overdamped parallel RLC circuit

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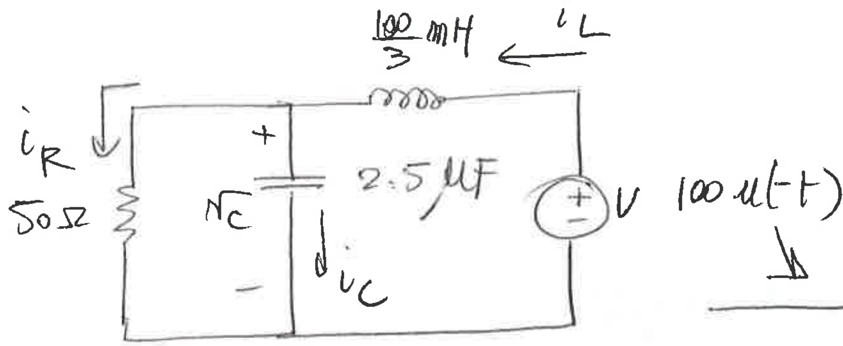
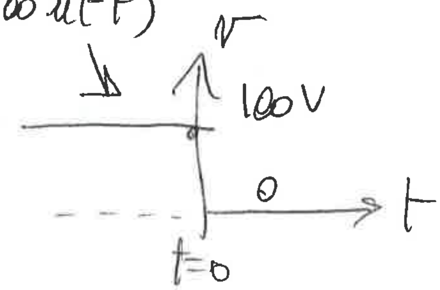
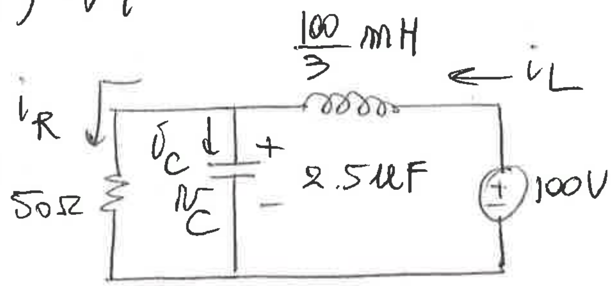




Find $i_L(t)\ \forall t$

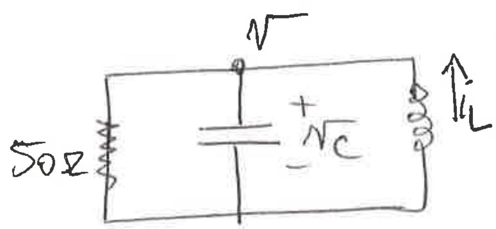
$t<0$



$\longrightarrow v_C(0_-) = 100V \qquad i_L(0) = i_R(0) = \frac{100}{50} = 2A$

$v_C(0_-) = v_C(0_+) = 100V$

$t>0$



$i_L(0_+) = 2A = i_L(0_-)$

$v_C(0_+) = 100V$

$$\boxed{C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0}$$

$v_C(t) = v(t)$?

$\alpha = \frac{1}{2RC} = 4000\ s^{-1}$ $\quad \alpha > \omega_0$

$\omega_0 = \frac{1}{\sqrt{LC}} = 3464\ s^{-1}$ $\quad$ overdamped

(over) →

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$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = \begin{cases} \text{---}\ s^{-1} \\ \text{---}\ s^{-1} \end{cases}$$

$$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_0^2}$$

$$i_L(t) = i_C(t) + i_R(t)$$

$$i_L(t) = C\frac{dv}{dt} + \frac{v(t)}{R} = C\left(s_1 A_1 e^{s_1 t} + s_2 A_2 e^{s_2 t}\right) + \frac{1}{R}\left(A_1 e^{s_1 t} + A_2 e^{s_2 t}\right)$$

$$v(t=0_+) = \boxed{A_1 + A_2 = 100}$$

$$\left[\frac{dv}{dt}\right]_{t=0_+} = s_1 A_1 + s_2 A_2$$

$$i_L(t=0_+) = 2A = 2.5\,10^{-6}\left(s_1 A_1 + s_2 A_2\right) + \frac{1}{50}\left(A_1 + A_2\right)$$

$$\cancel{2A} = 2.5\,10^{-6}\left(s_1 A_1 + s_2 A_2\right) + \cancel{2A}$$

$$\rightarrow \boxed{s_1 A_1 + s_2 A_2 = 0}$$

$$A_2 = 100 - A_1$$

$$s_1 A_1 + s_2(100 - A_1) = 0 \rightarrow (s_1 - s_2) A_1 = -100 s_2 =$$

$$\Rightarrow \begin{cases} A_1 = \dfrac{-100\left(-\alpha - \sqrt{\alpha^2 - \omega_0^2}\right)}{2\sqrt{\alpha^2 - \omega_0^2}} = 50\left(\dfrac{\alpha + \sqrt{\alpha^2 - \omega_0}}{\sqrt{\alpha^2 - \omega_0^2}}\right) \\ A_2 = 100 - A_1 \end{cases}$$

