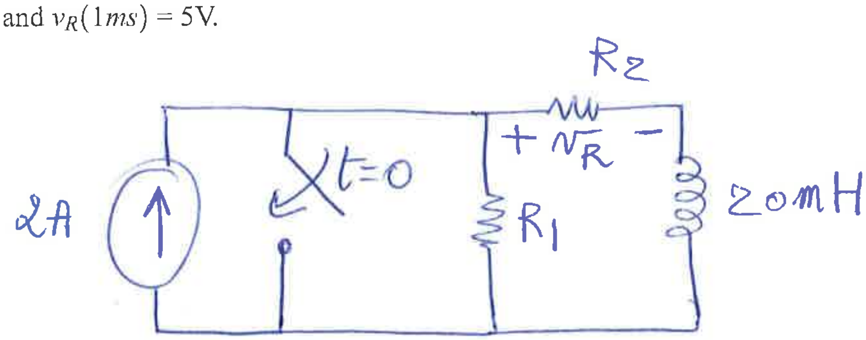
Final - Fall 2010 (200 pts max) - M. Cahay
Name:

NETWORK ANALYSIS I
Friday, December 10, 2010

1. (20 pts) Select the values of R_1 and R_2 in the circuit below so that $v_R(0)^+ = 10$ V and $v_R(1ms) = 5$V.



$t<0 \qquad v_R = \dfrac{2\ R_1 R_2}{R_1+R_2}$

$i_L(0) = \dfrac{2R_1}{R_1+R_2}$

$t>0 \qquad i_L(t) = \dfrac{2R_1}{R_1+R_2}\, e^{-50R_2 t}$

$\rightarrow v_R(t) = \dfrac{2R_1R_2}{R_1+R_2}\, e^{-50R_2 t}$

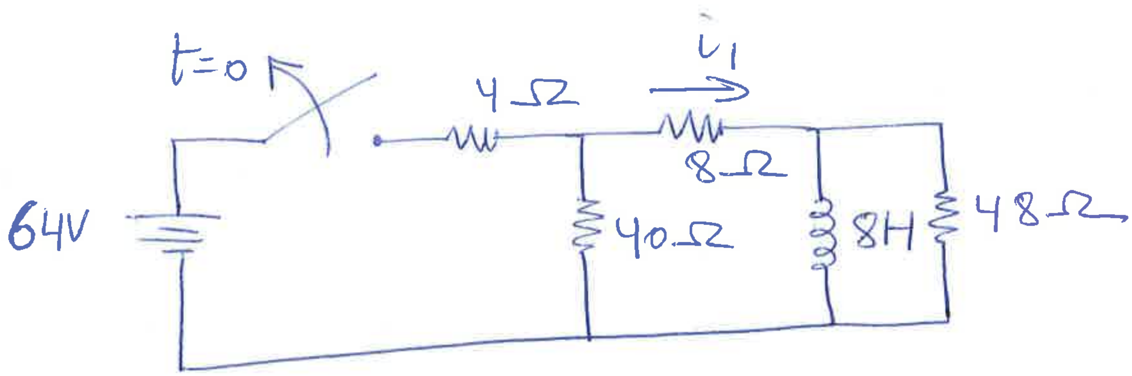
$v_R(0^+) = 10 = \dfrac{2R_1R_2}{R_1+R_2} \rightarrow R_1 // R_2 = 5\Omega$

$v_R(1ms) = 5 = 10\, e^{-\frac{50R_2}{1000}} \rightarrow 0.05R_2 = 0.6931$

$\rightarrow \boxed{R_2 = 13.86\Omega}$

$\rightarrow \boxed{R_1 = 7.821\Omega}$

2. (20 pts) Determine i_1 at t = -0.1s (the circuit has been set for a long time) and $i_1(t = 0.1s)$ in the circuit below.



