Chapter 8 (Basic RL & RC Circuits) pp 261

§8.1 Source free RL circuit

→ Homogeneous linear differential equation
a diff. eq. in which every term is of the first degree in the dependent variable or one of its derivatives.

Natural response: The response depends on the general "nature" of the circuit.
when we consider independent sources acting on a circuit, part of the response will resemble the nature of the particular solution (This is referred to the forced response).

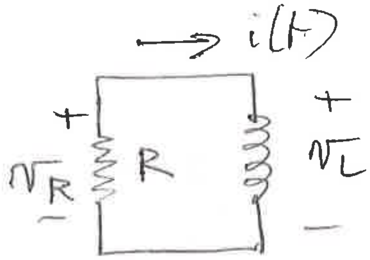


Figure 8.1

$v_R = v_L$
$-Ri = L\frac{di}{dt}$

passive sign Convention

Assume $i(t)$ at $t=0$ is $I_0 = i(t=0)$

$$Ri + v_L = Ri + L\frac{di}{dt} = 0$$

or

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Direct approach

$$\frac{di}{i} = -\frac{R}{L}dt$$

$$\int_{I_0}^{i(t)} \frac{di'}{i'} = \int_0^t -\frac{R}{L}dt'$$

$$\ln i'\Big]_{i_0}^{i} = -\frac{R}{L}(t-0)$$

$$i(t) = I_0 e^{-\frac{R}{L}t}$$

A more general solution approach p.258

Assume $\quad i(t) = A e^{s_1 t}$

where A, s_1 are constants to be determined.

Substitute $i(t)$ in $\frac{di}{dt} + \frac{R}{L} i = 0$

$$\rightarrow A s_1 e^{s_1 t} + A \frac{R}{L} e^{s_1 t} = 0$$

$$\rightarrow \left(s_1 + \frac{R}{L}\right) A e^{s_1 t} = 0$$

This is satisfied if $\underbrace{A = 0, \text{ or } s_1 = -\infty}_{\Downarrow \text{ response is zero!}}$, or $s_1 = -\frac{R}{L}$

Therefore we must choose $s_1 = -\frac{R}{L}$

$$\rightarrow i(t) = A e^{-\frac{R}{L}t}$$

But $i(0) = I_0 = A$

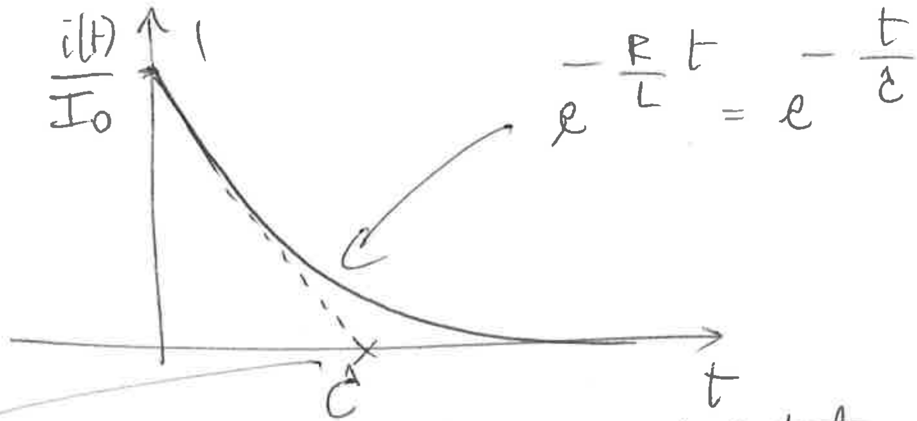
$$\rightarrow \boxed{i(t) = I_0 e^{-\frac{R}{L}t}}$$

$$\boxed{s_1 + \frac{R}{L} = 0}$$

is called a characteristic equation

It characterizes the natural response of the system

§8.2 Properties of the exponential response (p. 262)



