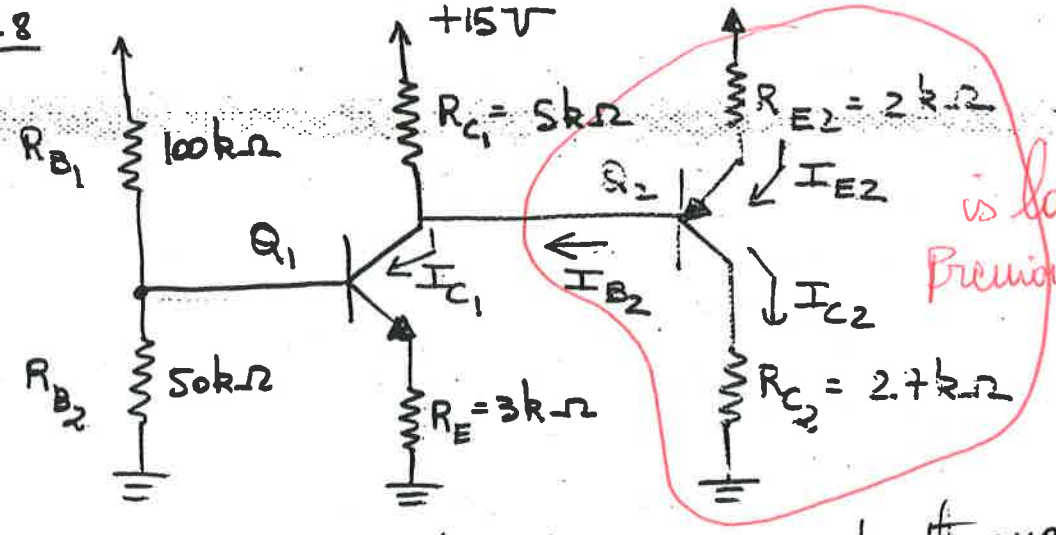
$$\Rightarrow \quad V_B = \underbrace{V_{BE}}_{=0.7V} + R_E I_E = 4.57V$$

Active mode $\Rightarrow$ $I_C = \alpha I_E = 0.99 \times 1.29 = 1.28\,mA$

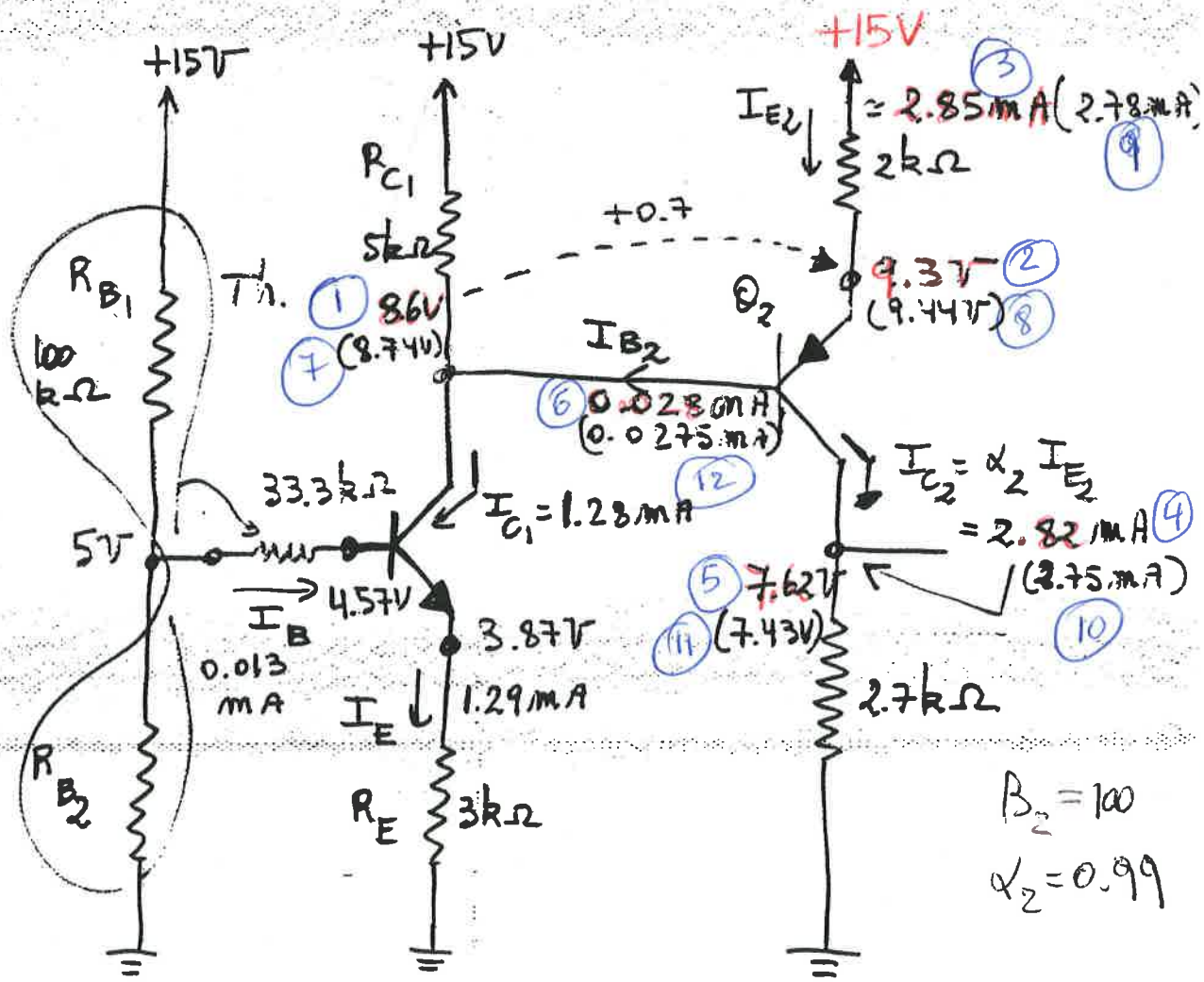
$$\Rightarrow \quad V_C = +15 - I_C R_C = 15 - 1.28 \times 5 = 8.6V > V_B$$