$$i_L(0) = \frac{64}{4+(40\|8)} \cdot \frac{40}{58} = 5A$$

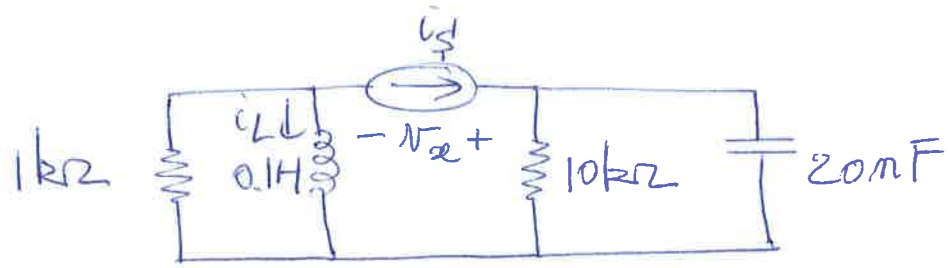
$$i_L(t) = 5e^{-\frac{24t}{8}} = 5e^{-\frac{t}{3}}\ A$$

$$\boxed{i_1(t) = 2.5e^{-3t}\ A \quad t>0}$$

$$\boxed{i_1(-0.1) = 2.5A}$$

$$\boxed{i_1(0.1) = 1.852A}$$

3. (30 pts) The value of i_s in the circuit below is 1 mA for $t < 0$, and zero for $t > 0$. Find v_x for (a) $t < 0$ and (b) $t > 0$.



$t<0$ $\quad i_s = 1\,mA$

$v_C(0) = 10\,V$

$i_L(0) = -1\,mA$

$v_x(0) = 10V, \ t<0$

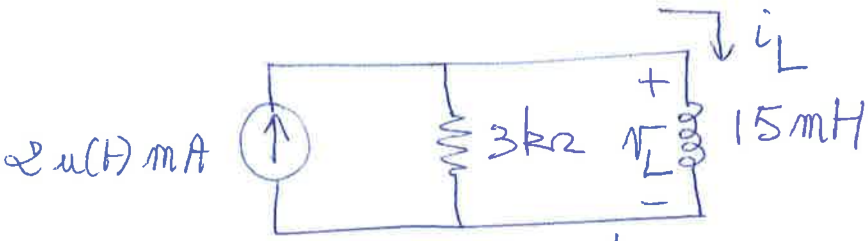
$t>0$ $\quad v_C(t) = 10\, e^{-\frac{t}{10^4 \times 20\, 10^{-9}}} = 10\, e^{-5000t}\ V$

$i_L(t) = -10^{-3}\, e^{-\frac{10^3 t}{0.1}} = -10^{-3}\, e^{-10^4 t}\ A$

$\rightarrow v_L(t) = e^{-1000\,t}\ V, \ t>0$

$$\boxed{v_x = v_C - v_L(t) = 10e^{-5000t} - e^{-10^4 t}, \quad t>0}$$

4. (30 pts) In the circuit below, find $i_L(t)$ for $t > 0$. Use that expression to find $v_L(t)$ for $t > 0$.



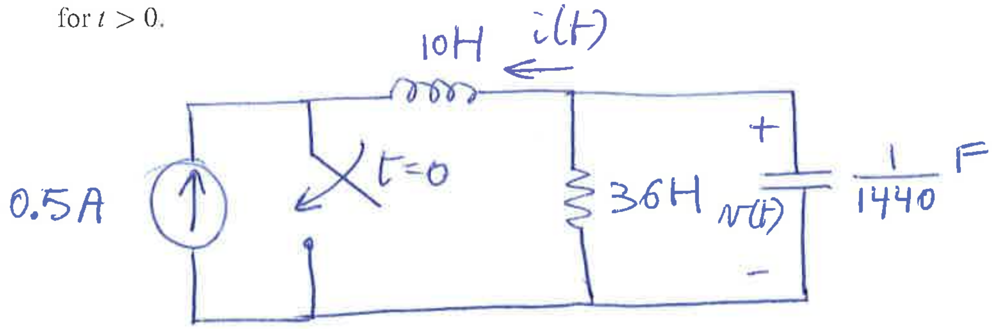
$$i_L(t) = \Big(\underset{\substack{\| \\ 2mA}}{i_f} + A e^{-\frac{Rt}{L}}\Big)\, u(t)\ mA \quad \Rightarrow A = -2\,mA$$

