1. (10 pt) Give a regular expression for the language

\[ L = \{ vuv \mid v, w \in \{0, 1\}^*, \ |v| = 2 \} \]  

(1)

2. (10 pt) Construct an NFA which will accept the language in (1).

3. (10 pt) Construct the transition diagram for a minimal DFA which will accept the language given in (1). (Note: You do not have to prove formally that your DFA is minimal, it will be obvious, when you draw the correct transition diagram).

4. (10 pt) If \( L_1 \) and \( L_2 \) are regular languages, prove that \( L_1 \cup L_2 \) is also a regular language.

5. (10 pt) Show that all languages would be regular if the following statement would be true: “If \( L_1 \) is regular and \( L_1 \cup L_2 \) is also regular then \( L_2 \) is regular.”
1. (10 pt) Given is the grammar

\[ S \rightarrow a \mid Sa \mid aS \mid bSS \mid SbS \mid SSb \]  

Use the general method for constructing a push down automaton that accepts the same language as the one generated by (2). By general method is meant that the construction can be applied to any context free grammar and it is not restricted to this particular example.

2. (10 pt) Convert the grammar in (2) into Chomsky normal form

3. (15 pt) Show how the string \( abaa \) is recognized by the CYK algorithm.

4. (15 pt) Are all productions in (2) needed in order to generate the given language? Which production(s) can be omitted? Justify your claim.