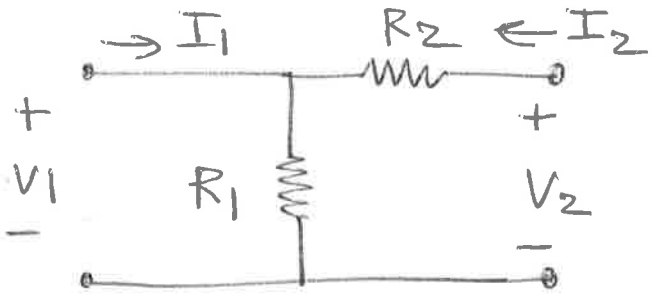
Homework #8 Solution

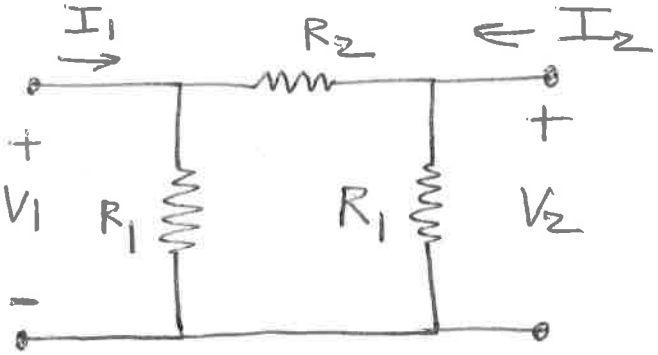
Due Thursday Feb. 2, 2012

Calculate ALL y, z, g, and h parameters for the following two networks

Network 1

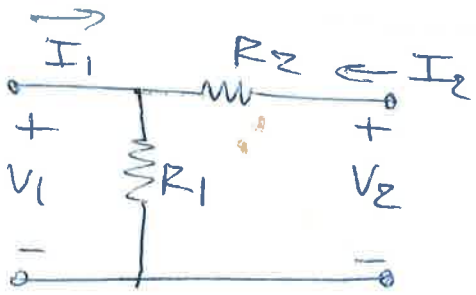


Network 2



10 pts for each set of parameters
40 Total

Small Signal parameters
2 port Network



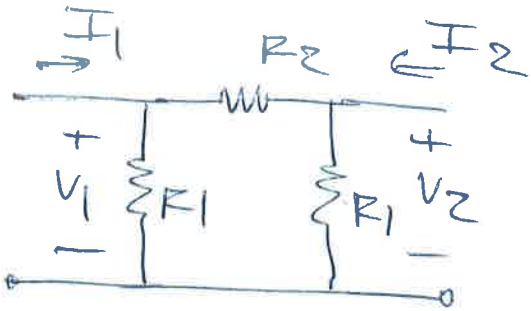
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R_1$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = R_1 + R_2$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = R_1$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = R_1$$



$$z_{11} = R_1 // (R_1 + R_2)$$

By symmetry

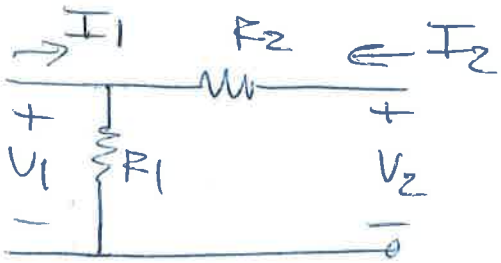
$$z_{22} = z_{11} = R_1 // (R_1 + R_2)$$

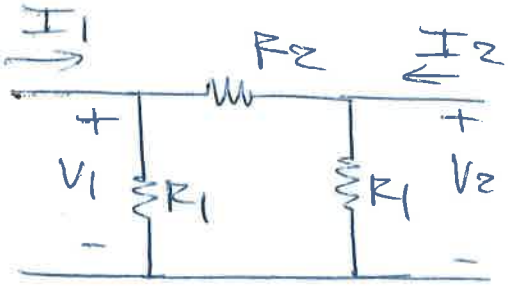
$$z_{12} = R_1 \underbrace{I'}_{\text{current divider}} = R_1 \frac{R_1}{2R_1 + R_2}$$

$$z_{12} = \frac{R_1^2}{2R_1 + R_2}$$

$z_{21} = z_{12}$ by symmetry.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$





