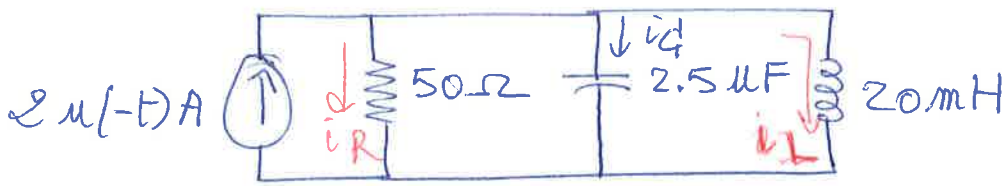
7. (25 pts) Find $i_C(t)$ for $t > 0$ in the circuit shown below.



$$\alpha = \frac{1}{2RC} = \frac{10^6}{100 \times 2.5} = 4000$$

$$\omega_0^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 2 \cdot 10^7$$

$$\omega_d = 2000$$

$$i_C = e^{-4000t}\left(B_1 \cos 2000t + B_2 \sin 2000t\right)$$

$i_L(0) = 2A$, $v_C(0) = 0 \rightarrow i_C(0^+) = -2A = B_1$

$$i_C'(0^+) = -i_L'(0^+) - i_R'(0^+)$$

$i_C(0^-) = 0$

$$i_C'(0^+) = -\frac{1}{L} v_C(0) - \frac{1}{R} v_C'(0^+) = 0 - \frac{1}{RC} i_C(0^+) = \frac{2 \cdot 10^6}{125}$$

$$\rightarrow B_1 = -2A; \quad \frac{2 \cdot 10^6}{125} = 16{,}000 = 2000 B_2 - 2(-4000)$$

$$\longrightarrow B_2 = 4$$

So $\boxed{i_C(t) = e^{-4000t}\left(-2\cos 2000t + 4 \sin 2000t\right), \; t > 0}$

nd solution for #7

$t > 0$

$\alpha = 4000 \qquad \omega_0^2 = 2\,10^7$

$\omega_d = 2000$

$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

$t < 0 \quad v_C(0-) = v_C(0+) = 0 \rightarrow B_1 = 0$

$v(t) = e^{-\alpha t} B_2 \sin \omega_d t$

$v'(t) = -\alpha e^{-\alpha t} B_2 \sin \omega_d t + B_2 \omega_d e^{-\alpha t} \cos \omega_d t$

$v'(0) = B_2 \omega_d$

$i_C = C v'(0) = C B_2 \omega_d$

$i_C(0_+) = -2A \rightarrow B_2 = \frac{-2}{C\omega_d}$

$i_L(0+) = 2A$

$v_C(0+) = 0$

$v(t) = \frac{-2}{C\omega_d} e^{-\alpha t} \sin \omega_d t$

$v'(t) = \frac{2\alpha e^{-\alpha t}}{C\omega_d} \sin \omega_d t - \frac{2}{C\omega_d} e^{-\alpha t} \omega_d \cos \omega_d t$

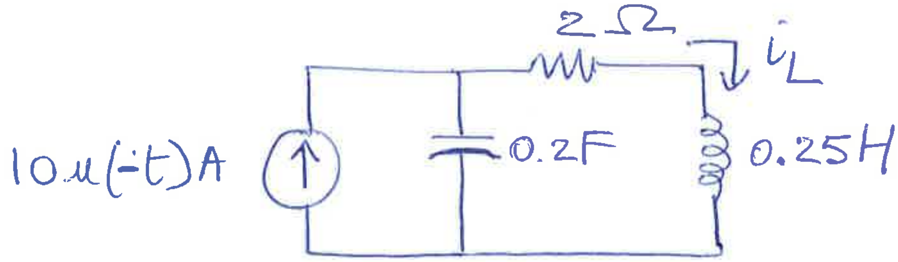
$= \frac{2}{C} \frac{4000}{2000} \sin \omega_d t\, e^{-\alpha t} - 2 \cos \omega_d t\, e^{-\alpha t}$

$v'(t) = (4 \sin 2000t - 2 \cos 2000t) \frac{e^{-\alpha t}}{C}$

OK.

agree

8. (25 pts) Find $i_L(t)$ for $t > 0$ in the circuit shown below.