Active mode indeed

Ex 4.8



Find voltages at all nodes & currents through all branches



$\beta_2 = 100$

$\alpha_2 = 0.99$

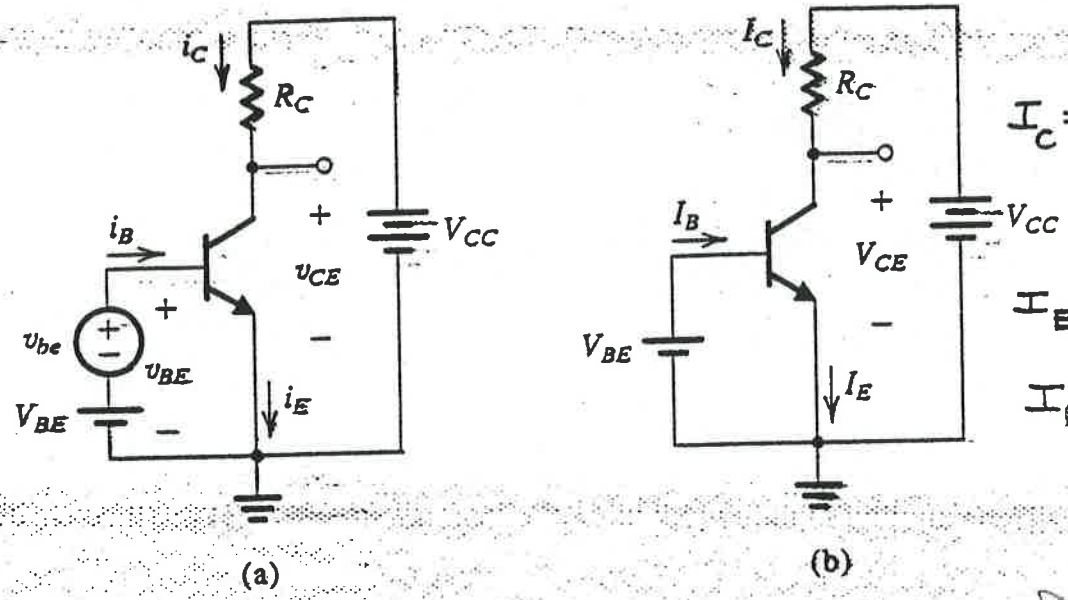
$$\begin{cases} V_{BB} = I_B R_{BB} + \underset{\substack{\parallel \\ 0.7}}{V_{BE}} + R_E I_E \\ I_B = \dfrac{I_E}{\beta+1} \end{cases}$$

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + [R_{BB}/(\beta+1)]} = 1.29\,mA$$

