Chapter 5: Useful Circuit Analysis Techniques [pp 121 → p.172]

Principle of superposition

The response (the desired current or voltage) in a **LINEAR CIRCUIT** having more than one independent source can be obtained by adding the responses caused by the separate independent sources acting alone.

Linear elements and linear circuits

linear element: a passive element with a linear current-voltage characteristic

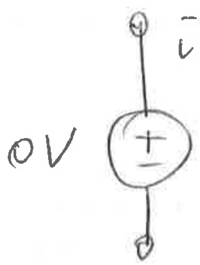
linear dependent source: is a dependent voltage or current source whose output current or voltage is proportional only to the first power of a specified current or voltage in another part of the circuit.

linear circuit: a circuit composed entirely of independent (voltage or current) sources, linear dependent sources, and linear elements.

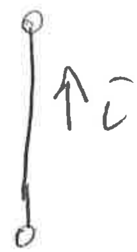
The Superposition Principle

Turning OFF or Zeroing OUT The Voltage and Current sources.

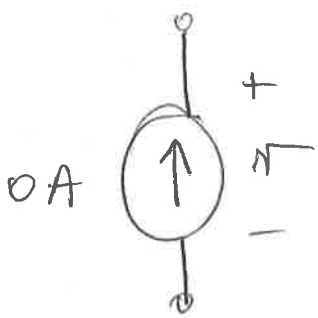
A voltage source set to zero acts like a **short circuit**



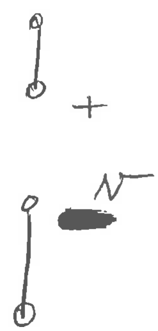
no voltage drop across terminals but current can flow



A current source set to zero acts like an **OPEN circuit**



No current flows but a voltage can appear across the terminals



Superposition Theorem p. 123
(middle of page)

In any linear resistive network, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits.

Summary of basic Superposition procedure

① Select one of the independent sources. Set all other dependent sources to zero

voltage source → SHORT

current source → OPEN

(Leave dependent sources alone) (untouched)

② Relabel voltages and currents using different notations each time you perform ①.

③ Analyze the simplified circuit to find the desired currents and/or voltages

④ Repeat steps 1 through 3 until each independent source has been considered.

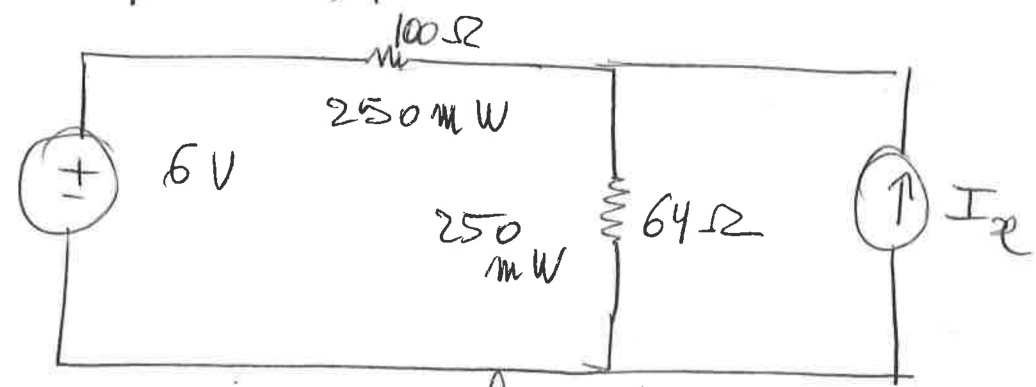
⑤ Add partial currents and/or voltages as found in steps ① through ④ above

WATCH OUT for SIGNS

⑥ DO NOT add power quantities ($p = vi$; this is not a linear quantity.

Calculate p after total v and total i have been obtained from step ⑤

Example 5.2 p 127



Use superposition to find the maximum positive current to which the source I_x can be set before any resistor exceeds its power rating.

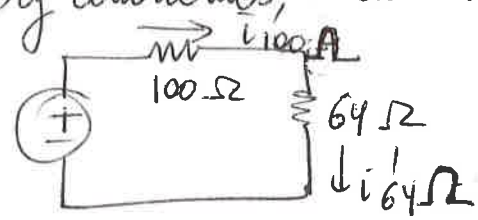
$P_{max} = 250\,mW$

The max. current allowed in the 100 Ω resistor is

$i_{MAX} = \sqrt{\frac{P_{max}}{R}} = 50\,mA$

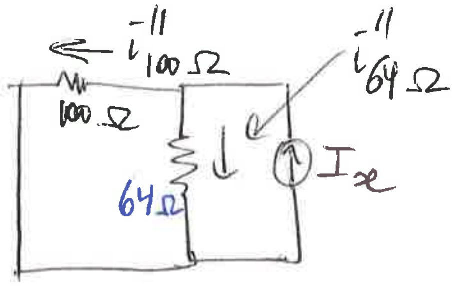
For the 64 Ω resistor, $\sqrt{\frac{P_{max}}{R}} = 62.5\,mA$