$$\alpha = \frac{R}{2L} = \frac{2}{1/2} = 4 \rightarrow \alpha^2 = 16$$

$$\omega_0^2 = \frac{1}{LC} = 20 \;, \quad \omega_d = \sqrt{20-16} = 2$$

$$i_L = e^{-4t}(A_1 \cos 2t + A_2 \sin 2t)$$

$$i_L(0) = 10A \,, \quad v_C(0) = 20V$$

$$A_1 = 10A \,; \quad i_L'(0^+) = \frac{1}{L} v_L(0^+) = 4(20-20) = 0$$

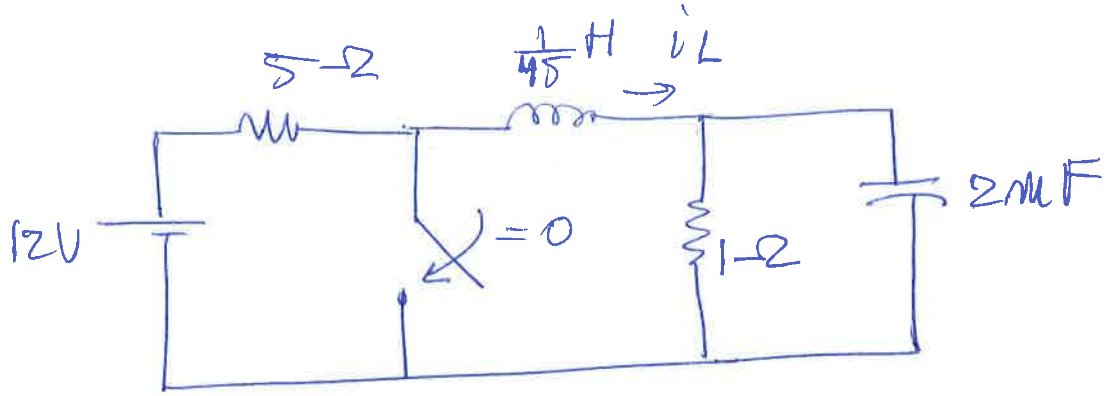
$$i_L'(0^+) = 2A_2 - 4 \times 10 = 0 \Rightarrow A_2 = 20$$

So

$$\boxed{i_L(t) = e^{-4t}(10 \cos 2t + 20 \sin 2t)A \,; \; t > 0}$$

#17

Find $i_L(t)$ for $t \geq 0$ in circuit below



$$i_L(0) = \frac{12}{5+1} = 2A, \quad v_C(0) = 2V$$

$$\alpha = 250, \quad \omega_0^2 = 22500$$

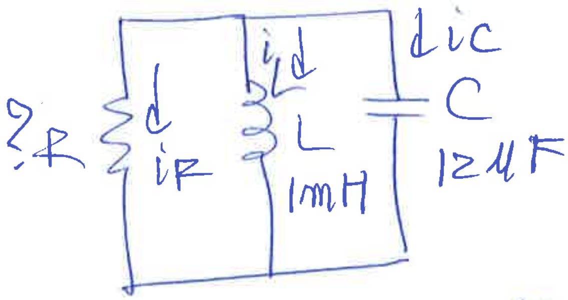
$$s_{1,2} = -50, -450 \ s^{-1}$$

$$i_L = A_1 e^{-50t} + A_2 e^{-450t}$$

$$A_1 + A_2 = 2 \qquad i_L'(0+) = \underbrace{45(-2) = -50A_1 - 450A_2}_{A_1 + 9A_2 = 1.8}$$

$$\rightarrow A_1 = 2.025$$
$$A_2 = -0.025$$

$$i_L(t) = 2.025e^{-50t} - 0.025e^{-450t} \ A, \ t \geq 0$$



R so that circuit is critically damped.
If $v_C(0^-) = 12V$ $\quad i_L(0^-) = 0 \rightarrow v_C(t)\ \forall t \geq 0$

critically $\quad \alpha = \omega_0 = \frac{1}{2RC} = \frac{1}{\sqrt{LC}} \rightarrow R_C = 4.564\ \Omega$

