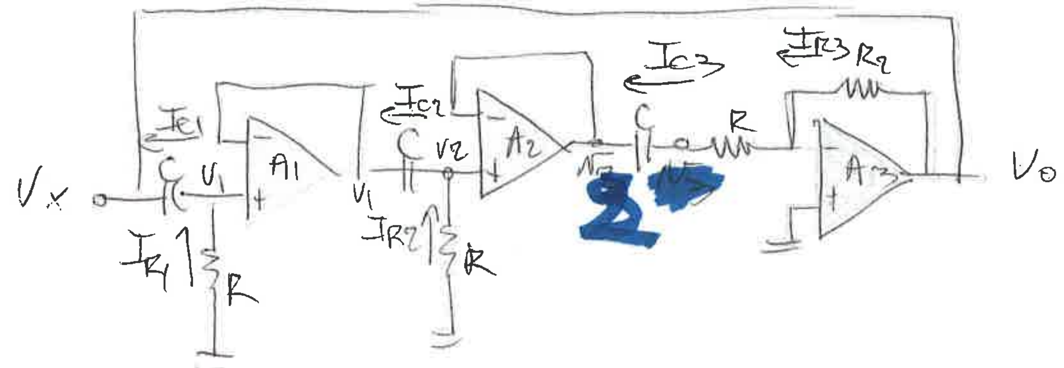
ECES 352 Electronics II
Assignment # 16
Due Monday February 26, 2007

Name:__SOLUTION_____

1. Consider the following phase shift oscillator where the op amps can be assumed to be **ideal**. Determine an expression for the loop gain L(s) = V_o / V_x in terms of the resistances R and R_2, the capacitances C (all the same size) and the s (=jω). For the intermediate currents and voltages, use the terminology shown on the figure. Hint: Write the voltage gain V_o / V_x as a set of three ratios, one for each op amp stage. Derive an expression for each voltage ratio from the circuit in terms of the R's and C. Determine the oscillation frequency ω_o in terms of the resistances and capacitance. Find the size of R_2 needed relative to R for oscillation.



For A_3 — Ideal op amps so $I_- = I_+ = 0$

So $I_{R_3} = I_{C_3} = \frac{v_o}{R_2}$ — for A_3 $v_- = v_+ = 0$

$$v_2 = 0 - (R + Z_c) I_{R_3} = -\frac{v_o}{R_2}\left(R + \frac{1}{sC}\right)$$

$$\frac{v_2}{v_o} = -\frac{sCR+1}{sCR_2} \quad \text{so} \quad \frac{v_o}{v_2} = \frac{-sCR_2}{1+sCR}$$

For A_2

$$I_{R_2} = -\frac{v_2}{R} = I_{c_2} = \frac{v_2 - v_1}{Z_c} = sC(v_2 - v_1)$$

so $v_2\left(-\frac{1}{R} - sC\right) = -sC\,v_1$

$$\frac{v_2}{v_1} = \frac{sCR}{1+sCR}$$

For A_1

$$I_{R_1} = -\frac{v_1}{R} = I_{c_1} = \frac{v_1 - v_x}{Z_c} = sC(v_1 - v_x)$$

$$v_1\left(-\frac{1}{R} - sC\right) = -sC\,v_x$$

$$\frac{v_1}{v_x} = \frac{sCR}{1+sCR}$$

$$\frac{v_o}{v_x} = \left(\frac{v_o}{v_2}\right)\left(\frac{v_2}{v_1}\right)\left(\frac{v_1}{v_x}\right)$$

So combining the three results we get

$$\frac{v_o}{v_x} = \left(\frac{-sCR_2}{1+sCR}\right)\left(\frac{sCR}{1+sCR}\right)\left(\frac{sCR}{1+sCR}\right)$$

$$= \frac{-s^3C^3R_2R^2}{(1+sCR)^3}$$

$$= \frac{-s^3C^3R_2R^2}{(1+2sCR+s^2C^2R^2)(1+sCR)}$$

$$= \frac{-s^3C^3R_2R^2}{1+3sCR+3s^2C^2R^2+s^3C^3R^3}$$

Assignment # 12 (continued)

Putting in $s = j\omega$ and simplifying we get

$$\frac{v_o}{v_x} = \frac{-(+j)^3\omega^3 C^3 R_2 R^2}{1 + 3j\omega CR - 3\omega^2 C^2 R^2 - j\omega^3 C^3 R^3} = \frac{j\omega^3 C^3 R_2 R^2}{\underbrace{(1 - 3\omega^2 C^2 R^2)} + j(3\omega CR - \omega^3 C^3 R^3)}$$