The 6V battery contributes a current $i'_{100\Omega} = \frac{6}{100+64} = 36.59\,mA$



$i'_{64\Omega} = 36.59\,mA$ also

For i_x



$i''_{64\Omega}$ will add to $i'_{64\Omega}$

$i''_{100\Omega}$ " subtract from $i'_{100\Omega}$

I_x can contribute $62.5 - 36.59 = 25.91\,mA$ to the 64 Ω

$50 - (-36.59) = 86.59\,mA$ to the 100 Ω

For 100 Ω $\quad I_{100\Omega} \overset{max}{\leq} \frac{64}{64+100} I_x$

$\uparrow$ 86.59 mA $\longrightarrow$ $I_x \leq \frac{164}{64} \cdot (86.59\,mA) = 221.9\,mA$

For 64Ω

$$I_{64\Omega}^{max} = \frac{100}{164} I_x$$

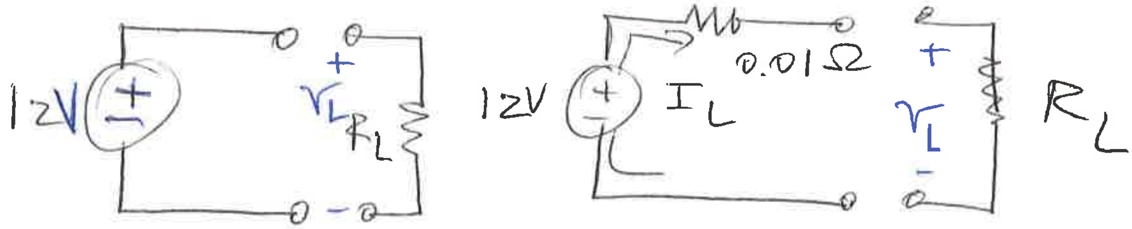
$$= 25.91 mA$$

$$\rightarrow I_x^{max} \leq \frac{164}{100} \cdot (25.91 mA) = 42.49 mA$$

So $I_x < 42.49 mA$ or 64Ω resistor will overheat before the 100Ω does.

The 64Ω (lowest of the 2) sets the limit on I_x!

5.2 Source transformations p.131

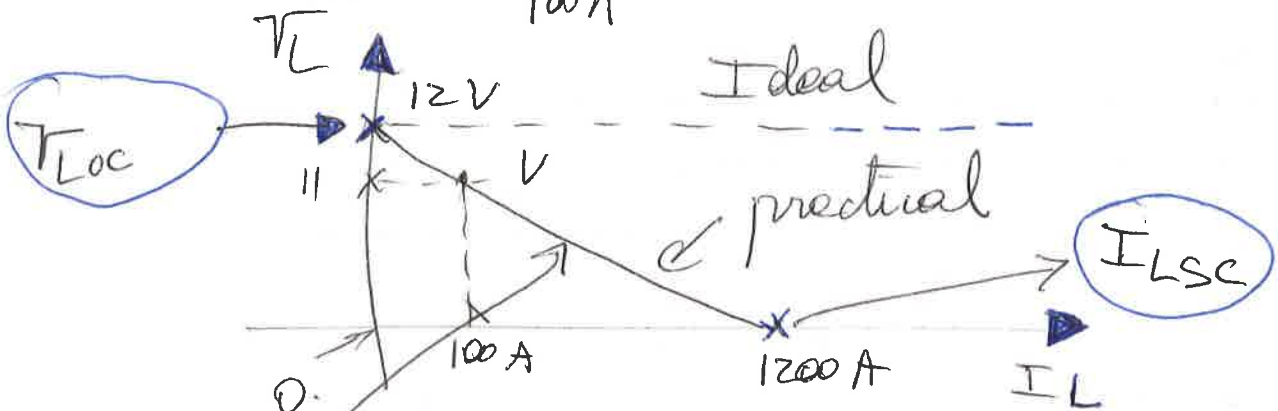


Ideal   Practical

$V_L = 12\,V$ if $I_L = 0$ $(R_L = \infty)$

$V_L = 11\,V$ if $I_L = 100\,A$

$$\frac{1\,V}{100\,A} = 0.01\,\Omega$$



$$12 = 0.01\,I_L + v_L$$

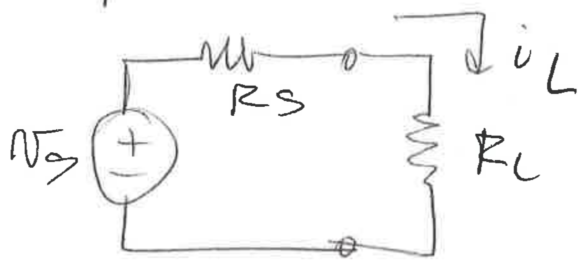
$$\boxed{v_L = 12 - 0.01\,I_L}$$ negative slope.