Underdamped Parallel RLC circuit

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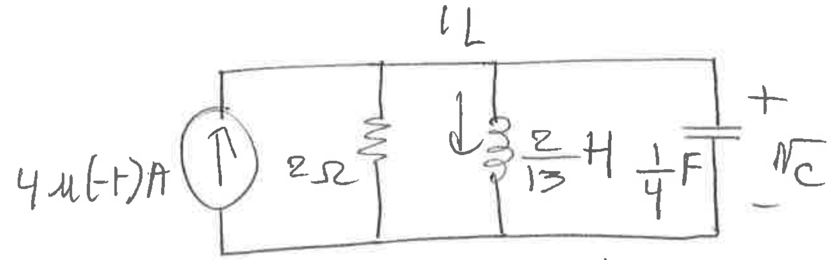
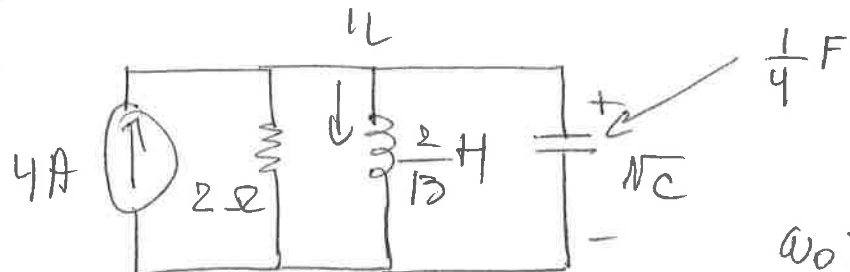


(a) $i_L(0_+)$; (b) $v_C(0_+)$; (c) $\left.\frac{di_L}{dt}\right]_{t=0_+}$

(d) $\left.\frac{dv_C}{dt}\right]_{t=0_-}$ (e) $v_C(t)$; sketch $v_C(t)$, $-0.1 < t < 2$ s.

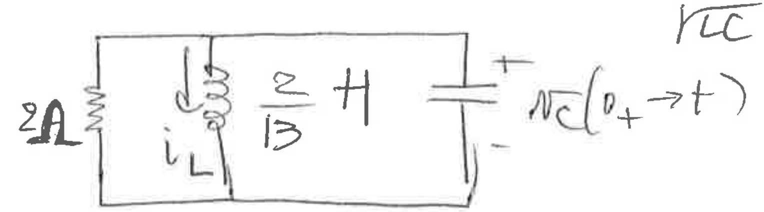
$t<0$



$$\omega_0 > \alpha = \frac{1}{2RC} = 1\ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5.1\ \text{rad/s}$$

$t>0$



$t<0$ $\quad i_L(0_-) = 4A$, $i_R(0_-) = 0 = i_C(0_-)$

(a) $\boxed{i_L(0_+) = 4A}$ $\quad v_C(0_-) = 0 \rightarrow \boxed{v_C(0_+) = 0}$ (b)

$\left.\frac{di_L}{dt}\right]_{0+}$? $\quad i_L(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 5\ \text{rad}\ s^{-1}$$

$$i_L(0+) = \boxed{4A = B_1} \longrightarrow i_L(t) = e^{-\alpha t}(4\cos\omega_d t + B_2 \sin\omega_d t)$$

$$v_L = v_C = L\frac{di_L}{dt} = L\left(-\alpha e^{-\alpha t} 4\cos\omega_d t - 4e^{-\alpha t}\omega_d \sin\omega_d t - \alpha e^{-\alpha t} B_2 \sin\omega_d t + B_2 e^{-\alpha t}\omega_d \cos\omega_d t\right)$$