To get this to go to one we need this term to go to zero, so

$$1 - 3\omega^2 C^2 R^2 = 0 \Rightarrow \omega_o = \frac{1}{\sqrt{3}RC}$$

Then at $\omega = \omega_o$ we have

$$\left.\frac{v_o}{v_x}\right|_{\omega=\omega_o} = \frac{\omega_o^3 C^3 R_2 R^2}{3\omega_o CR - \omega_o^3 C^3 R^3} = \frac{\omega_o^2 C^2 R_2 R}{3 - \omega_o^2 C^2 R^2} = 1$$

$$\omega_o^2 C^2 R_2 R = 3 - \omega_o^2 C^2 R^2$$

$$\cancel{C^2} R_2 \cancel{R} \frac{1}{3R^{\cancel{2}}\cancel{C^2}} = 3 - \cancel{C^2 R^2}\frac{1}{3\cancel{R^2 C^2}}$$

$$\frac{R_2}{3R} = 3 - \frac{1}{3} = \frac{8}{3}$$

so $\boxed{\frac{R_2}{R} = 8}$

$$\frac{\omega^3 C^3 R_2 R^2}{(3\omega CR - \omega^3 C^3 R^3) + j(3\omega^2 C^2 R^2 - 1)}$$

Test 3 - Spring 2003 (100pts max) - M. Cahay
Name: SOLUTION

ELECTRONICS II
May 27, 2003

I. (20 pts): A second order filter has its poles at

$$s = -\frac{1}{4} \pm j\frac{\sqrt{8}}{4}.$$

The transfer function is zero at ω = 3 rad/s and is equal to 4 at DC.

Find the analytical expression for the transfer function T(s).

$$T(s) = k\,\frac{(s+j3)(s-j3)}{\left(s+\frac{1}{4}+j\frac{\sqrt{8}}{4}\right)\left(s+\frac{1}{4}-j\frac{\sqrt{8}}{4}\right)}$$

$$T(s) = \frac{k\,(s^2+9)}{\left(s+\frac{1}{4}\right)^2 + \left(\frac{\sqrt{8}}{4}\right)^2} = k\,\frac{s^2+9}{s^2+\frac{s}{2}+\frac{1}{16}+\frac{8}{16}}$$

$$T(s) = k\,\frac{s^2+9}{s^2+\frac{s}{2}+\frac{9}{16}} = \frac{16k\,(s^2+9)}{[16s^2+8s+9]}$$

as $s \to 0$ (DC)

$$T(0) = 16k = 4$$

$$\Rightarrow \boxed{k = 1/4}$$

so $$T(s) = \frac{1}{4}\,\frac{(s^2+9)}{\left[s^2+\frac{s}{2}+\frac{9}{16}\right]}$$

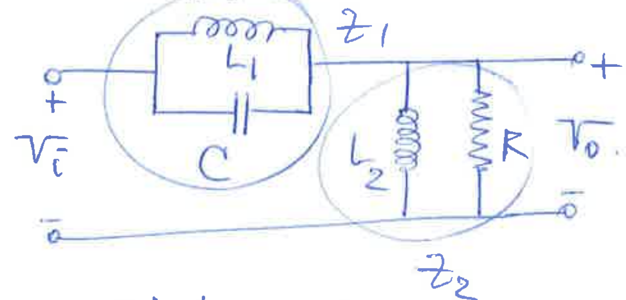
IV. (30 pts): The filter below is said to be of the high-pass notch type, i.e,

$$T(s) = a_2 \frac{s^2 + \omega_n{}^2}{s^2 + s(\frac{\omega_o}{Q}) + \omega_o{}^2} \tag{4}$$

with ω_n lesser than ω_o.

Derive the experssion for T(s) from the circuit diagram and give analytical expressions for ω_n, ω_o and Q in terms of R, L_1, L_2, and C.

The notch in the transfer function occurs at the angular frequency ω_n (see attached table for the second-order high-pass notch filter. For what ratio of L_1/L_2 does ω_n = 0.9 ω_o ?



$$T(s) = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{Rj\omega L_2}{R+j\omega L_2}}{\left[\frac{j\omega L_1 \frac{1}{j\omega C}}{j\omega L_1 + \frac{1}{j\omega C}}\right] + \left(\frac{Rj\omega L_2}{R+j\omega L_2}\right)} = \frac{\frac{sRL_2}{R+sRL_2}}{\frac{\left(\frac{L_1}{C}\right)(R+sRL_2) + RsL_2\left(sL_1+\frac{1}{sC}\right)}{\left(sL_1+\frac{1}{sC}\right)(R+sL_2)}}$$