$R_L = \infty$ $I_L = 0$ practical source is open-circuited $\rightarrow$ $v_{Loc} = 12V$

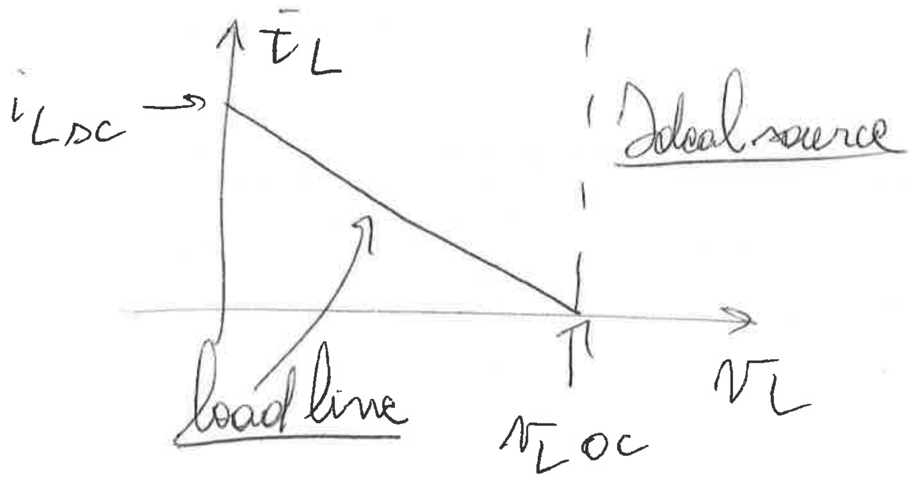
$R_L = 0$ (short circuiting the load terminals)
$\rightarrow I_L = I_{Lsc} = 1200\,A.$

v_{Loc} & I_{Lsc} determine the entire $I_L - v_L$ curve

General practical voltage source



R_S = internal resistance or output resistance

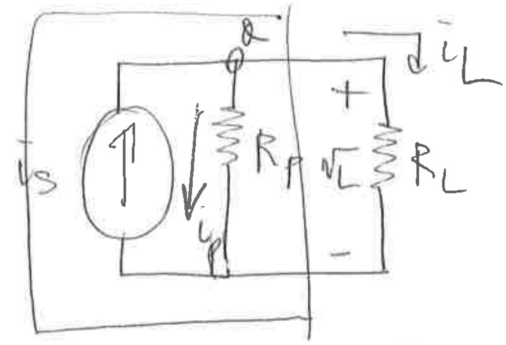


$$i_L = \frac{v_S - v_L}{R_S}$$

$$i_L = \frac{-1}{R_S} v_L + \frac{v_S}{R_S}$$

Referred to as LOAD LINE

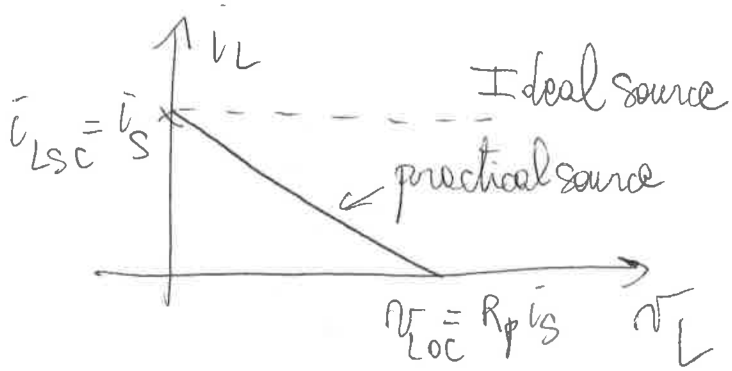
P.133 Practical Current sources



KCL at node (a) $\quad i_L = i_s - \dfrac{v_L}{R_p}$

$i_{Lsc} = i_s$ short-circuit current

$v_{Loc} = R_p i_s$ open-circuit voltage



Equivalent practical circuits

2 Sources are equivalent if they produce identical values of i_L & v_L when they are connected to identical values of R_L

$R_L = \infty$ & $R_L = 0$ are such values.

⇒ Equivalent sources provide the same open-circuit voltage and short-circuit current.