$$v_C(t=0_+) = L\left(-4\alpha + B_2\omega_d\right)$$

$$v_C(t=0_+) = v_C(t=0_-) = 0$$

$$\longrightarrow \quad -4\alpha + B_2\omega_d = 0$$

$$\boxed{B_2 = \frac{4\alpha}{\omega_d}}$$

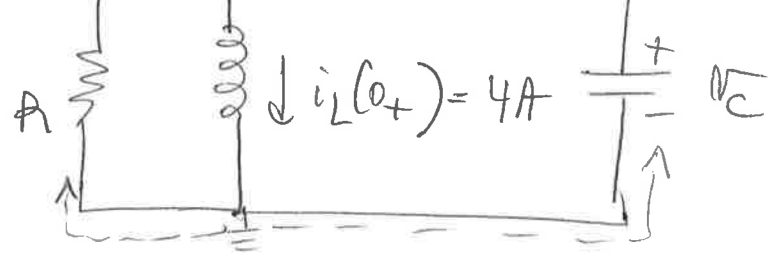
$$\left[\frac{dv_C}{dt}\right]_{t=0_-} = 0 \quad ; \quad v_C(t) = L\Big(-4\alpha e^{-\alpha t}\cos\omega_d t - 4\omega_d e^{-\alpha t}\sin\omega_d t - B_2\alpha e^{-\alpha t}\sin\omega_d t + B_2\omega_d e^{-\alpha t}\cos\omega_d t\Big)$$

$$v_C(t) = L e^{-\alpha t}\Big(\cancel{-4\alpha\cos\omega_d t} + \cancel{B_2\omega_d\cos\omega_d t} - 4\omega_d\sin\omega_d t - B_2\alpha\sin\omega_d t\Big)$$

$$v_C(t) = -L e^{-\alpha t}\Big(4\omega_d\sin\omega_d t + B_2\alpha\sin\omega_d t\Big)$$

$$v_C(t) = \ominus L e^{-\alpha t}\sin\omega_d t\left(4\omega_d + \frac{4\alpha^2}{\omega_d^2}\right)$$

why is it<0?



$\boxed{\text{v goes negative}}$

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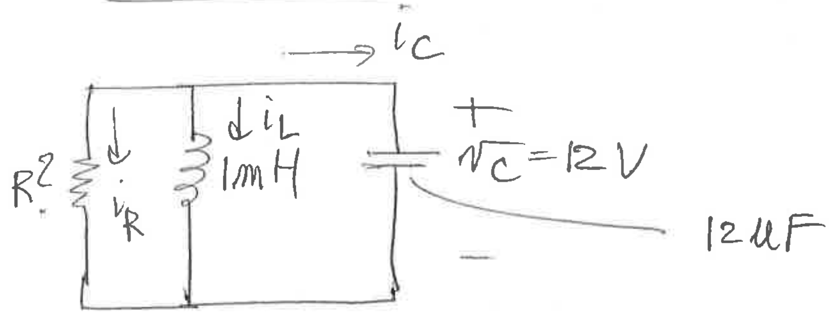
#24

A parallel RLC circuit is constructed using a 1mH inductor and a 12 uF capacitor.

(a) Select R such that the circuit response is critically damped.

(b) If $v_C(0_-) = 12V$ $i_L(0_-) = 0$, find $v_C(t)$ for $t > 0$.

How can this be?



(a) $\alpha = \omega_0 \longrightarrow \frac{1}{2RC} = \frac{1}{\sqrt{LC}} \Rightarrow R = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$

$$R = \frac{1}{2}\sqrt{\frac{10^{-3}}{12\,10^{-6}}} = \frac{1}{2}\sqrt{\frac{10^{3}}{12}}$$

$$\boxed{R_c = 4.56\,\Omega}$$

(b) critically damped $v(t) = e^{-\alpha t}(A_1 t + A_2)$

$$v_C(0_-) = 12V = v_C(0_+) = A_2$$

$$\longrightarrow \boxed{A_2 = 12V}$$

$i_L(0_-) = 0 \Rightarrow i_L(0_+) = 0A$ $\qquad i_R(0_-) = \frac{12V}{R_c} \simeq 2.63\,A$

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$$i_R(0+) + \underbrace{\cancel{i_L(0_+)}}_{=0} + i_C(0_+) = 0$$

$$i_C(0_+) = -i_R(0_+) = \frac{-12}{R_C}\ A = -2.63\,A$$

$$v(t) = e^{-\alpha t}(A_1 t + A_2)$$

$$i_C = C\frac{dv}{dt} = C\left[-\alpha e^{-\alpha t}(A_1 t + A_2) + e^{-\alpha t} A_1\right]$$

$$i_C(0_+) = C\left[-\alpha A_2 + A_1\right]$$

$$\| $$

$$\frac{-12}{R_C} = C\left[-\alpha\, 12 + A_1\right]$$

$$\longrightarrow \quad A_1 - 12\alpha = \frac{-12}{R_C C}$$

$$A_1 = 12\left[\alpha - \frac{1}{R_C C}\right] = 12\left[\frac{1}{2R_C C} - \frac{1}{R_C C}\right]$$