$$T(s) = \frac{sRL_2\left(sL_1 + \frac{1}{sC}\right)}{\frac{L_1}{C}R + s\frac{RL_1L_2}{C} + s^2L_1L_2R + R\frac{L_2}{C}}$$

$$T(s) = \frac{s^2L_1L_2R + \frac{RL_2}{C}}{s^2L_1L_2R + s\frac{RL_1L_2}{C} + \frac{R(L_1+L_2)}{C}}$$

$$\Longrightarrow T(s) = \frac{s^2 + \frac{1}{L_1C}}{s^2 + \frac{s}{C} + \left(\frac{L_1+L_2}{L_1L_2}\right)\frac{1}{C}} = \frac{s^2 + \frac{1}{L_1C}}{s^2 + \frac{s}{C} + \frac{1}{(L_1/\!/L_2)C_1}}$$

so notch is at $\omega_n^2 = \frac{1}{L_1C} \rightarrow \omega_n = \frac{1}{\sqrt{L_1C}}$ $\quad a_2 = 1$

$\omega_o = \frac{1}{\sqrt{(L_1/\!/L_2)C_1}}$

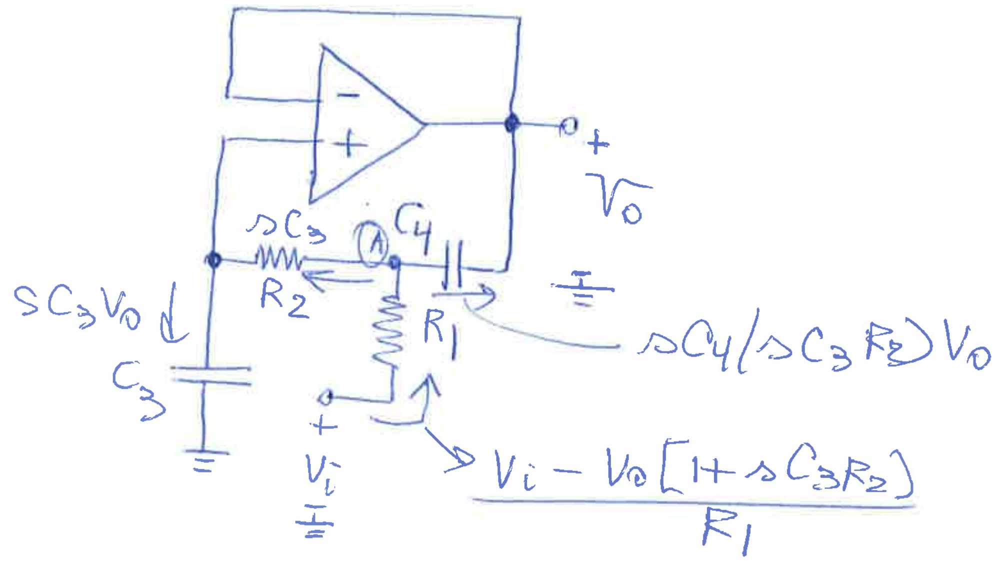
$\omega_n = \frac{1}{\sqrt{L_1C_1}}$ $\Longrightarrow \omega_o^2 = \left(\frac{L_1+L_2}{L_2}\right)\omega_n^2$ for $\omega_n^2 = (0.9\omega_o)^2$

$\longrightarrow \boxed{\frac{L_1}{L_2} = 0.2346}$

III. (30 pts): Consider the circuit below in which the op-amp is assumed to be ideal (i.e, infinite input resistance looking into the positive and negative terminals). Show that the transfer function for that filter T(s) = V_o/V_i is of second order and can be expressed as follows

$$T(s) = \frac{a_0}{s^2 + (\frac{\omega_o}{Q})s + \omega_o{}^2} \tag{3}$$

Give analytical expressions for ω_o and Q in terms of R_1, R_2, and C_3, C_4.



Potential at node (A) $V_o(1+sC_3R_2)$

Node equation at (A)

$$\frac{V_i - V_o(1+sC_3R_2)}{R_1} = sC_3V_o + s^2C_3C_4R_2V_o$$

$$\rightarrow \frac{V_i}{R_1} = V_o\left[s^2C_3C_4R_2 + sC_3 + \frac{sC_3R_2}{R_1} + \frac{1}{R_1}\right]$$

$$\text{So } \frac{V_o}{V_i} = \frac{1/C_3C_4R_1R_2}{s^2 + s\frac{1}{R_2C_4}\left(1+\frac{R_2}{R_1}\right) + \frac{1}{C_3C_4R_1R_2}}$$

$$\Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{C_3C_4R_1R_2}}}$$

$$\boxed{Q = \frac{1}{\sqrt{C_3C_4R_1R_2}} \; C_4 \; \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$