Careful
p. 134 Fig 5.15(b)

$$v_L = v_S \frac{R_L}{R_S + R_L} \qquad v_L = \left[i_S \frac{R_P}{R_P + R_L} \right] \cdot R_L$$

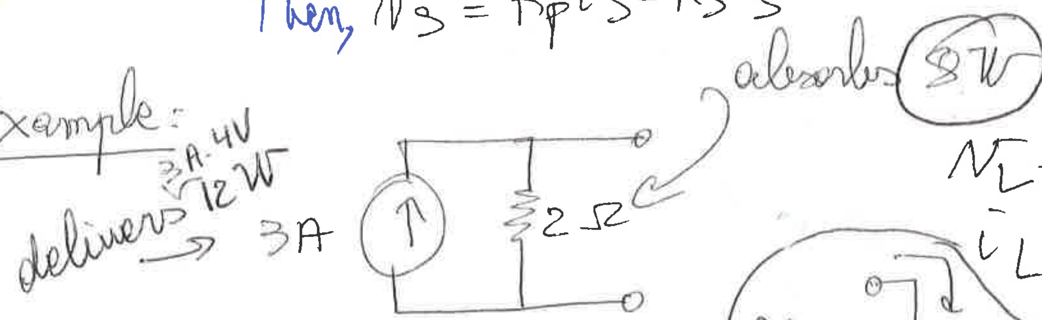
The two practical sources are electrically equivalent

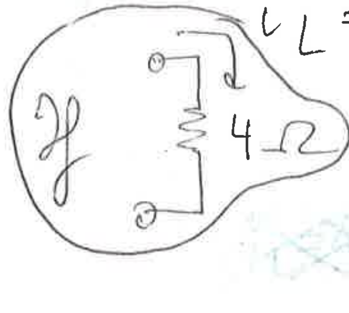
If

$$R_S = R_P$$

Then, $v_S = R_P i_S = R_S i_S$

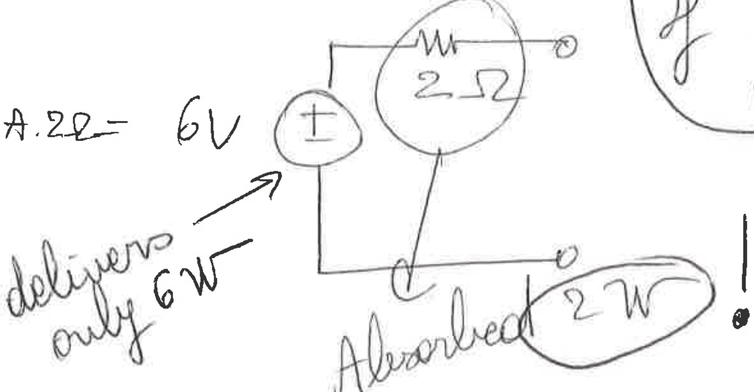
Example:





$v_L = 4V$
$i_L = 1A$
$P_L = 4W$

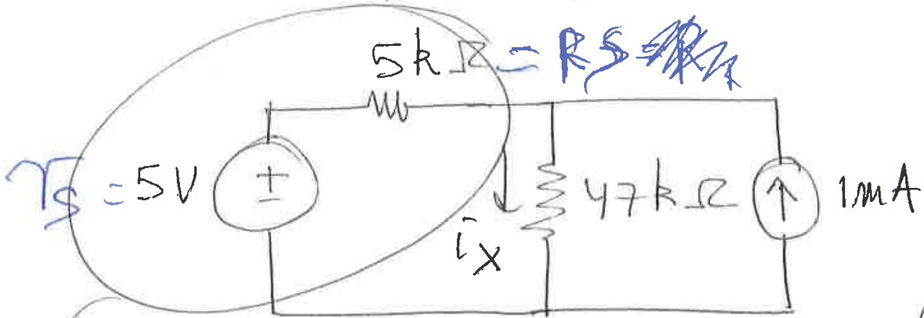
3A·2Ω = 6V



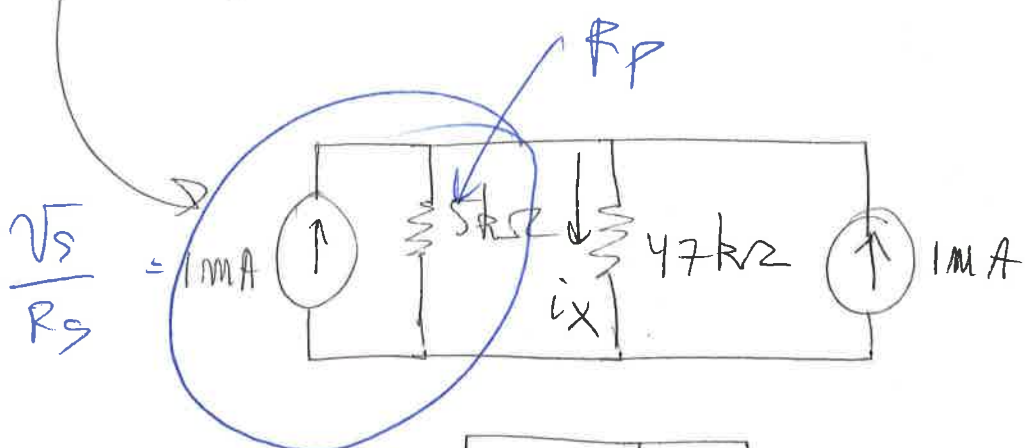
⚠ The two practical sources are NOT equivalent internally.