$$A_1 = \frac{-12}{2R_C C} = \frac{-6}{R_C C}$$

$$v_C(t) = e^{-\alpha t}\left(\frac{-6}{R_C C} t + 12\right)$$

$$12 - \frac{6t}{R_C C} = 0 \rightarrow t^* = \frac{12 R_C C}{6} = 2R_C C$$

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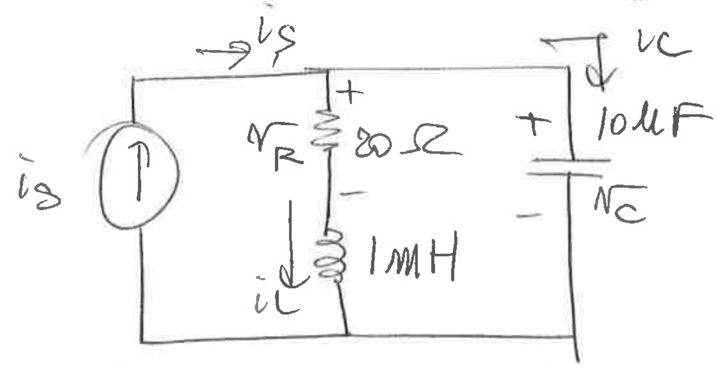
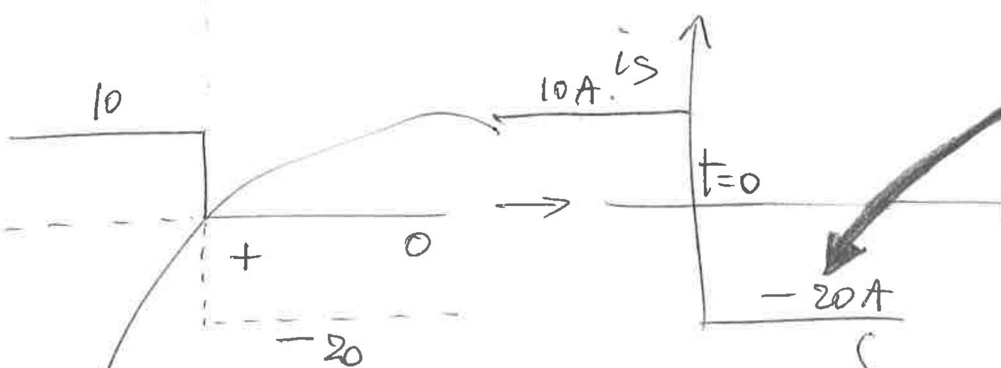


$$i_s = [10\,u(-t) - 20\,u(t)]\,A.$$

$i_L(t)$

(a) $i_L(0_-)$ (b) $v_C(0_+)$; (c) $v_R(0_+)$; (d) $i_{L,\infty}$, (e) $i_L(0.1\,ms)$?



we have a non-zero current source acting for t>0

$i_L(0_-) = 10\,A$!

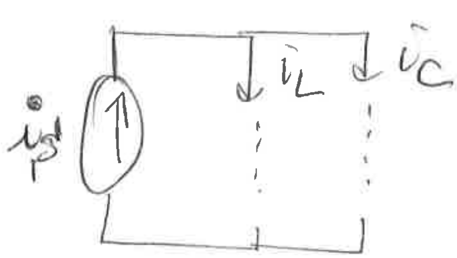
$v_C(0_+) = v_C(0_-) = 10A\ 20\Omega = 200\,V$

$v_R(0_+) = v_C(0_+)$ $\Leftarrow$ $i_L(0_+) = i_L(0_-)$
$= 200\,V$

$i_L(\infty) = -20A$

$$v_C(t) = v_R + L\frac{di_L}{dt}$$

$$v_C(t) = R\,i_L + L\frac{di_L}{dt}$$



9.30.

(1880)

$$i_L + i_C = i_g$$

$$i_C = C\frac{dv_C}{dt} = -20 - i_L \,; \quad v_C = R\,i_L + L\frac{di_L}{dt}$$

$$\frac{dv_C}{dt} = R\frac{di_L}{dt} + L\frac{d^2 i_L}{dt^2}$$

$$\frac{-20 - i_L}{C}$$

$$\boxed{L\frac{d^2 i_L}{dt^2} + R\frac{di_L}{dt} + \frac{i_L}{C} = \frac{-20}{C}}$$

$$L\frac{d^2 i_L}{dt^2} + R\frac{di_L}{dt} + \frac{i_L}{C} = 0$$ Natural response

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{20}{2\,10^{-3}} = 10^4\, s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{10^{-3}\,10\,10^{-6}}} = \frac{1}{\sqrt{10^{-8}}} = 10^4 \text{ rad } s^{-1}$$

$\alpha = \omega_0$ critically damped

$$i_L(t) = e^{-\alpha t}(A_1 t + A_2)$$

~~$t=0 \quad i_L(0^+) = i_L(0^-) = 10A = A_2$~~

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(18%)

~~$i_L(t) = e^{-\alpha t}(10 + A_1 t)$~~

~~$t \to \infty \quad i_L(\infty) = -20\,A$~~

~~$i_L = i_f + e^{-\alpha t}(10 + A_1 t)$~~

~~i_f?~~

Total response

$$\boxed{i_L(t) = i_f + e^{-\alpha t}(A_1 t + A_2)}$$

$t \to \infty \quad i_L(\infty) = i_f = -20\,A$

$i_L(0) = i_L(0_+) = 10 = -20 + A_2$

$$\boxed{A_2 = 30\,A}$$

A_1?