$\tau = \frac{L}{R}$. if $\tau \uparrow$, the exponential takes longer to decay

Initial rate of decay

$$\left.\frac{d}{dt}\left(\frac{i}{I_0}\right)\right]_{t=0} = -\frac{R}{L} e^{-\frac{Rt}{L}}\Bigg|_{t=0} = -\frac{R}{L}$$

can be determined from the display on an oscilloscope!

other interpretation of τ

$$\frac{i(\tau)}{I_0} = e^{-1} = 0.3679$$

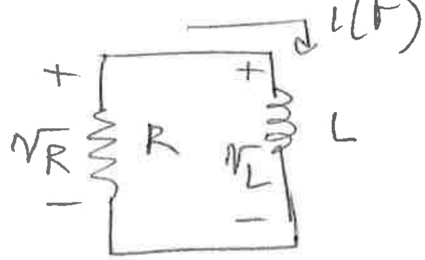
Thus, in one time constant, the response has dropped to $\simeq 36.8\%$ of its initial value.

$$\frac{i(t)}{I_0} = 0.1353 \text{ at } t = 2\tau$$
$$0.04979 \text{ at } t = 3\tau$$
$$0.01832 \text{ at } t = 4\tau$$
$$0.006738 \text{ at } t = 5\tau$$

If we are asked "How long does it take for the current to decay to zero?", we can answer "about five time constants".

At that point, the current is less than 1% of its original value.

Back to Fig. 8.1



Energy

Instantaneous power; $P_R = R i^2 = I_0^2 R e^{-2\frac{R}{L}t}$

Total energy turned into heat in resistor is

$$w_R = \int_0^\infty P_R\, dt = I_0^2 R \int_0^\infty e^{-2\frac{R}{L}t}\, dt$$

$$w_R = I_0^2 R \left(\frac{-L}{2R}\right) e^{-2\frac{R}{L}t}\Big]_0^\infty = \frac{L}{2} I_0^2$$

This is the total energy stored initially in the inductor at $t=0$; the current through the inductor has dropped to zero at $t=\infty$ and the energy must appear as dissipated in the resistor.

Example 8.2 (p. 260)

For the circuit below, find $v(t = 200\,ms)$

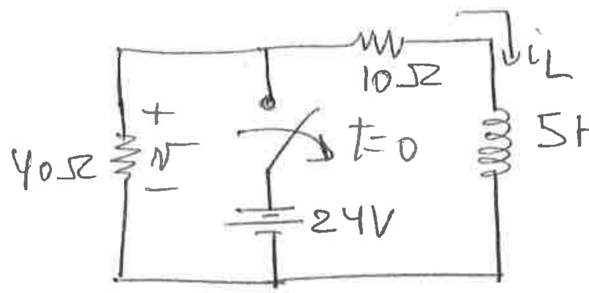


Fig. 8.5(a)

(a)

t<0

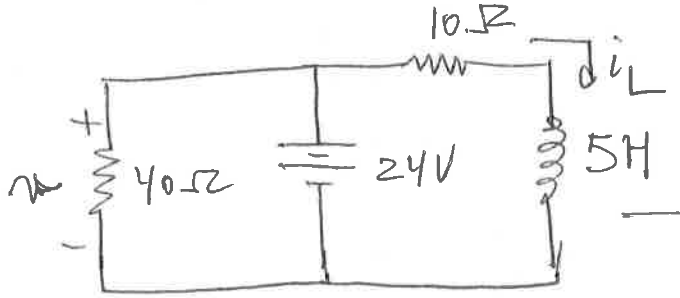


Fig 8.5(b)

→ $v(0) = 24V$ just before the switch opens.

t≥0

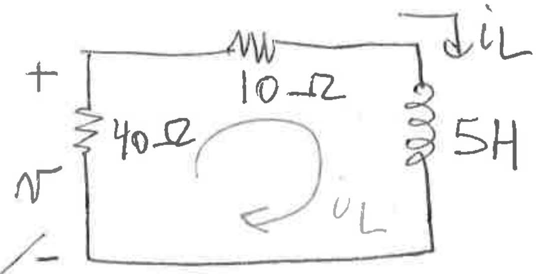


Fig. 8.5(c)