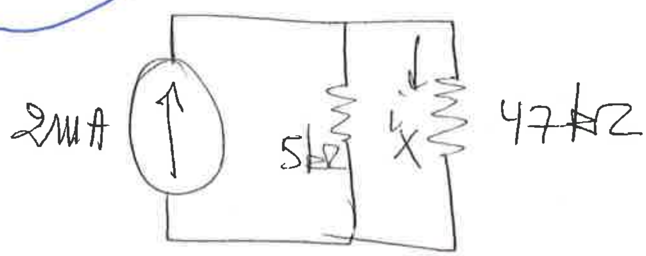
The two practical sources are equivalent only in terms of what is happening in load circuit.

Practice problem 5.3



i_x? after performing a source transformation

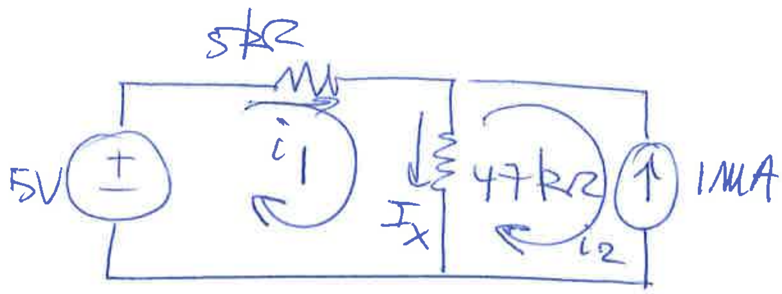




$$i_x = \frac{5}{52}\,2mA = 192\,\mu A$$

EXTRA CREDIT SHOW THAT YOU GET SAME i_x using either MESH OR NODE Analysis.

Practice 5.3



$$-5 + 5{,}000 i_1 + 47{,}000(i_1 - i_2) = 0$$

$$i_2 = -0.001 A$$

$$-5 + 52{,}000 i_1 + 47 = 0 \rightarrow i_1 = \frac{-42}{52{,}000} = \frac{-42}{52{,}000}$$