$i_L(t) = -20 + (30 + A_1 t)e^{-\alpha t}$

$v_C(t) = v_R(t) + L\dfrac{di_L(t)}{dt}$

$v_C(0_+) = v_R(0_+) = 200\,V \Rightarrow \left.\dfrac{di_L}{dt}\right]_{0+} = 0$

$\rightarrow \left(A_1 e^{-\alpha t} + (30 + A_1 t)(-\alpha)e^{-\alpha t}\right]_{t=0_+} = 0$

$A_1 - 30\alpha = 0$

$A_1 = 30\alpha$

$$\rightarrow \boxed{i_L(t) = -20 + 30(1 + \alpha t)e^{-\alpha t}} \rightarrow \underline{i_L(0.1\,ms) = \ldots}$$

Complete response

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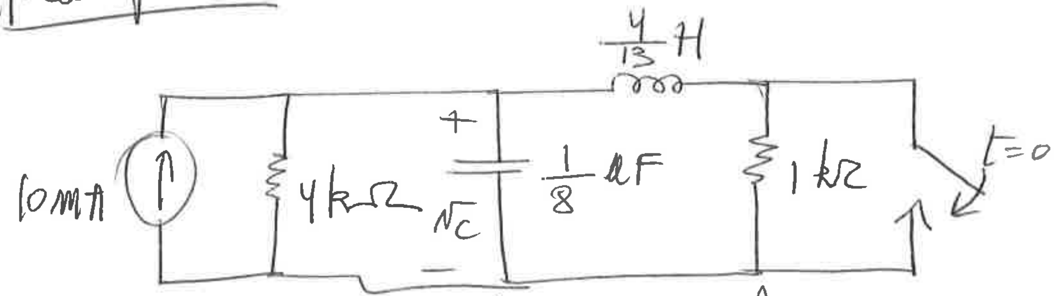
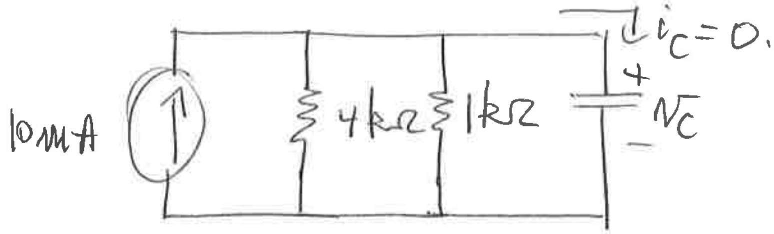


$v_C(t)$ for $t > 0$; sketch $v_C(t)$ for $-0.1 < t < 2$ ms

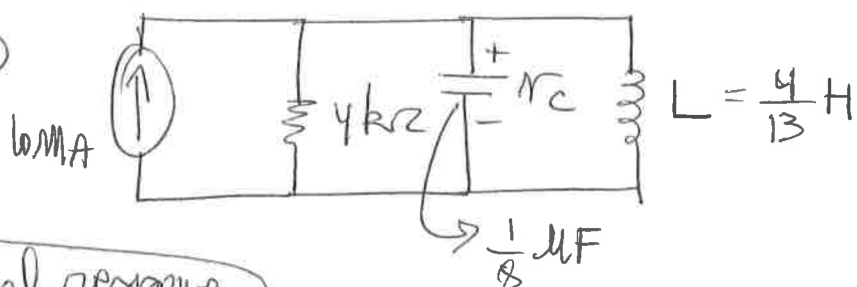
t<0



$$i_L(t=0) = \frac{4}{5} 10\,mA = 8\,mA$$

$$v_C = 10\,mA\,(4k\Omega // 1k\Omega) = 10\,mA\,\frac{4k\Omega}{5} = 8V$$

t>0



$$i_L(t=\infty) = 10\,mA$$

Natural response without 10 mA (R, L, C) // network

$$\alpha \gtreqless \omega_0 \qquad \alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 4\,10^{+3} \frac{1}{8} 10^{-6}} = 10^{3}\,s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{4}{13} \cdot \frac{10^{-6}}{8}}}$$

$$\omega_0 = \frac{10^3}{\sqrt{\frac{1}{26}}} = \sqrt{26}\,10^3\,rad\,s^{-1}$$

$$\omega_0 = 5.1\,10^3\,s^{-1}$$

$\omega_0 > \alpha$ underdamped.