KVL ↳ $$-v + 10\,i_L + 5\frac{di_L}{dt} = 0$$

$$i_L = -v/40$$

$$\frac{5}{40}\frac{dv}{dt} + \left(\frac{10}{40} + 1\right)v = 0$$

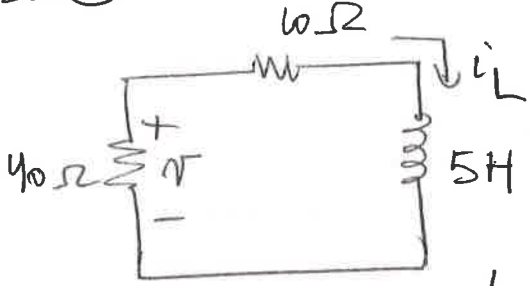
$$\frac{dv}{dt} + 10v = 0$$

↳ when switch is thrown, only the current through the inductor must remain unchanged.

In Fig 8.5b, $i_L = \frac{24}{10} = 2.4A$

(The inductor acts as a short to a DC current).

From Fig 8.5(c)



$v(0^+) = 40(-2.4) = -96\,v$!

$$\frac{dv}{dt} + 10v = 0$$

$$v(t) = A e^{st}$$

$\longrightarrow$ $s + 10 = 0$ is the characteristic equation

$\longrightarrow s = -10$

$$v(t) = A e^{-10t}$$

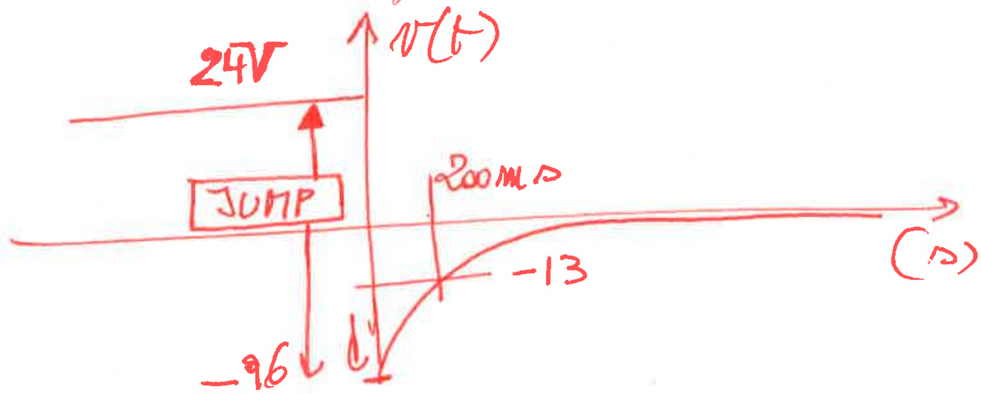
at $t = 0^+$ $\quad v(0^+) = -96V$

$\longrightarrow$ $\boxed{v(t) = -96\, e^{-10t}}$ $\Rightarrow v(200ms) = -96\, e^{-10(0.2)}$