$$\rightarrow i_x = i_1 - i_2$$

$$\boxed{i_x = 0.001 - \frac{42}{52{,}000} = 192 \mu A}$$

§5-3. Thévenin & Norton equivalent circuits

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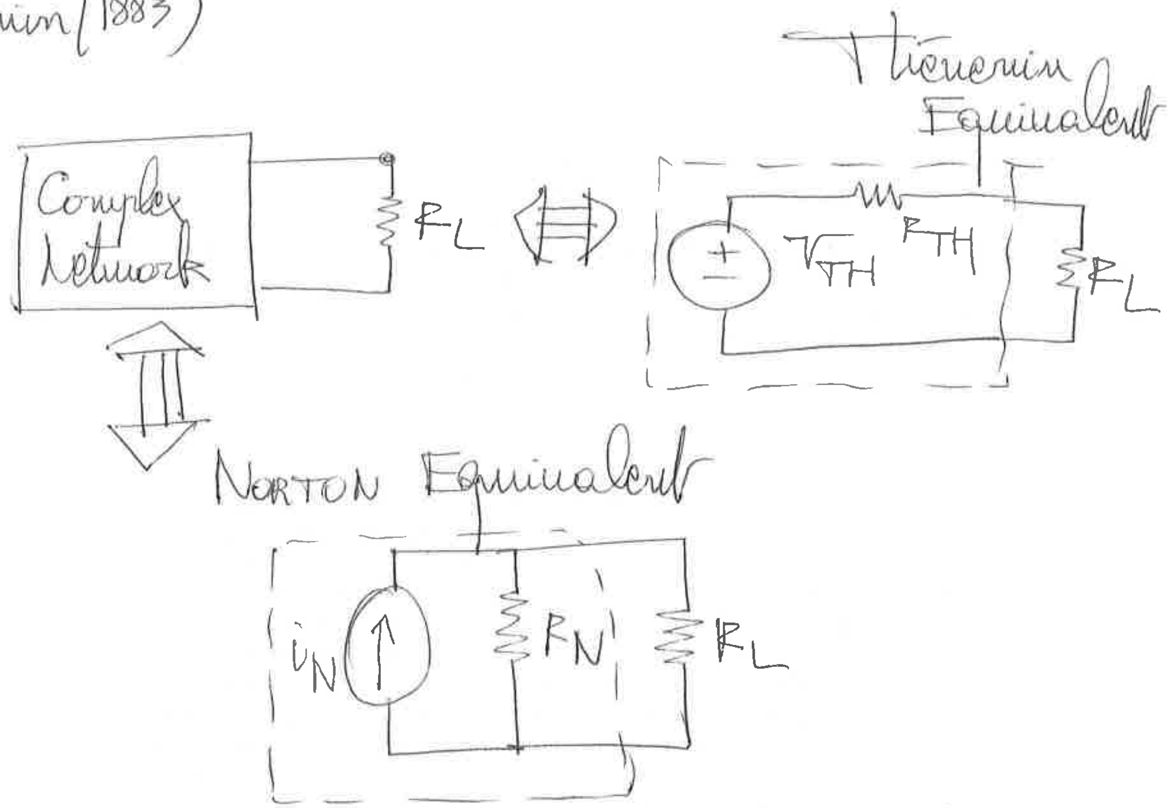
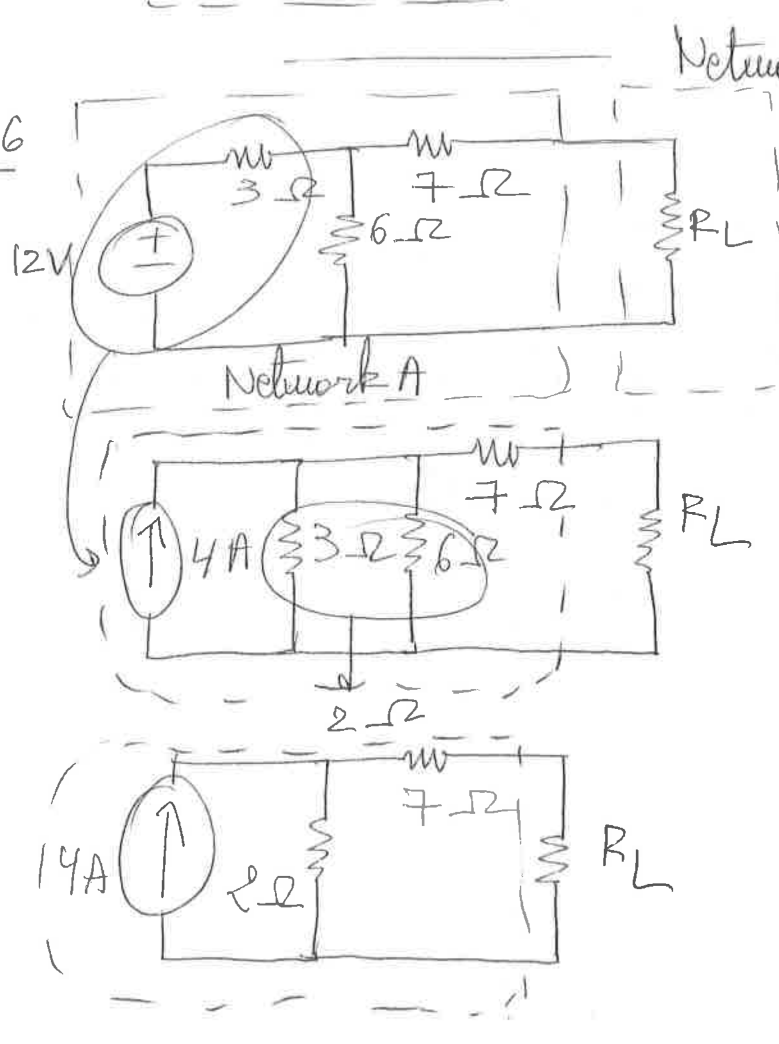
Thévenin (1883)



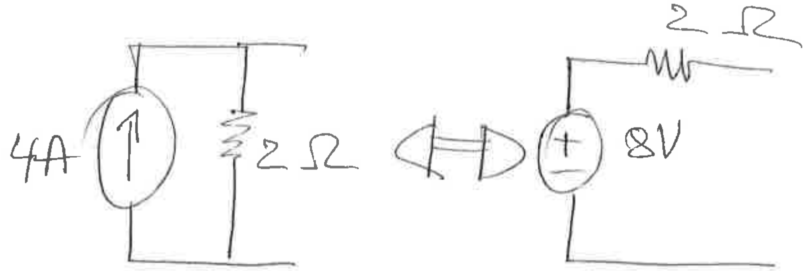
Example 5.6

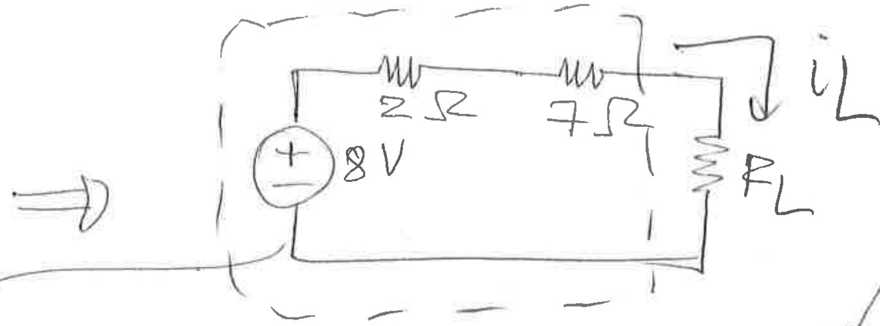


Questions

(a) max. voltage across load if $R_L = \infty$

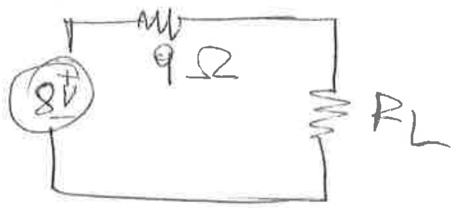
(b) max current that can be delivered to load if $R_L = 0$.