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18P

So $v_C(t) = e^{-\alpha t}\left[B_1 \cos\omega_d t + B_2 \sin\omega_d t\right]$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(26-1)}\ 10^3$$

$$\omega_d = 5\ 10^3 \text{ rad s}^{-1}$$

Forced response v_f is response as $t \to \infty$

$t \to \infty$ $v_L = 0$ L is a short

10 mA goes through L

$$v_C(\infty) = v_f = 0$$

Total response $v_C(t) = v_f + e^{-\alpha t}\left[B_1 \cos\omega_d t + B_2 \sin\omega_d t\right]$

$$v_C(t=0_+) = v_C(t=0_-) = \boxed{8\,V = B_1}$$

B_2? $v_C = v = L \dfrac{di_L}{dt}$

$$i_L(t) = i_L(0_+) + \frac{1}{L}\int_0^t v_C(t')\,dt'$$

$$i_L(t) = i_L(0_-) + \frac{1}{L}\int_0^t v_C(t')\,dt'$$

$$v_C(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$$

$$\int_0^t v_C(t')\,dt' = ?$$

$$\int e^{az} \cos bz \, dz = \frac{e^{az}}{a^2+b^2}(a \cos bz + b \sin bz)$$

$$\int e^{az} \sin bz \, dz = \frac{e^{az}}{a^2+b^2}(a \sin bz - b \cos bz)$$

$$\int_0^{t\to\infty} v_C(t')\,dt' = B_1 \left[-\frac{a}{a^2+b^2} \right] + B_2 \left[-\left(\frac{-b}{a^2+b^2} \right) \right]$$

$$= \frac{bB_2 - aB_1}{a^2+b^2}$$

$$\rightarrow \quad \underset{\substack{\| \\ 10mA}}{i_L(t=\infty)} = \underset{\substack{\| \\ 8mA}}{i_L(0_-)} + \frac{1}{L}\left[\frac{bB_2 - aB_1}{a^2+b^2} \right]$$

$$\rightarrow \quad \left[\frac{bB_2 - aB_1}{a^2+b^2} \right] = L(2mA)$$

$\rightarrow$ $\boxed{\text{$B_1$ is known} \\ \text{Solve for } B_2}$

chapter 9

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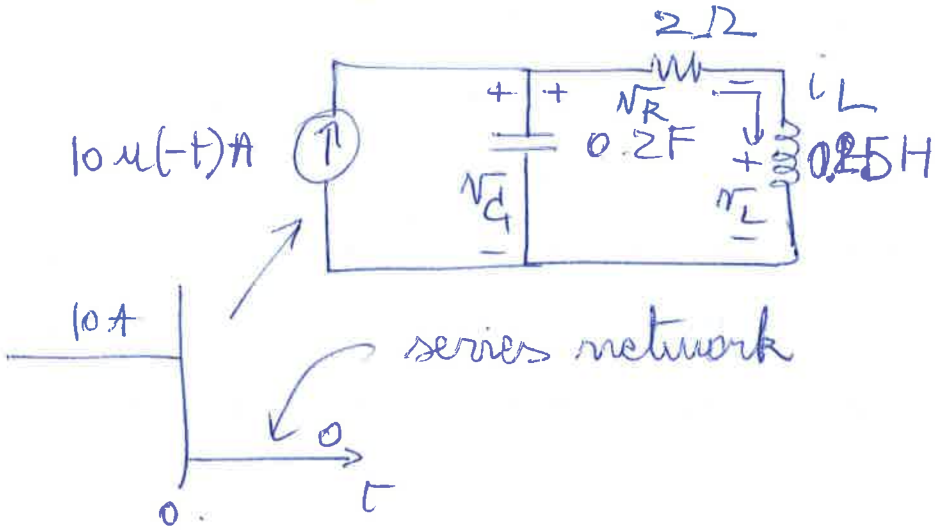
Find $i_L(t)$ for $t > 0$ in the circuit below



$$v_R + v_L = v_C$$

$$v_C(0^+) = v_R(0^+) + v_L(0^+)$$

$v_R(0^+) = v_R(0^-)$ because $i_L(0^-) = i_L(0^+)$

$$\alpha = \frac{R}{2L} = \frac{2}{1/2} = 4 \quad ; \quad \omega_0^2 = \frac{1}{LC} = \frac{4}{0.2} = 20$$