$$i_L(t) = \left(2 - 2\, e^{-2\,10^5 t}\right) u(t)\ mA$$

$$v_L = L\, i_L' = 15\,10^{-3}\,10^{-3}(-2)\left(-2\,10^5 e^{-2\,10^5 t}\right) V$$

$$\boxed{v_L(t) = 6\, e^{-2\,10^5 t}\, u(t)\ V}$$

5. (25 pts) The circuit below was put together a long time ago and has reached steady-state (no more time variation) before the switch is closed. Find v(t) and i(t) for $t > 0$.



$$\alpha = \frac{1}{2RC} = \frac{1440}{72} = 20 \ ; \quad \omega_0^2 = \frac{1440}{10} = 144$$

$$s_{1,2} = -20 \pm \sqrt{400-144} = -4, -36$$

$$v = A_1 e^{-4t} + A_2 e^{-36t}$$

$$v(0) = \boxed{18 = A_1 + A_2} \ (1), \quad v'(0) = 1440\left(\frac{1}{2} - \frac{18}{36}\right) = 0$$

$$\boxed{-4A_1 - 36A_2 = 0} \ (2)$$

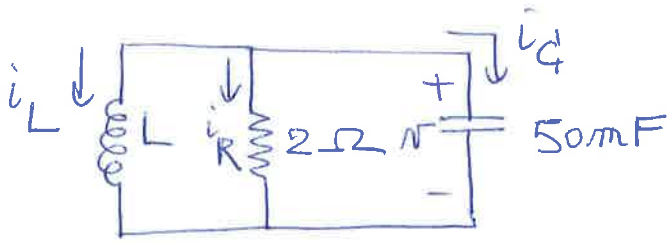
$$(1)(2) \Rightarrow A_1 = 20.25 \ ; \ A_2 = -2.25$$

$$v(t) = \left(20.25 e^{-4t} - 2.25 e^{-36t}\right) V \text{ for } t>0$$

$$i(t) = \frac{v}{36} + \frac{v'}{1440} = 0.5625 e^{-4t} - 0.0625 e^{-36t} - 0.05625 e^{-4t} + 0.05625 e^{-36t}$$

$$\rightarrow \boxed{i(t) = 0.50625 e^{-4t} - 0.00625 e^{-36t}}$$

6. (25 pts) Referring to the circuit below, what value of L will result in a transient response of the form $v = Ae^{-4t} + Be^{-6t}$? Find A and B if $i_R(0^+) = 10A$ and $i_C(0^+) = 15A$.



(a) we want $v = Ae^{-4t} + Be^{-6t}$

$$\alpha = \frac{1}{2RC} = 5\,s^{-1}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4 = -5 + \sqrt{25 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6 = -5 - \sqrt{25 - \omega_0^2}$$

$$\longrightarrow \omega_0 = 4.899 \text{ rad/s}$$

$$\Downarrow$$

$$\omega_0^2 = \frac{1}{LC} \rightarrow \boxed{L = \frac{1}{\omega_0^2 C} = 833.3\,mH}$$