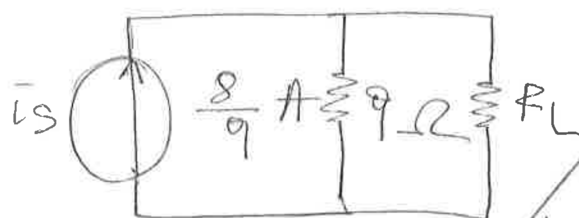




Power delivered to load is $P_L = \left(\frac{8}{9+R_L}\right)^2 R_L = R_L i_L^2$ (where $i_L = \frac{8}{9+R_L}$)

Max voltage across R_L is 8 V when $R_L = \infty$



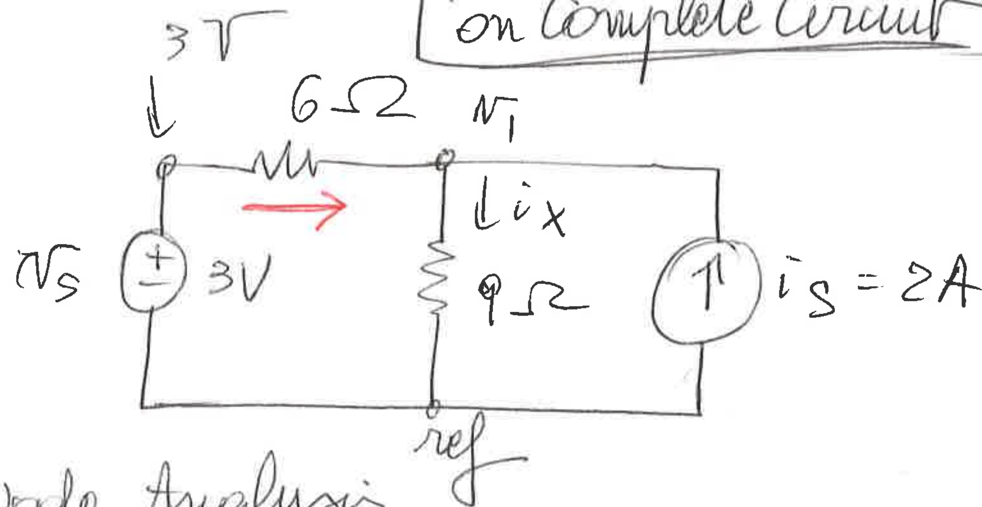


These answers are not obvious when looking at the original network

Maximum current that can be delivered to load is $\frac{8}{9}$ A when $R_L = 0$

Do practice 5-5

NODE ANALYSIS on Complete Circuit



Node Analysis

$$\frac{3 - v_1}{6} - i_x + 2 = 0.$$

$$i_x = \frac{v_1}{9}$$

$$\Rightarrow \frac{3 - v_1}{6} - \frac{v_1}{9} + 2 = 0$$

$$3(3 - v_1) - 2v_1 + 36 = 0$$

$$9 - 3v_1 - 2v_1 + 36 = 0$$

$$-5v_1 + 45 = 0$$

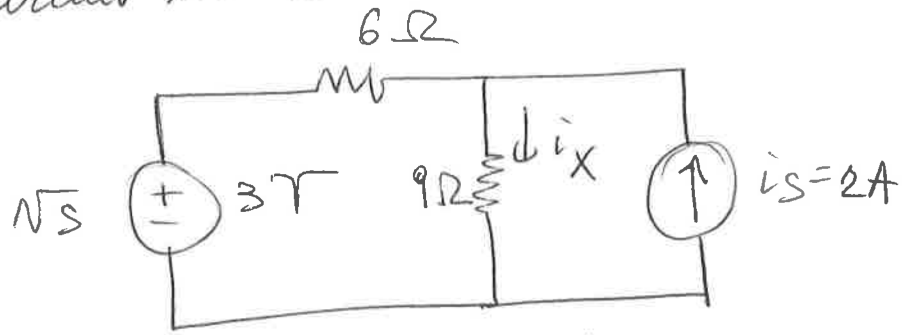
~~$v_1 = 45$ V~~

$$v_1 = 9V$$

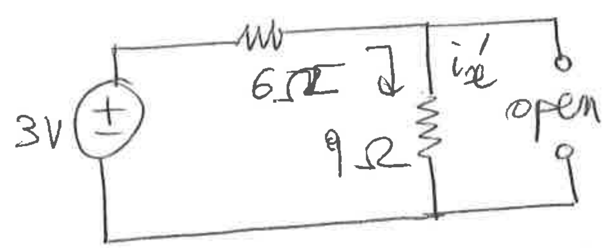
$$\rightarrow \boxed{i_x = 1A}$$