$\alpha^2 < \omega_0^2$

$\alpha < \omega_0$

$\alpha < \omega_0$ underdamped

$$\omega_d = \sqrt{20-16} = \sqrt{\omega_0^2 - \alpha^2} = 2$$

$$i_L = e^{-4t}(A_1 \cos 2t + A_2 \sin 2t)$$

$i_L(0) = 10A$, $v_C(0) = 20V$

$\Downarrow$

$A_1 = 10$

$$i_L'(0^+) = \frac{1}{L} v_L(0^+) = \frac{1}{L}(v_C(0^+) - v_R(0^+))$$

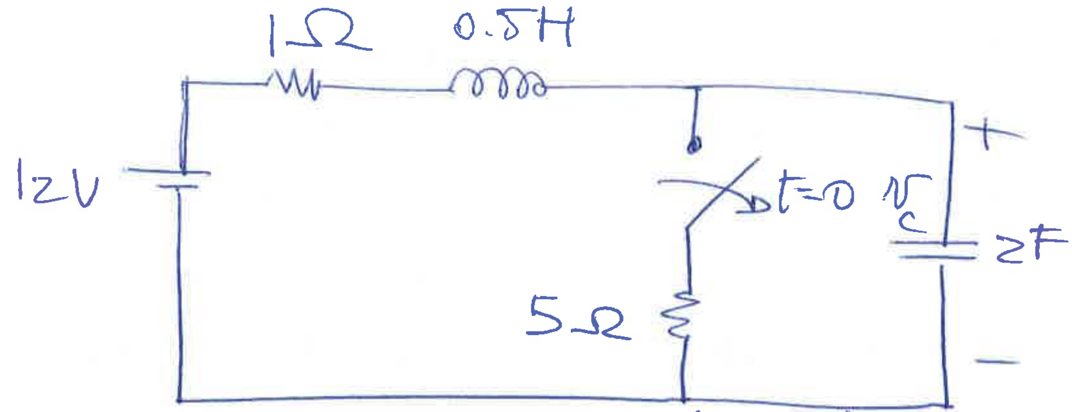
$$i_L'(0^+) = 4(20-20) = 0$$

$$i_L' = -4e^{-4t}(A_1 \cos 2t + A_2 \sin 2t) + e^{-4t}(-2A_1 \sin 2t + 2A_2 \cos 2t)$$

$$i_L'(0) = -4A_1 + 2A_2 = 2A_2 - 40 = 0 \Rightarrow A_2 = 20$$

So $\boxed{i_L(t) = e^{-4t}(10 \cos 2t + 20 \sin 2t)\ A, \quad t > 0}$

Problem 61



The switch in the circuit above has been closed for a long time. It opens at $t=0$. Find $v_C(t)$ for $t>0$.

$$\alpha = \frac{R}{2L} = \frac{1}{1} = 1 \;, \quad \omega_0^2 = \frac{1}{LC} = 1 \qquad \alpha = \omega_0$$

Critically damped

$$v_C(0^+) = v_C(0^-) = \frac{5}{6}\,12 = 10V \;, \quad i_L(0^-) = i_L(0^+) = 2A$$

Force response (12V battery)

$$v_C(t) = v_{Cf} + e^{-\alpha t}(A_1 t + A_2) = 12 + e^{-t}(A_1 t + A_2)$$

$$t \to \infty \quad v_{Cf} = 12$$

$$v_C(0^+) = 10V \Rightarrow A_2 = -2$$

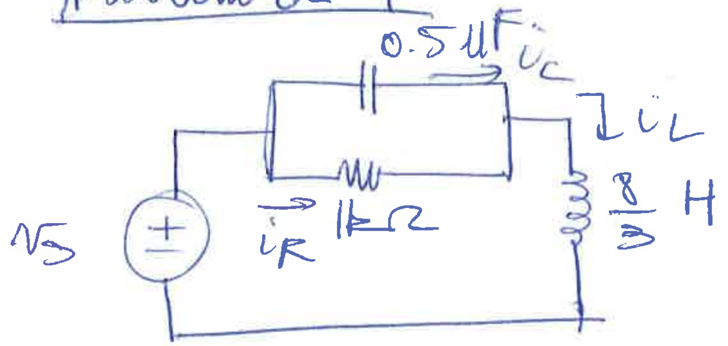
$$v_C'(0^+) = \frac{1}{C}\, i_C(0^+) = \frac{1}{2}\, i_L(0^+) = 1$$

$$v_C'(t) = -e^{-t}(A_1 t + A_2) + e^{-t} A_1 \Big|_{t=0^+} = A_1 - A_2 = A_1 + 2 = 1$$