$= -96\, e^{-2}\, V$

$= -13V$

$v(0^-) = 24V$



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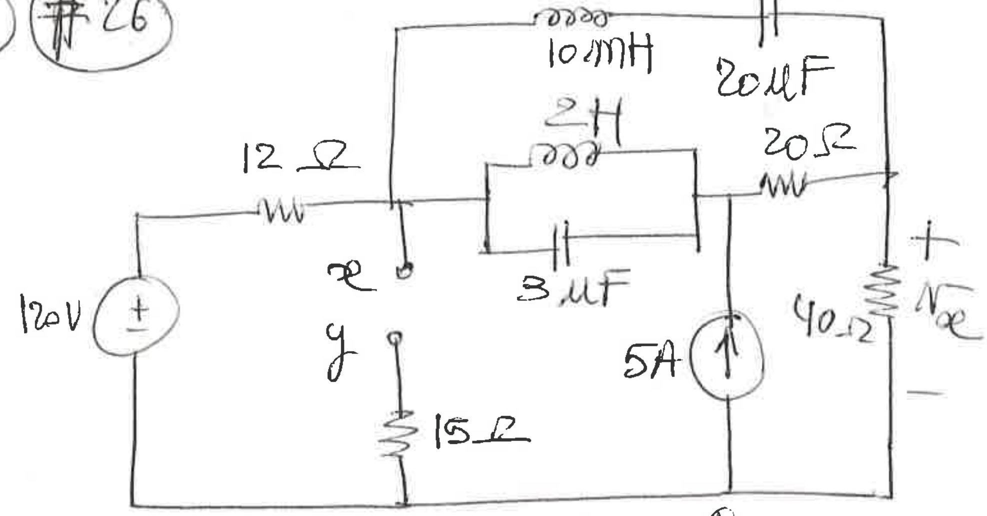
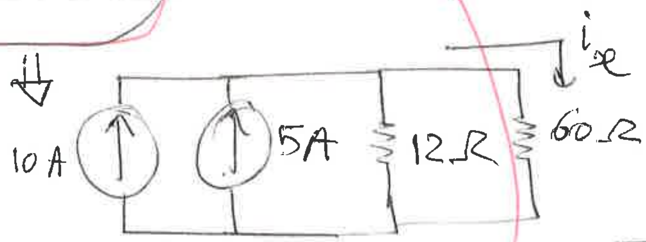


A long time after all connections have been made in the circuit shown above, find v_x if

(a) a capacitor is present between x and y

(b) an inductor is present between x and y.

(a)



$i_x = \frac{12}{72}$ 15A

$i_x = \frac{\cancel{15A}}{6}$

$\boxed{v_X = \frac{40 \cdot 15}{6} = 100V}$

(b)

$$i_x = \frac{\frac{1}{60}}{\frac{1}{12} + \frac{1}{15} + \frac{1}{60}} \quad 15$$

$$i_x = \frac{15}{5 + 4 + 1} = 1.5A$$

$\boxed{v_x = 60V}$

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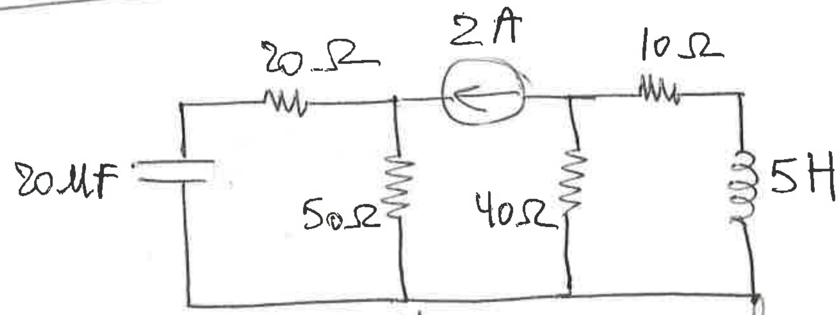
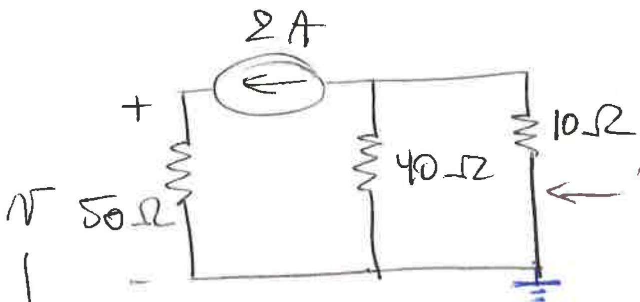
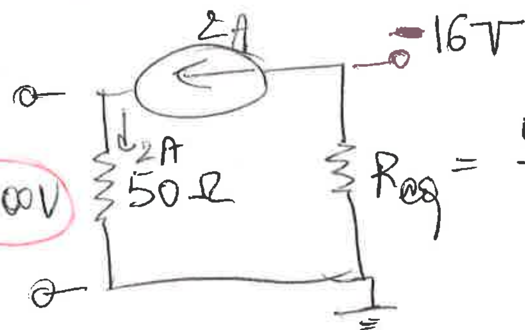