Example 5.1

Use the superposition theorem to calculate i_x in the circuit below

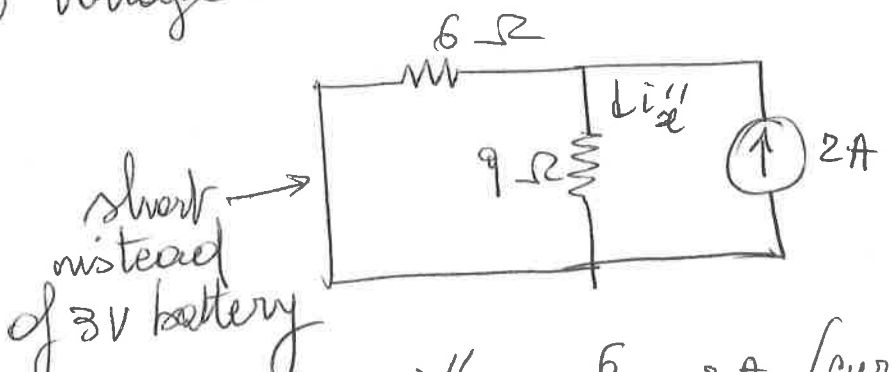


(a) Same circuit with current source as open-circuited



$$i_x' = \frac{3V}{15} = 0.2A$$

(b) Voltage source zeroed OUT



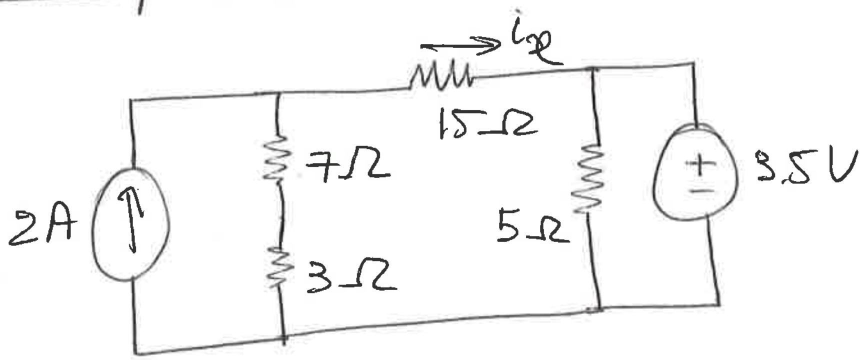
$$i_x'' = \frac{6}{6+9}\, 2A \quad \text{(current divider rule)}$$

$$i_x'' = 0.8A$$

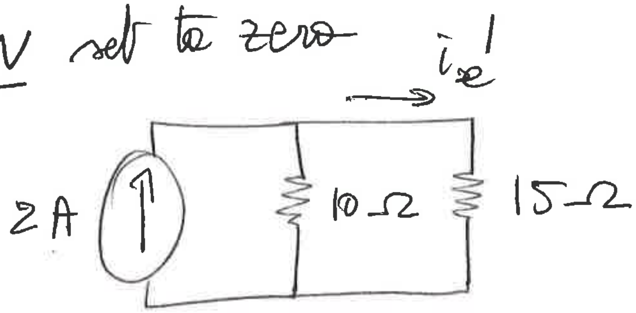
Total current $i_x = i_x' + i_x'' = \underline{\underline{1A}}$

Practice problem 5.1 p125

#1



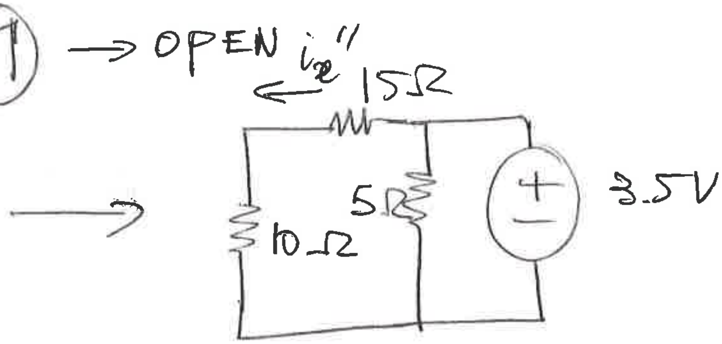
(a) 3.5V set to zero



$$i_x' = \frac{10}{25} \cdot 2A$$

$$i_x' = \frac{2}{5}\, 2A = 0.8A$$

(b) 2A (↑) → OPEN



$$i_x'' = \frac{3.5V}{25} = 0.14\ A$$

⚠ i_x' and i_x'' are in opposite directions

$i_x = 0.8 - 0.14 = 660\ mA$
in the circuit at the top