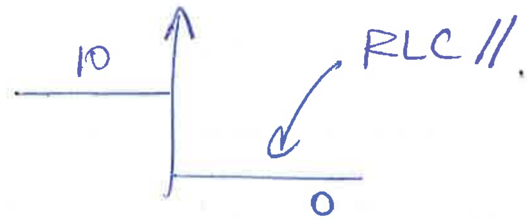
$$\Rightarrow A_1 = -1$$

$$\boxed{v_C(t) = 12 - e^{-t}(t+2)\,,\ t>0}$$

Problem 62



a) $i_R(t)$ $t>0$ if $v_S(t) = 10u(-t)$



$$\alpha = \frac{1}{2RC} = \frac{10^6}{2000 \times 0.5} = 10^3$$

$\alpha > \omega_0$ $\quad \omega_0^2 = \frac{1}{LC} = 0.75\,10^6 \rightarrow s_{1,2} = -500, -1500$

overdamped

$$v_C = A_1 e^{-500t} + A_2 e^{-1500t}$$

$v_C(0^-) = 10V$, $i_L(0^-) = 10mA$

$v_C(0^+) = A_1 + A_2 = 10$ (1) $\quad i_R(0^-)$ because $v_C(0^+) = v_C(0^-)$

$$v_C' = \frac{1}{C} i_C = 2\,10^6 \left[i_L(0^+) - i_R(0^+) \right] = 2\,10^6 [10mA - 10mA] = 0$$

$$v_C' = -500 A_1 e^{-500t} - 1500 A_2 e^{-1500t}$$

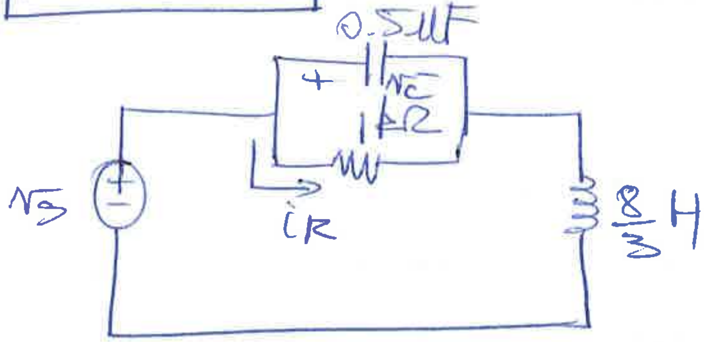
$$v_C'(0^+) = -500 A_1 - 1500 A_2 = 0.$$

$$-A_1 - 3A_2 = 0 \quad (2)$$

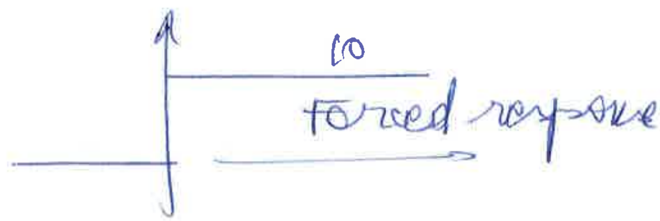
(1) & (2) $\rightarrow$ $\boxed{A_1 = 15 , A_2 = -5}$

$v_C(t) = (15e^{-500t} - 5e^{-1500t})\,V \rightarrow$ $\boxed{i_R = (15e^{-500t} - 5e^{-1500t})\ mA}$

Problem 62b



$v_S(t) = 10\,u(t)$



$V_{C,f} = 10V$

$\longrightarrow v_C = V_{C,f} + A_1 e^{-500t} + A_2 e^{-1500t}$

$v_C(0^+) = v_C(0^-) = 0 \qquad i_L(0) = 0.$

$A_1 + A_2 = -10V \quad (1)$

0 because $v_C(0^+) = v_C(0^-)$

$v_C'(0^+) = \frac{1}{C} i_C(0^+) = 2\cdot 10^6 \left(\underset{i_L(0^-)=0}{i_L(0^+)} - i_R(0^+) \right) = 0$

$v_C' \Big|_{0^+} = -500A_1 - 1500A_2 = 0 \quad (2)$

$\longrightarrow \boxed{\begin{aligned} A_1 + 3A_2 &= 0. \\ A_1 + A_2 &= -10 \end{aligned}} \begin{matrix} (2') \\ (1) \end{matrix}$

$\longrightarrow A_1 = -15 \quad A_2 = 5$

$$\boxed{v_C(t) = 10 - 15e^{-500t} + 5e^{-1500t} \quad t > 0}$$

$$\boxed{\therefore\ i_R(t) = 10 - 15e^{-500t} + 5e^{-1500t}\ mA;\ t > 0}$$