11.30 •• The probability of a radiative transition $(n, l, m \to n', l', m')$ induced by unpolarized isotropic radiation is given by an average of (11.52) and the two corresponding expressions with x replaced by y and by z. Prove the selection rule that this probability is zero unless $m' = m$ or $m \pm 1$ (that is, $\Delta m = 0$ or ± 1). [*Hint:* Use the form (8.98) of the wave functions and write the integral of (11.52) in terms of spherical polar coordinates. You will need to examine only the integral over ϕ.]

11.31 ••• The selection rule (11.46) that radiative transitions are forbidden unless $\Delta l = \pm 1$ is related to the fact that photons have spin 1, as we mentioned in Section 11.8. The details of the argument are surprisingly complicated and require a greater knowledge of quantum mechanics than we have developed. Nevertheless, you can reach the right conclusion arguing semiclassically, as follows: To be definite, consider a hydrogen atom whose electron has initial orbital angular momentum **L** and spin **S** and which is excited by a photon to a state with **L**′ and **S**′. Conservation of angular momentum requires that

$$\mathbf{L}' + \mathbf{S}' = \mathbf{L} + \mathbf{S} + \mathbf{L}_\gamma + \mathbf{S}_\gamma \quad (11.53)$$

where $\mathbf{L}_\gamma$ and $\mathbf{S}_\gamma$ are the orbital and spin angular momenta of the incoming photon. Notice first that the electric field has no effect on the electron's spin,* so that $\mathbf{S}' = \mathbf{S}$ and these two terms cancel from (11.53). **(a)** Next you can show that $\mathbf{L}_\gamma = 0$, as follows: If it is to interact with the atom, the incoming photon must come within a distance of order a_B of the nucleus; thus, to interact with the atom, its orbital angular momentum must be of order $a_B p$ or less, where p is the linear momentum of the photon. Given that the photon's energy has to be of order E_R, show that L_γ must be of order $\alpha\hbar$ or less, where α is the so-called fine-structure constant $\alpha = ke^2/\hbar c \approx 1/137$. Since the only possible values of L are $\sqrt{l(l+1)}\hbar$ with l equal to an integer, this shows that $l = 0$ and $L_\gamma = 0$. Thus, (11.53) reduces to

$$\mathbf{L}' = \mathbf{L} + \mathbf{S}_\gamma \quad (11.54)$$

(b) The maximum magnitude of **L**′ occurs when **L** and $\mathbf{S}_\gamma$ are parallel. By taking the z component of (11.54) and remembering that the maximum value of L_z is $l\hbar$ (and similarly for all angular momenta), show that the maximum possible value of l' is $l + 1$. Show similarly that the minimum possible value is $l - 1$. This leaves only three possibilities, $l' = l + 1$ or l or $l - 1$, and Problem 11.26 has shown that $l' = l$ is not allowed, so you're home.

SECTION 11.10 (Further Properties of Lasers*)

11.32 • A single-mode He–Ne laser has a beam of diameter 3 mm. As the beam propagates away from the laser, its diameter increases because of diffraction. **(a)** Given that this effect simply adds to the initial beam size, find the distance at which the beam diameter is doubled. **(b)** At what distance will the beam have a diameter of 1 m?

*Recall that the spin enters the energy through the term $-\boldsymbol{\mu} \cdot \mathbf{B}$, which involves the magnetic field, and that the effects of B are usually negligible compared to those of $\mathscr{E}$.

11.33 •• In three of the Apollo lunar experiments, astronauts left reflector panels on the moon so that laser beams from the earth could be reflected off the panels and back to earth. Lasers with $\lambda = 532$ nm send pulses of 0.3 J to the moon, and the round-trip time is measured within $\delta t \approx 0.4$ ns. In this way the one-way distance to the reflector on the moon is determined regularly with-in $\delta l = c\,\delta t/2 \approx 6$ cm. The beams from the lasers have a diameter of 5 km on the moon. (This is the result of spreading due to diffraction and to atmospheric turbulence.) The reflector panels contain 300 mirrors, each of diameter 4 cm. **(a)** Find the number of photons sent from the earth in a single pulse. **(b)** Find the fraction of these photons that strike any one of the small mirrors. **(c)** Find the angular divergence $\delta\theta \approx \lambda/d$ of the reflected beam from this mirror (where d is the mirror's diameter). **(d)** What is the diameter of the reflected beam when it returns to earth? **(e)** The return light is measured by a photomultiplier at the focus of a telescope. What fraction of the return light is captured by the telescope, whose diameter is 1 m? **(f)** For a single pulse, find the total number of photons captured by the telescope from all 300 mirrors. (The actual number is somewhat smaller because of losses in the atmosphere and in the telescope.) This experiment monitors the moon's orbit within a few centimeters (compared to the earth–moon distance of 4×10^8 m), allowing stringent tests of competing theories of gravity.

COMPUTER PROBLEMS

11.34 ••• (Section 11.4) To illustrate the property (11.20) of completeness, consider an infinite square well of width $a = 1$, centered on the origin, with stationary-state wave functions $\psi_n(x)$ as given in (11.34) and (11.35). Let $\psi(x)$ be the flat wave function equal to 1 everywhere inside the well (and 0 outside). **(a)** Find the coefficients A_m in the expansion (11.20) of $\psi(x)$. **(b)** To reproduce $\psi(x)$ exactly, you would have to include all of the terms in the infinite series (11.20), but you can get a surprisingly good approximation by including just the first few terms. Use a suitable plotting program to plot the sum of the first 5 nonzero terms of the series. **(c)** Repeat with the first 10 nonzero terms.

11.35 ••• (Section 11.4) Do the same exercises as in Problem 11.34 for the same square well with width 1, but using the triangular wave $\psi(x) = 1 - 2|x|$. [In part (b) plot just the first term.]

11.36 ••• (Section 11.5) Consider the electron of Problem 11.22, which is initially in the $n = 2$ state ψ_2 of the infinite square well with width $a = 1$. When an electric field $\mathscr{E}$ is switched on for a brief time Δt, the wave function picks up a small component proportional to $x\psi_2(x)$. **(a)** Use a suitable plotting program to make plots of $\psi_2(x)$ and $x\psi_2(x)$. **(b)** It is probably not immediately clear what the extra wave function $x\psi_2(x)$ (multiplied by a small constant) signifies. To clarify this, expand $x\psi_2(x)$ as in (11.20), and find the coefficients A_m of this expansion. Use your answer to argue that, to a good approximation, the extra wave function has the form $x\psi_2(x) \approx A_1\psi_1(x) + A_3\psi_3(x)$. Plot the right-hand side of this approximation, and compare with your plot of $x\psi_2(x)$. What is the physical significance of this result?