(a) w_L? (b) w_C?, (c) voltage across each circuit element
(d) current in each circuit element.

circuit at steady state

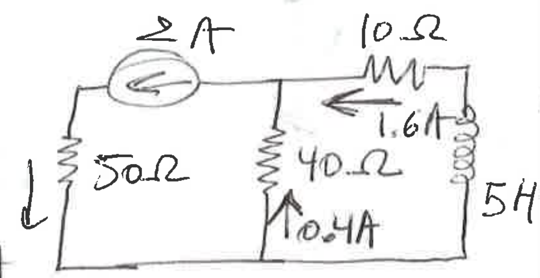


v ↓ is also voltage across capacitor



$v_C = 100V$

$R_{eq} = \frac{400}{50} = 8\,\Omega$



16 V across 10 Ω resistor

→ 1.6A flowing through 10 Ω & 5H inductance

$$\boxed{w_L = \frac{1}{2} L i^2} = \frac{1}{2} 5 (1.6)^2 = 6.4\,J$$

$$\boxed{w_C = \frac{1}{2} C v^2} = \frac{1}{2} 20\, 10^{-6} (10^2)^2 = 10^{1-6+4} = 0.1\,J$$

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$v_S = 400t^2$ V for $t>0$

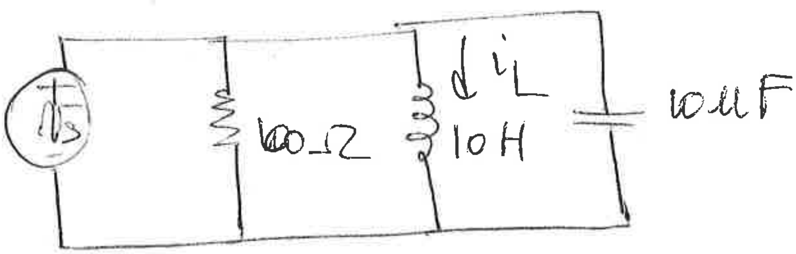
$i_L(0) = 0.5$ A

at $t = 0.4$ s, find values of energy:

(a) stored in the capacitor

(b) stored in the inductor

(c) dissipated by the resistor since $t=0$.



(c) $P_R = \frac{dw_R}{dt}$

$$\underbrace{w_R(t) - w_R(0)}_{(c)} = \int_0^t P_R\,dt' = \int_0^t dt'\, R\, i_R^2(t') = \int_0^t dt' \frac{1}{R} v_S^2(t')$$

$$i_R(t') = \frac{v_S(t')}{R}$$

calculate (c) for $\boxed{t = 0.4\text{ s}}$ → $W_R(0.4\text{s}) = 3.277$ J

(b) $w_C(t) = \frac{1}{2} C v_S^2(t) = \frac{10 \cdot 10^{-6}}{2}\left[400t^2\right]^2$ for $t = 0.4$ s (20.48 mJ)

(a) $\cancel{w_L(t) - w_L(0) = \frac{1}{2}L(i^2(t) - i^2(0))}$

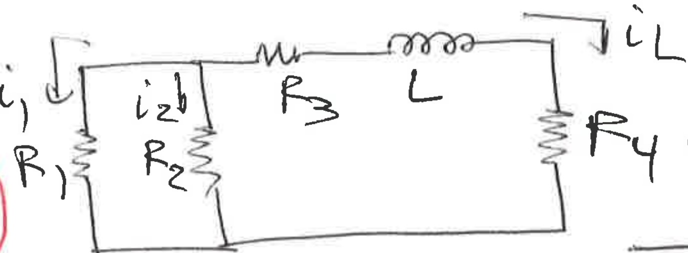
$$w_L(t)\Big|_{t=0.4\text{s}} = \frac{1}{2} L i^2(t) = 9.158 \text{ J}$$

$$\boxed{i_L(t) = i_L(0) + \frac{1}{L}\int_0^t dt'\, v_S(t')} \rightarrow \boxed{i_L(0.4) = 1.3533\text{ A}}$$