$$I_{B2} = \frac{I_{E2}}{\beta_2 + 1} = 0.028\,mA$$

§4.7

123



$I_C = I_S e^{\frac{V_{BE}}{V_T}}$

$I_E = \frac{I_C}{\alpha}$

$I_B = \frac{I_C}{\beta}$

Fig. 4.23

Graphically

Typical value

$I_C \sim mA$

$V_T = 25mV$

$\rightarrow g_m \sim 40 mS$

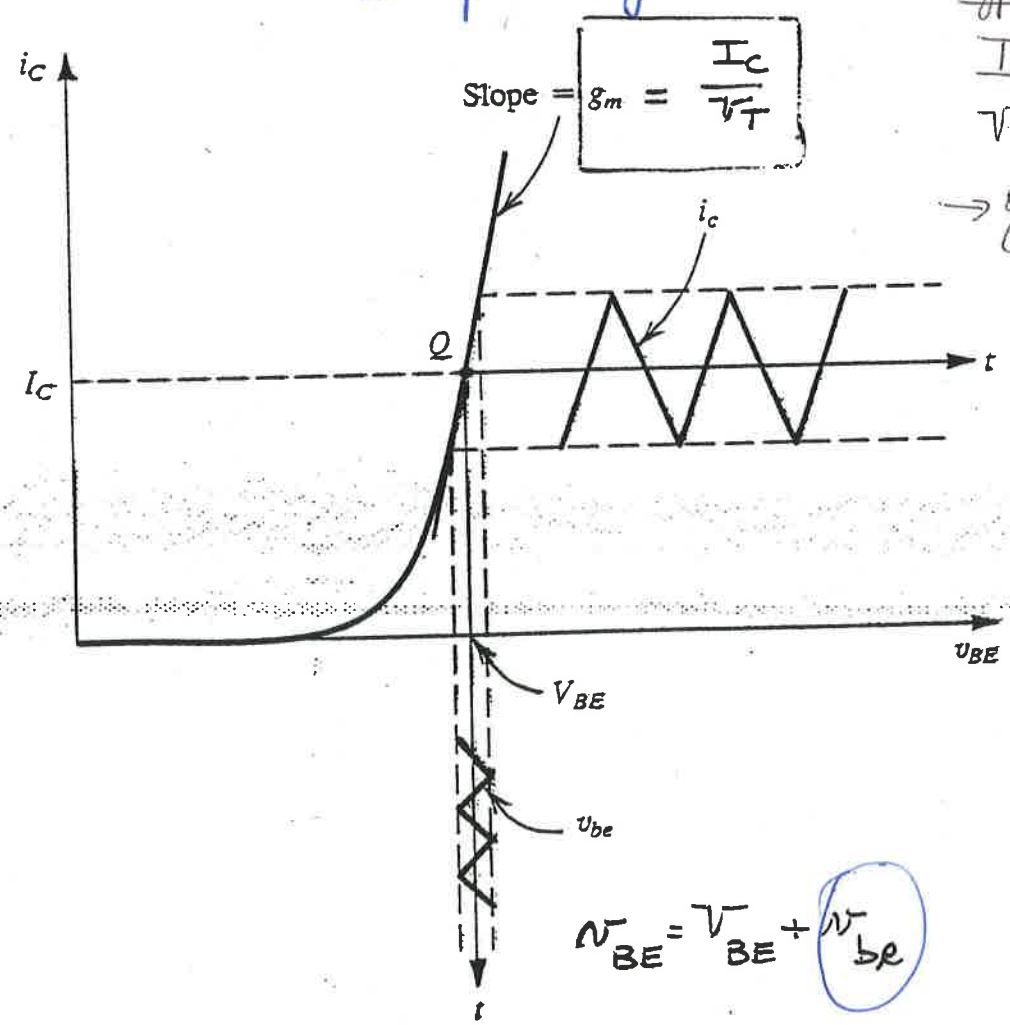


Fig. 4.24

$v_{BE} = V_{BE} + v_{be}$

Collector current

Mathematically

$$v_{BE} = V_{BE} + v_{be}$$

$$i_C = I_S e^{(V_{BE}+v_{be})/V_T}$$

$$i_C = I_S e^{V_{BE}/V_T} e^{v_{be}/V_T}$$

Taylor expansion

$$i_C = \underbrace{I_S e^{V_{BE}/V_T}} \left(1 + \frac{v_{be}}{V_T} + \cdots\right)$$

$$i_C = I_C + \left(\frac{I_C}{V_T}\right) v_{be} = I_C + i_c$$

Define g_m such that $i_c = g_m \cdot v_{be}$

$$\boxed{g_m \triangleq \frac{I_C}{V_T}}$$

Base Current

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \underbrace{\frac{1}{\beta}\left(\frac{I_C}{V_T}\right) v_{be}}_{i_b}$$

or $$i_b = \frac{g_m}{\beta} v_{be}$$

Define $$\boxed{r_\pi \triangleq \frac{v_{be}}{i_b} = \frac{\beta}{g_m}}$$

Also, $$\boxed{r_\pi = \frac{V_T}{I_B}}$$

Emitter Current

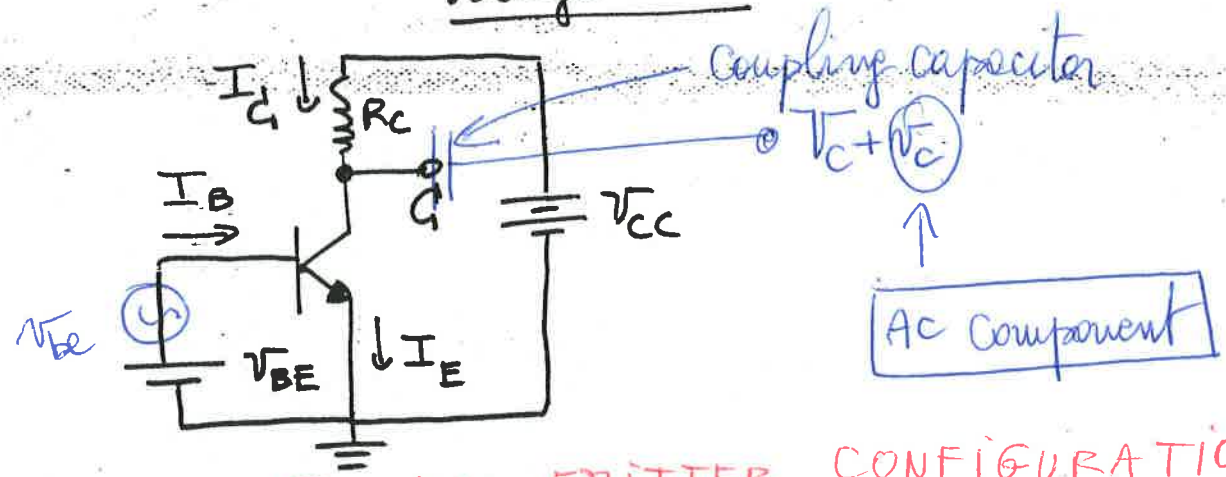
$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha} = \frac{I_C}{\alpha} + \underbrace{\left(\frac{g_m}{\alpha}\right) v_{be}}_{i_e}$$

$$\Rightarrow \quad i_e = \frac{i_c}{\alpha} = \frac{I_E}{V_T} v_{be} = \left(\frac{I_C}{\alpha}\frac{1}{V_T}\right) v_{be}$$