(b) $i_R(0^+) = 10A \quad i_C(0^+) = 15A$

$$i_R(0^+) = 10A, \; v_R(0^+) = v(0^+) = v_C(0^+) = 20V$$

$$v(0) = A + B = 20$$

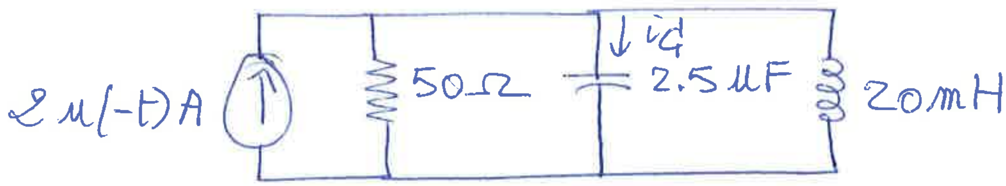
$$i_C = C\frac{dv}{dt} = 50 \cdot 10^{-3}\left(-4Ae^{-4t} - 6Be^{-6t}\right)$$

$$i_C(0^+) = 50 \cdot 10^{-3}(-4A - 6B) = 15$$

$$\Longrightarrow A = 210V \; ; \; B = -190V$$

$$\Longrightarrow \boxed{v(t) = 210e^{-4t} - 190e^{-6t} \; ; \; t > 0}$$

7. (25 pts) Find $i_C(t)$ for $t > 0$ in the circuit shown below.



$$\alpha = \frac{1}{2RC} = \frac{10^6}{100 \times 2.5} = 4000$$

$$\omega_0^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 2\,10^7$$

$$\omega_d = 2000$$

$$i_C = e^{-4000t}\left(B_1 \cos 2000t + B_2 \sin 2000t\right)$$

$$i_L(0) = 2A, \quad v_C(0) = 0 \rightarrow i_C(0^+) = -2A$$

$$i_C'(0^+) = -i_L'(0^+) - i_R'(0^+)$$

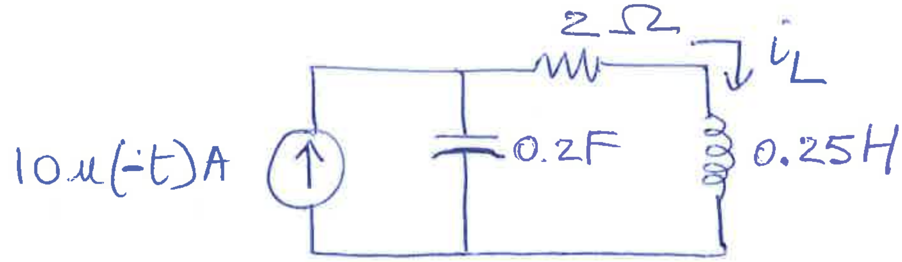
$$i_C'(0^+) = -\frac{1}{L} v_C(0) - \frac{1}{R} v_C'(0^+) = 0 - \frac{1}{RC} i_C(0^+) = \frac{2\,10^6}{125}$$

$$\rightarrow B_1 = -2A; \quad \frac{2\,10^6}{125} = 16{,}000 = 2000 B_2 - 2(-4000)$$

$$\longrightarrow B_2 = 4$$

So $$\boxed{i_C(t) = e^{-4000t}\left(-2\cos 2000t + 4 \sin 2000t\right), \; t > 0}$$

8. (25 pts) Find $i_L(t)$ for $t > 0$ in the circuit shown below.



$$\alpha = \frac{R}{2L} = \frac{2}{1/2} = 4$$

$$\omega_0^2 = \frac{1}{LC} = 20 \quad , \quad \omega_d = \sqrt{20-16} = 2$$

$$i_L = e^{-4t}\left(A_1 \cos 2t + A_2 \sin 2t\right)$$

$$i_L(0) = 10A \, , \quad v_C(0) = 20V$$

$$A_1 = 10A \, ; \quad i_L'(0^+) = \frac{1}{L} v_L(0^+) = 4(20-20) = 0$$

$$i_L'(0^+) = 2A_2 - 4 \times 10 = 0 \Rightarrow A_2 = 20$$

So

$$\boxed{i_L(t) = e^{-4t}\left(10 \cos 2t + 20 \sin 2t\right) A \, ; \, t > 0}$$