The distinction between 0^+ and 0^-

Natural response

$i_2(t)$? $i_1(t)$? $t>0$



$\Longleftrightarrow$

$$\boxed{R_{eq} = R_3 + R_4 + (R_1 // R_2)}$$

Assume at $t = 0_-$, $i_L(0_-)$

$$\rightarrow i_L(t) = i_L(0_-)e^{-\frac{t}{\tau}} = i_L(0_+)e^{-\frac{t}{\tau}}$$

i_2? using current divider rule.

$$i_2(t) = \left[\frac{-R_1}{R_1+R_2}\right] i_L(t) \; ; \; i_1(t) = \left[\frac{-R_2}{R_1+R_2}\right] i_L(t)$$

$$\boxed{i_2(t) = i_1(t)\frac{R_1}{R_2}}$$

$\underline{t>0}$

$$i_2(t) = \left(\frac{-R_1}{R_1+R_2}\right) i_L(0_+)e^{-t/\tau}$$

$$i_1(t) = \left(\frac{-R_2}{R_1+R_2}\right) i_L(0_+)e^{-t/\tau}$$

All currents have the same time dependence

Example 8.4 Determine i_1 & i_L for $t>0$

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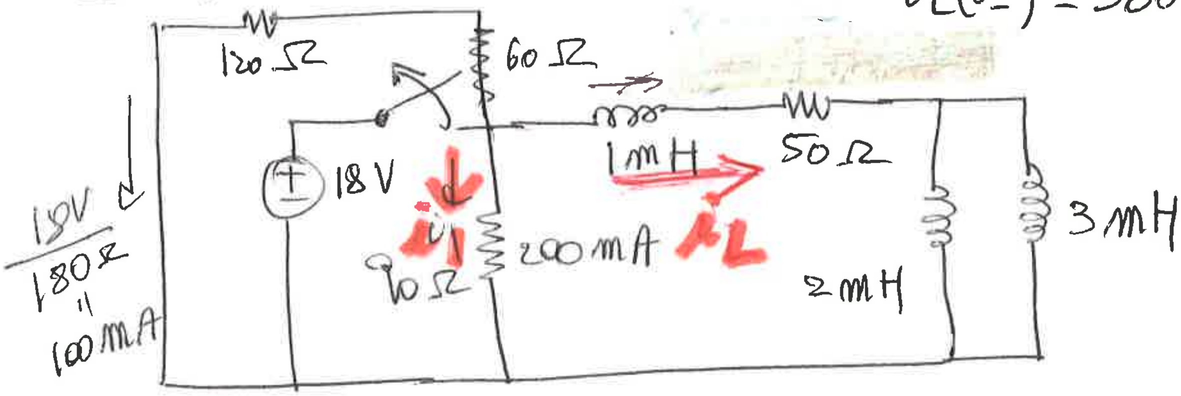
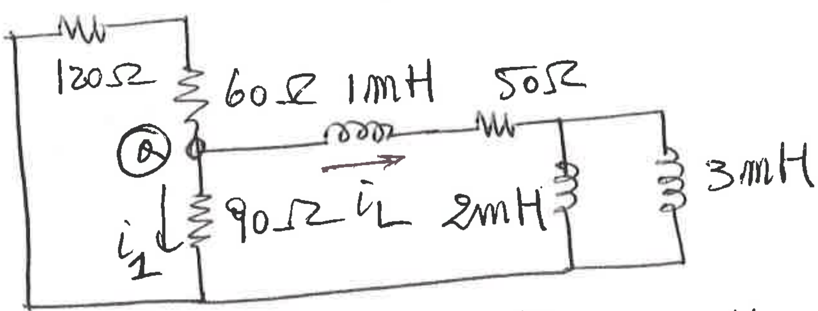
$i_L(0_-) = 360mA \leftarrow$ Show that.