Define $$\boxed{r_e \triangleq \frac{v_{be}}{i_e} = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \simeq \frac{1}{g_m}}$$

$$v_{be} = i_b r_\pi = i_e r_e \Rightarrow \boxed{r_\pi = \left(\frac{i_e}{i_b}\right) r_e = (\beta+1) r_e}$$

Voltage Gain



COMMON EMITTER CONFIGURATION

$$v_C = V_{CC} - R_C i_C$$

$$= V_{CC} - (I_C + i_c) R_C$$

$$= (V_{CC} - R_C I_C) - i_c R_C$$

$$= V_C \underbrace{- i_c R_C}_{v_c}$$

$$\Rightarrow \quad v_c = -i_c R_C = -g_m v_{be} R_C$$

$$v_c = (-g_m R_C) v_{be}$$

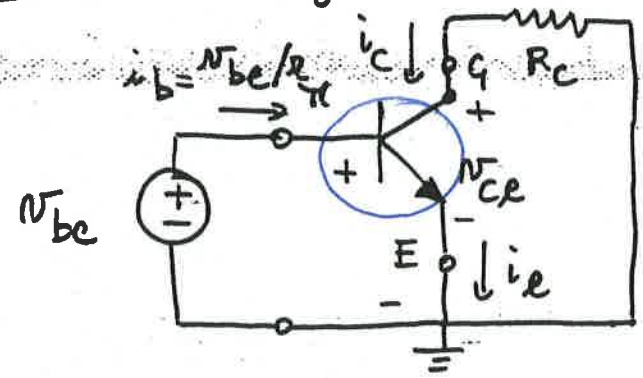
$$\Rightarrow \boxed{\text{Voltage Gain} \triangleq \frac{v_c}{v_{be}} = -g_m R_C}$$

Typical value

$g_m \simeq 40\,mS$

$R_C \simeq$ few $k\Omega$ $\rightarrow \dfrac{v_c}{v_{be}} \simeq \ominus$ few hundred

§4.8 Small Signal Equivalent Circuit.



we have found.

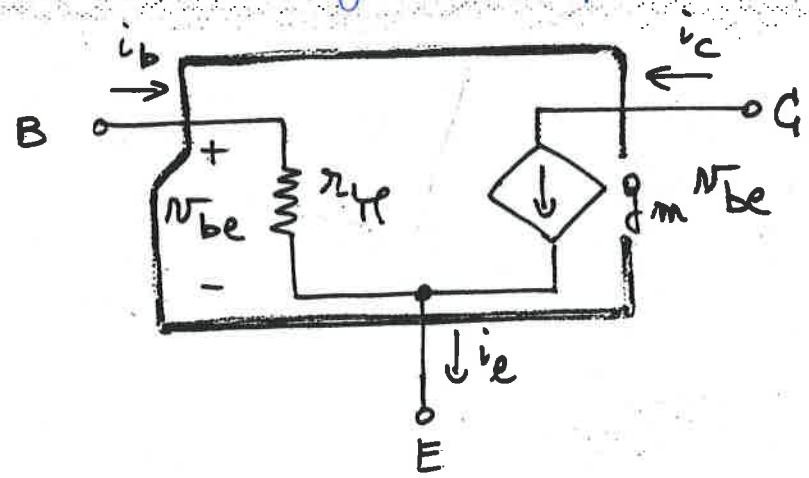
$$i_c = g_m v_{be}$$

$$i_e = \frac{v_{be}}{r_e}$$

$$r_e = \frac{V_T}{I_E};\quad r_\pi = \frac{V_T}{I_B};\quad g_m = \frac{I_C}{V_T}$$

Equivalent Transistor Model for small signal analysis

Hybrid-π Model: for AC Spice simulations

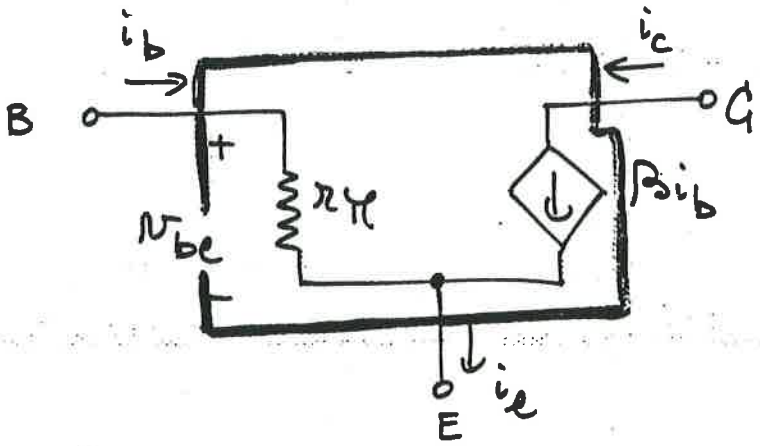


Correct i_e?