$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{(R_1 \| R_2)}$$

$$y_{11} = y_{22} = \frac{1}{(R_1 \| R_2)}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\left(\frac{I_2}{V_2}\right)_{V_1=0} = \frac{-1}{R_2} \qquad (I_1 = -I_2)$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$I_1 = -I' = -\frac{R_1}{R_1+R_2} \cdot I_2$$

$$V_2 = I_2 (R_1 \| R_2)$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-1}{R_2}$$

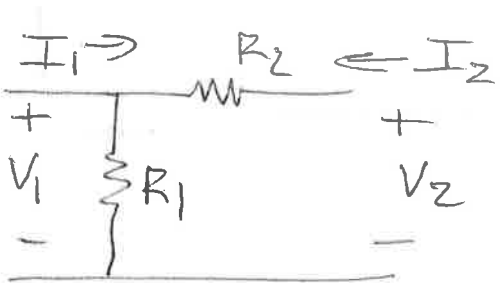
$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{R_2}$$

So $$y_{12} = y_{21} = \frac{-R_1}{R_1+R_2} \frac{1}{(R_1 \| R_2)}$$

$$y_{12} = y_{21} = \frac{-R_1}{R_1+R_2} \frac{R_1+R_2}{R_1 R_2}$$

$$y_{12} = \frac{-1}{R_2} = y_{21}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

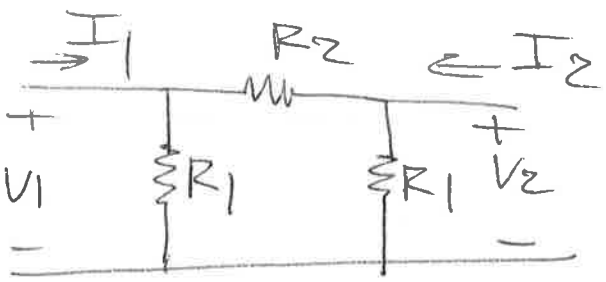


$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{1}{R_1}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = -1$$

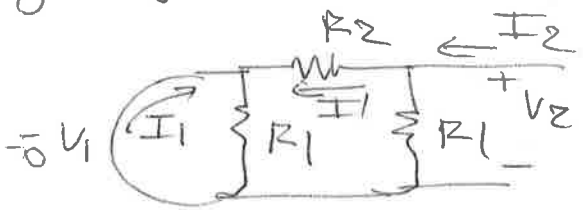
$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = +1$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = R_2$$



$$g_{11} = g_{22} = \frac{1}{R_1 /\!/ (R_1 + R_2)}$$

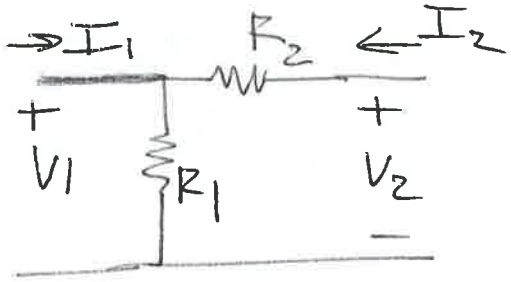
$g_{12} = g_{21} = ?$



$$I_1 = -I' = -\frac{R_1}{R_1 + R_2} I_2$$

$$\rightarrow \begin{matrix} g_{12} \\ \| \\ g_{21} \end{matrix} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{-R_1}{R_1 + R_2}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

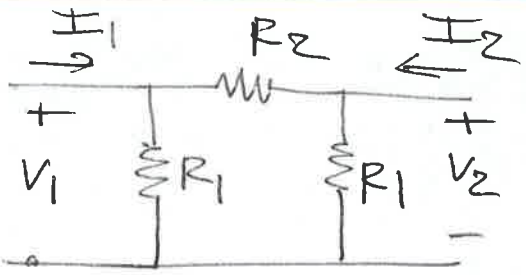


$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = (R_1 // R_2)$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{R_1}{R_1+R_2}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{-R_1}{R_1+R_2}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{R_1+R_2}$$



$$h_{11} = h_{22} = (R_1 // R_2)$$

by symmetry

$$h_{12} = h_{21} = \frac{R_1}{R_1+R_2}$$