After $t=0$



$$L_{eq} = 1\,mH + \frac{2mH\ 3mH}{(2+3)\,mH} = 2.2mH$$

$$R_{eq} = (90 /\!/ 180) + 50 = 110\,\Omega$$

$$\tau = \frac{L_{eq}}{R_{eq}} = 20\,\mu s$$

natural response is $Ke^{-\frac{t}{\tau}}$

Prior to opening switch $\quad i_L = \frac{18V}{50\Omega} = \frac{18}{50}A = 360mA$

$i_L(0_+) = i_L(0_-)$

$\rightarrow i_L(0_-) = 360mA \ ; \ i_L(t \geq 0) = 360mA\,e^{-t/\tau}$

Current divider rule at node a

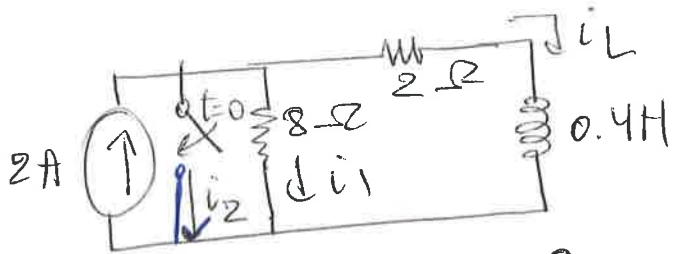
$i_1(t=0_-) = \frac{18}{90} = 200\,mA$ // $i_1(0_+) = \left[\frac{-(120+60)}{120+60+90}\right] i_L(0_+)$

abrupt change in current through resistor

$i_1(0_+) = -240\,mA \ ; \ i_1(t>0) = -240mA\,e^{-t/\tau}$

Practice 8.5 p272

At $t = 0.15s$, find i_L, i_1, i_2

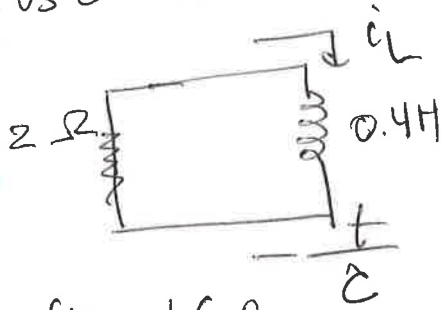


$t<0$ $\quad i_L(0_-) = \frac{8}{10} 2A = 1.6A$

$i_1(0_-) = \frac{2}{10} 2A = 0.4A$

$t \geq 0$ $\quad i_L(0_+) = 1.6A$

$t=0$ switch is closed $\Rightarrow v$ across 8Ω is zero.



Natural response

$i_L(t) = 1.6 e^{-\frac{t}{\tau}}$

$\tau = \frac{L}{R} = \frac{0.4}{2} = 0.2s$

$$\boxed{\bullet\ i_L(0.15s) = 1.6 e^{-\frac{0.15}{0.2}} = 0.756A}$$

$i_1(0_-) = 0.4A \Rightarrow i_1(0_+) = 0 \Rightarrow i_1(0.15) = 0$ (8 Ω resistor is shorted)

$i_2(0_+) = 0.4A$ the short takes on all the current which was flowing in 8 Ω resistor

$\bullet\ i_2(0.15s) + i_L(0.15s) = 2A\,!$

$i_2(0.15s) = 2 - i_L(0.15s)$

$= 1.244A$

$$\boxed{\begin{aligned} & i_L(+\infty) \Rightarrow 0 \\ & i_2(+\infty) = 2A \end{aligned}}$$

$i_1(t) = 0 \quad \forall t > 0$

$$\boxed{i_2(t) = 2 - i_L(t) = 2 - 1.6 e^{-t/\tau}}$$