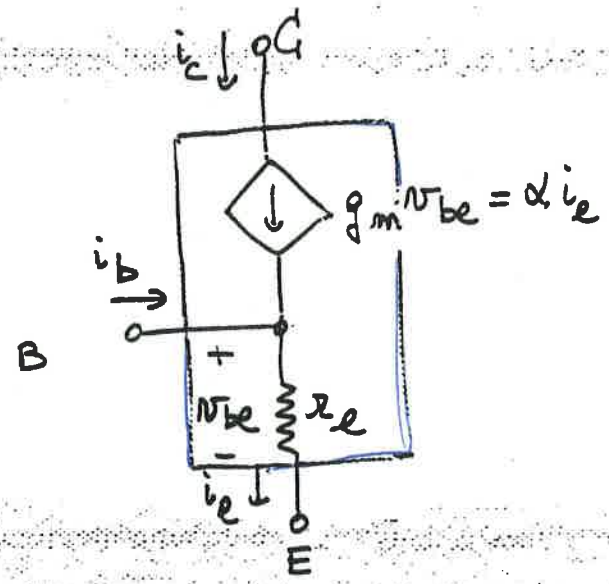
$$i_e = \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi}\left(1 + \underbrace{g_m r_\pi}_{\beta}\right)$$

$$i_e = \frac{v_{be}}{r_\pi}(1+\beta) = v_{be} \Big/ \left(\frac{r_\pi}{1+\beta}\right) = v_{be}/r_e$$

Also $\quad g_m v_{be} = g_m(i_b r_\pi) = (g_m r_\pi) i_b = \beta i_b$



The T-model



Voltage controlled current source

i_c, i_e are correct. How about i_b?

$$i_b = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}(1 - g_m r_e)$$

$$i_b = \frac{v_{be}}{r_e}(1-\alpha) = \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{1+\beta}\right)$$

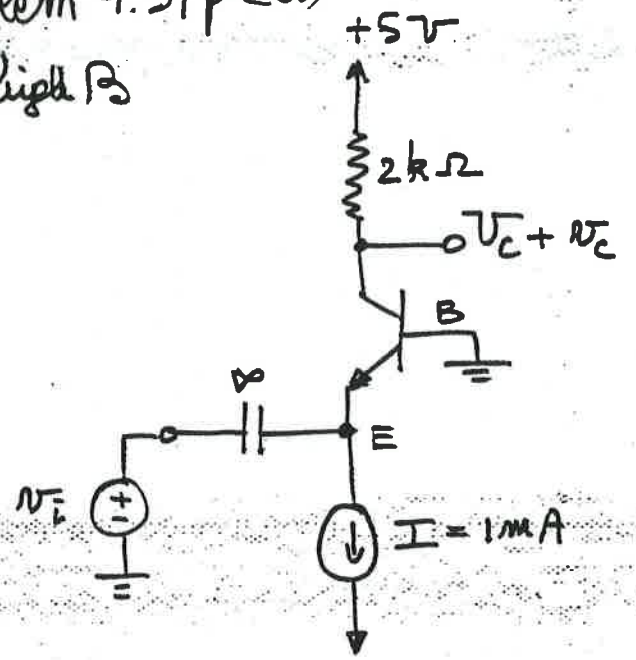
$$i_b = v_{be}/(\beta+1)r_e = \frac{v_{be}}{r_\pi}$$

<u>Procedure</u>: ① DC operating point is determined
→ calculate model parameters
g_m, r_π, r_e

② DC sources are removed (short DC voltage sources, open circuit DC current sources)
→ Proceed with AC analysis using { hybrid-π model / T-model }
→ voltage gain of transistor

Problem 4.51 p 263)

Very high β



(a) Find

V_C, g_m

(b) Use hybrid-π model

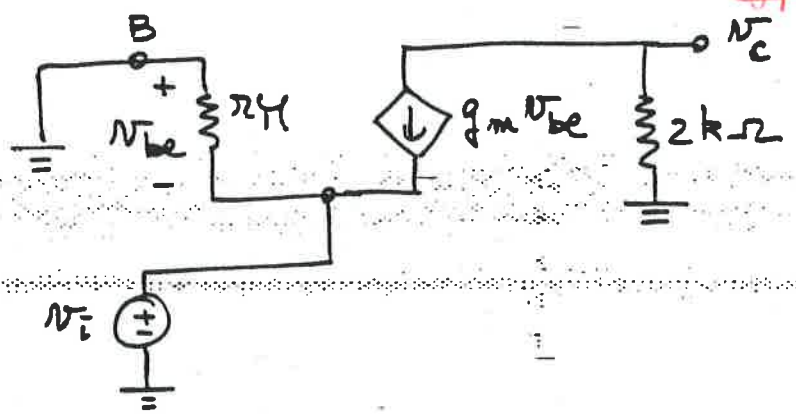
Find v_C/v_i

β high → $\alpha \simeq 1$ $\quad I_C = I_E = 1mA$

$V_C = 5 - 2\times1 = \underline{\underline{+3V}}$

$g_m = \frac{I_C}{V_T} = \frac{1mA}{0.025V} = 40\ mA/V$ Typical

→ Transistor is in Forward Active Mode of Operation.