$\alpha = 9.129\ 10^3\ s^{-1}$

$$v(t) = e^{-\alpha t}(A_1 t + A_2)$$

$t=0 \quad v = A_1 0 + \boxed{A_2 = 12}$

$$\frac{dv}{dt} = e^{-\alpha t}[A_1) - \alpha e^{-\alpha t}(A_1 t + A_2)$$

$$e^{-\alpha t}[-\alpha A_1 t + A_1 - \alpha A_2)$$

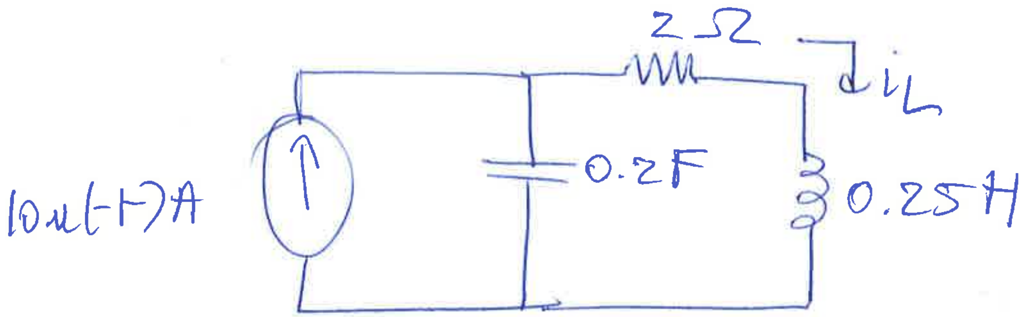
$$i_C = -(i_R + i_L)$$

$$A_1 - \alpha 12 = \left.\frac{dv}{dt}\right|_{t=0} = -\frac{1}{C}\left(\frac{v_C(0)}{R} + 0\right) = -\frac{1}{12\ 10^{-6}}\left(\frac{12}{4.565} + 0\right)$$

$$\boxed{A_1 = -109.6\ 10^3\ V}$$

$$\boxed{v(t) = e^{-\alpha t}(12 - 109.6\ 10^3 t)}$$

Find $i_L(t)$ for $t>0$ in the circuit below



$\alpha = \frac{R}{2L} = 4$ $\quad \omega_0^2 = 20$ $\quad \omega_d = \sqrt{20-16} = 2$

$i_L = e^{-4t}(A_1 \cos 2t + A_2 \sin 2t)$

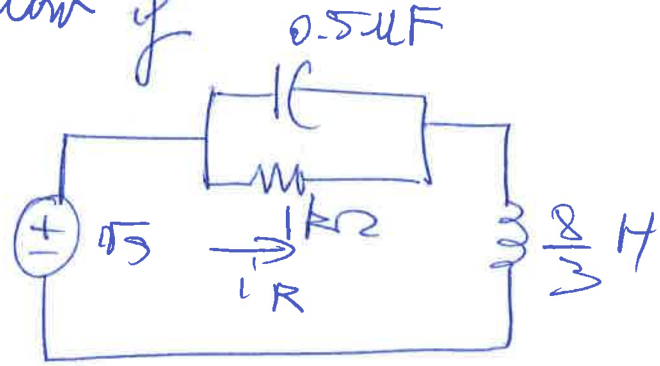
$i_L(0) = 10$, $v_C(0) = 20$

$A_1 = 10$ $\quad i_L'(0^+) = \frac{1}{L} v_L(0^+) = 4(20-20) = 0$

$i_L'(0^+) = 2A_2 - 4 \times 10 = 0 \rightarrow A_2 = 20$

$$\boxed{i_L(t) = e^{-4t}(10\cos 2t + 20 \sin 2t)}$$

Find $i_R(t)$ in the circuit below if



a $v_s(t) = 10\,u(-t)$

$$\alpha = \frac{1}{2RC} = 1000$$

$$\omega_0^2 = 0.75\,10^6$$

$$\alpha > \omega_0$$

$$s_{1,2} = -500, -1500$$

$$v_C(t) = A_1 e^{-500t} + A_2 e^{-1500t}$$

$$\boxed{\begin{matrix} v_0(0) = 10\,V \\ \| \\ A_1 + A_2 \end{matrix}}\quad , \; i_L(0) = 10\,mA$$

$$v_C'(0^+) = 2\,10^6\left[i_L(0) - i_R(0^+)\right]$$

$$= 2\,10^6\left[0.01 - \frac{10}{1000}\right] = 0$$

$$-500A_1 - 1500A_2 = 0$$

$$\boxed{A_1 + 3A_2 = 0}$$

$$\boxed{A_1 + A_2 = 10}$$

$$\longrightarrow v_C(t) = 15e^{-500t} - 5e^{-1500t}$$

$$i_R(t) = \left(15e^{-500t} - 5e^{-1500t}\right) mA$$

(b) $v_s(t) = 10u(t)$

$v_{cf} = 10$

$v_C = 10 + A_3 e^{-500t} + A_4 e^{-1500t}$

$v_C(0) = 0$, $i_L(0) = 0$

$\to A_3 + A_4 = -10$

$v_C'(0^+) = 2\,10^6 \left[i_L(0) - i_R(0^+) \right]$

$= 2\,10^6 [0 - 0] = 0$

$= -500 A_3 - 1500 A_4$

$\longrightarrow A_3 = -15 \quad A_4 = 5$

$v_C(t) = 10 - 15 e^{-500t} + 5 e^{-1500t}, \; t > 0$

$$\boxed{i_R(t) = 10 - 15 e^{-500t} + 5 e^{-1500t} \text{ mA}}$$