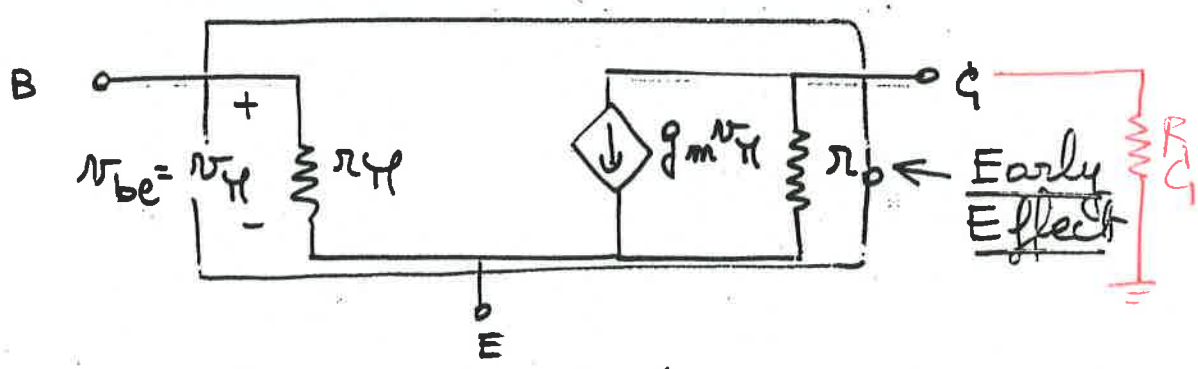
$v_{be} = -v_i \longrightarrow v_C = -g_m v_{be} \times 2k\Omega$

$v_C = g_m v_i \cdot 2k\Omega$

$$\rightarrow \boxed{\frac{v_C}{v_i} = 40\frac{mA}{V}\, 2k\Omega = 80\ V/V}$$

only are v_C & v_i in phase?

Augmenting the Hybrid-π model to account for the Early Effect.



If E is grounded r_o is then in // with R_C

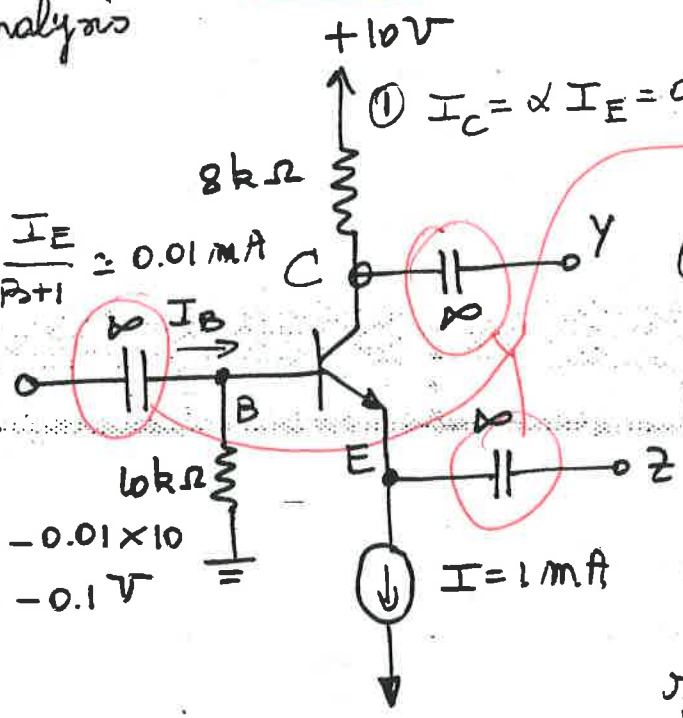
$$\Rightarrow v_o = -g_m v_{be}(R_C // r_o) \qquad \left[r_o = \frac{V_A}{I_C}\right]$$

In general, r_o can be neglected if > $10 R_C$.

Exercise 4.24

g_m, r_π, r_o?

DC Analysis



Coupling Capacitors $Z_C \simeq 0$ at frequency of operation.

① $I_C = \alpha I_E = 0.99\,mA$

② $V_C = 10 - 0.99 \times 8 = +2V$

③ $I_B = \frac{I_E}{\beta+1} \simeq 0.01\,mA$

④ $V_B = -0.01 \times 10 = -0.1V$

⑤ $V_E = -0.1 - 0.7 = -0.8V$

Therefore

$$g_m = \frac{I_C}{V_T} = \frac{0.99\,mA}{25\,mV} \simeq 40\,mA/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\,mA/V} = 2.5\,k\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100V}{0.99\,mA} \simeq 100\,k\Omega$$

$\beta = 100$

$V_A = 100V$

[Neglect V_A in DC analysis but not in AC analysis]

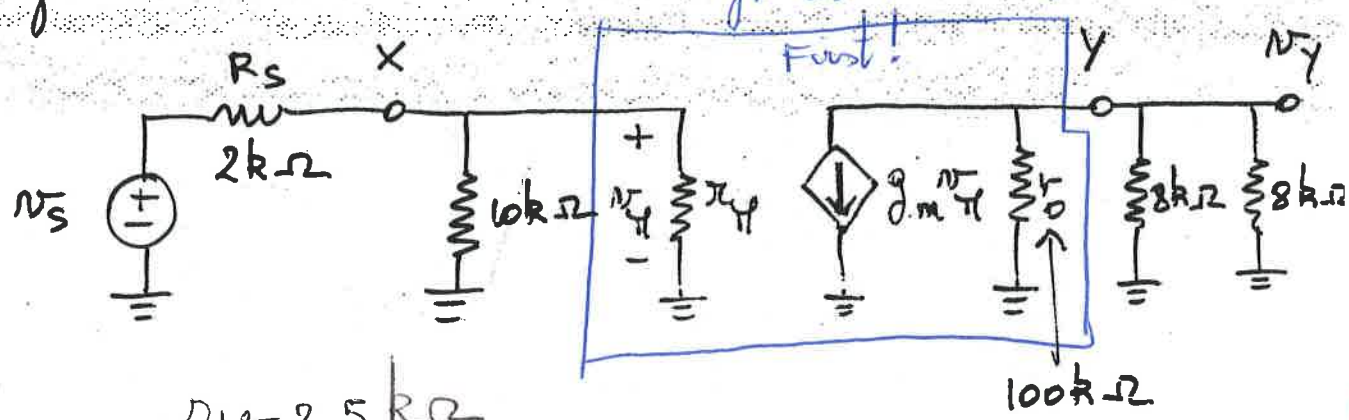
Z is connected to ground → common emitter configuration

X to signal source v_s, $R_s = 2k\Omega$

Y to a $8k\Omega$ load

- Use hybrid π-model and draw small-signal equivalent circuit of transistor.
- Calculate v_Y/v_S
- If r_o is neglected, what is the error in estimating the gain.

Draw Hybrid-π with r_o First!



$r_\pi = 2.5\,k\Omega$

$$\frac{v_\pi}{v_S} = \frac{(10 // 2.5)}{(10 // 2.5) + 2} = \frac{2}{2+2} = 0.5$$

$$v_Y = -g_m v_\pi (r_o // 8 // 8)$$

$$v_Y = -40\,(100 // 8 // 8)\, v_\pi = -153.8\, v_\pi$$

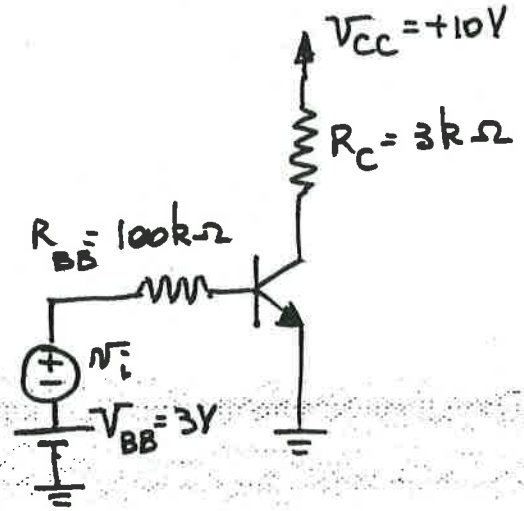
$$\Rightarrow \boxed{v_Y / v_S = -77\ V/V}$$

Why − sign?

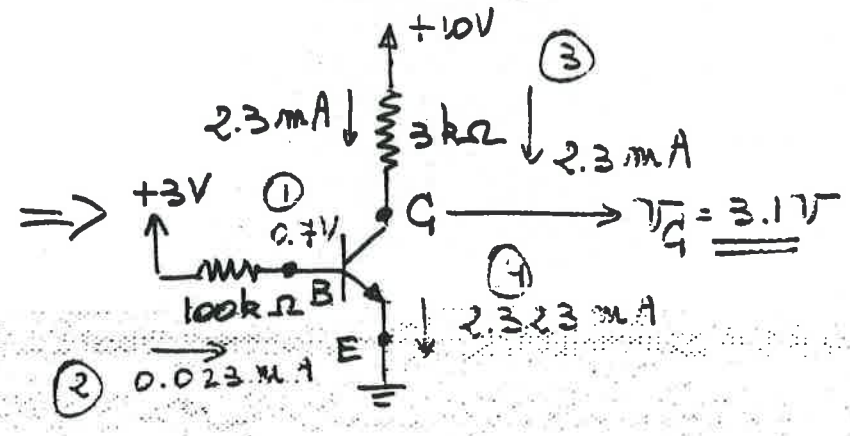
Neglecting r_o, $\dfrac{v_Y}{v_S} = -0.5 \times 40\,(8//8) = -80\ V/V$

error of 3.9% only

Example 4.9 Assume $\beta = 100$.
What is the voltage gain of the following transistor amplifier?

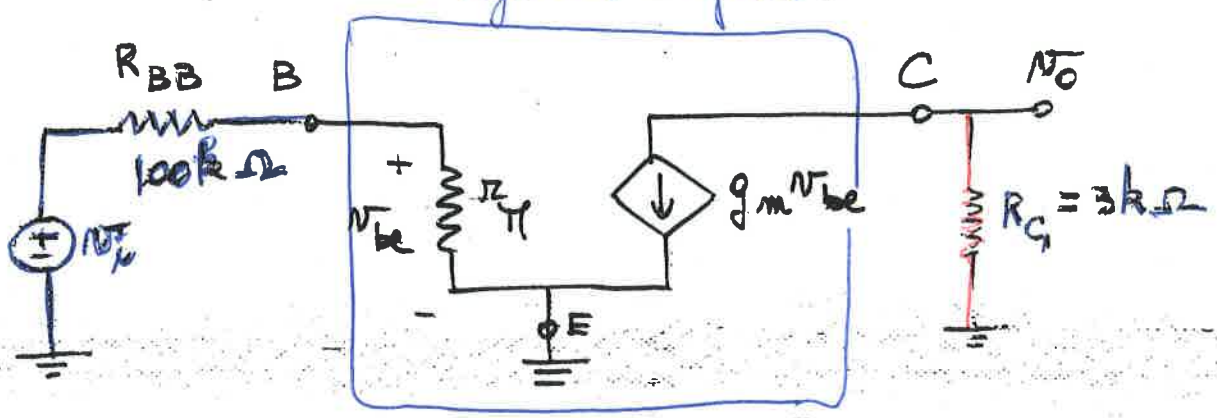


DC Analysis



AC Analysis

Draw Hybrid-π first!



$$g_m = \frac{I_C}{V_T} = \frac{2.3\,mA}{25\,mV} = 92\,mA/V \;;\quad r_\pi = \frac{\beta}{g_m} = \frac{100}{92} = 1.09\,k\Omega$$

$$v_{be} = v_i \left\{ \frac{r_\pi}{r_\pi + R_{BB}} \right\} = v_i \left\{ \frac{1.09}{101.09} \right\} = 0.011\, v_i$$

$$v_o = -g_m v_{be} R_C = -92 \times 0.011 v_i \times 3 = -3.04 v_i$$

$$\Rightarrow \boxed{\text{gain} = \frac{v_o}{v_i} = -3.04}$$

Figure 4.29 Signal waveforms.

$v_{be} = v_i \left\{ \frac{r_\pi}{r_\pi + R_{BB}} \right\} \Rightarrow v_{be}$ = 10 mV amplitude

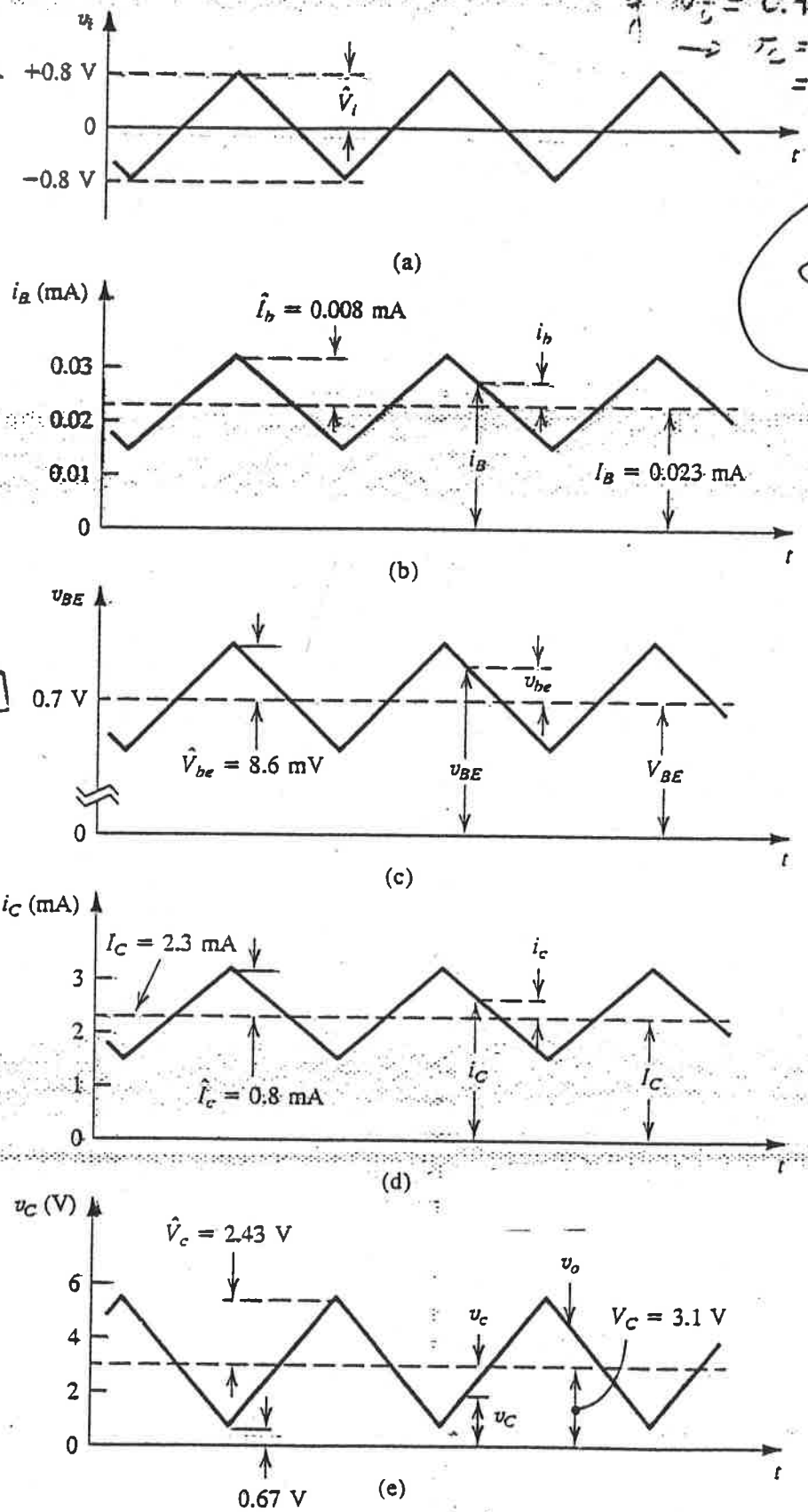
& $v_i = 0.91$ V.

$\rightarrow v_i = 0.91 \times$ gain $= 2.77$ V

DC at v_i

$\downarrow$ v_i min

$v_i = \frac{3.1 - 0.7}{\text{gain}}$



$\hat{I}_b = \frac{\hat{V}_i}{[R_{BB} + r_\pi]}$

$\hat{V}_{be} = \hat{V}_i \frac{r_\pi}{[r_\pi + R_{BB}]}$

$\hat{I}_c = \beta \hat{I}_b$

SKIP

Fig. 4.29

Read Section 4.9 Skip exercises 4.25 & 4.26