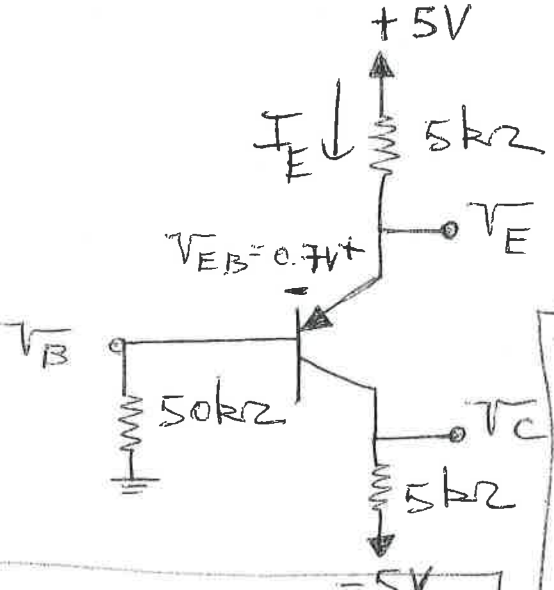
Example Questions - Final

Final Exam (200 pts) - Fall 2010 - M. Cahay

Name:

ELECTRONICS I
December 8, 2010

1. (20 pts) In the circuit below, a measurement shows that the emitter voltage is 1.2V. Find V_B, V_C, I_E, I_B, I_C, and the value of β and α. Neglect the effect of the Early voltage (λ=0).



$$I_E = \frac{5-1.2}{5} = 0.76mA$$

$$V_B = 1.2 - 0.7 = 0.5V$$

$$I_B = \frac{0.5-0}{50k\Omega} = 0.01mA$$

$$I_C = I_E - I_B = 0.75mA$$

+10

+5

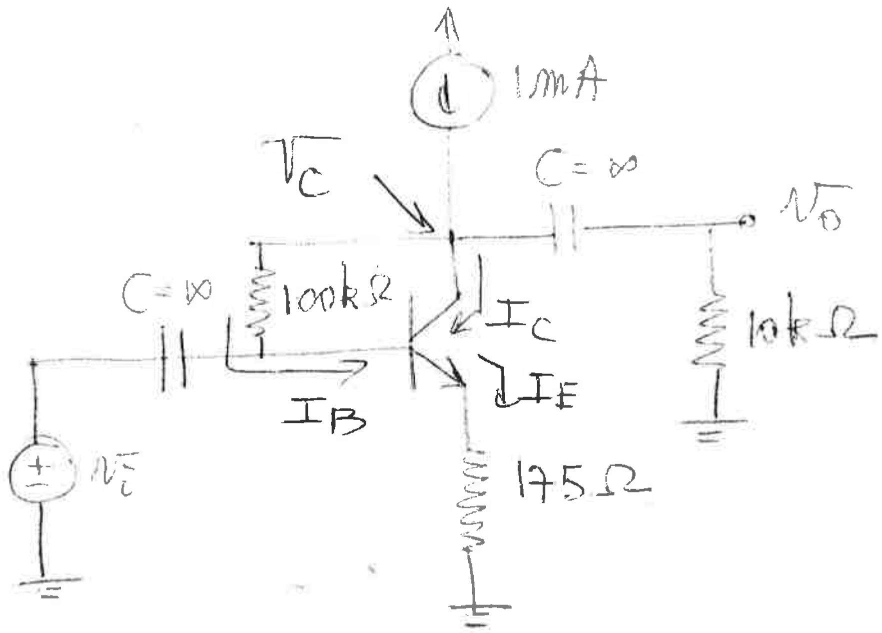
$$V_C = -5 + 0.75 \times 5 = -1.25V$$

$$\beta = \frac{I_C}{I_B} = \frac{0.75}{0.01} = 75$$

+5

$$\alpha = \frac{\beta}{\beta+1} = \frac{75}{76} = 0.987$$

III. (30pts) For the circuit below, find the DC collector current and the DC voltage at the collector. Assume β is 100 and that V_{BE} is 0.7 V. Use the T-model and draw the small signal equivalent circuit of the amplifier. Neglect the effect of r_O. Determine the voltage gain v_O/v_i.



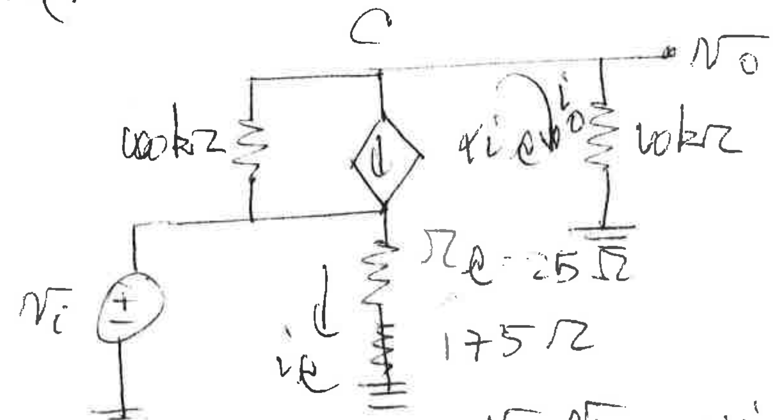
DC

$I_C = 0.99\ mA$

$V_C = R_E I_E + V_{BE} + R_B I_B$

$= (1 \times 0.175) + 0.7 + (0.01 \times 100) = \underline{1.875 V}$

AC



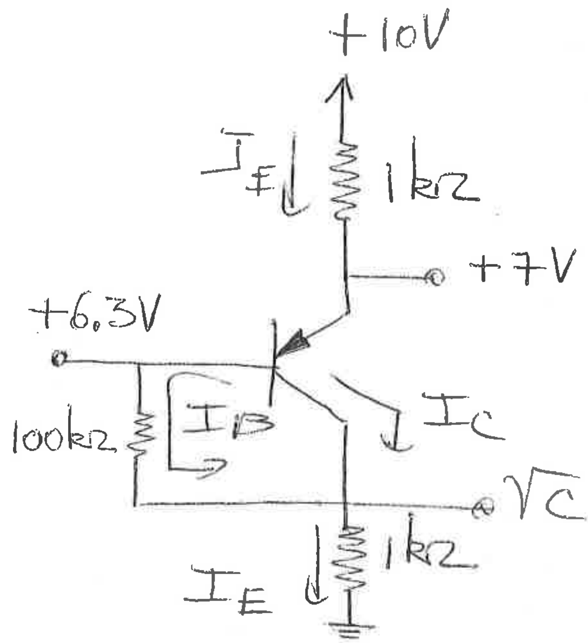
$i_e = \frac{v_i}{r_e + R_E} = \frac{v_i}{0.2 k\Omega}$

at C: $\frac{v_o - v_i}{100k\Omega} + \alpha i_e + \frac{v_o}{10k\Omega} = 0$

$0.01 v_o - 0.01 v_i + 0.99 \frac{v_i}{0.2} + 0.1 v_o = 0$

$\Rightarrow \boxed{\frac{v_o}{v_i} = -44.9\ V/V}$

2. (20 pts) Measurements on the circuit below produce the labeled voltages as indicated. Find the value of β for the transistor.



$$I_E = \frac{10-7}{1k} = 3\,mA \quad (+4)$$

$$I_E = I_C + I_B = 3\,mA \quad (+4)$$

$$V_C = 3\,mA\,(1k) = 3V \quad (+4)$$

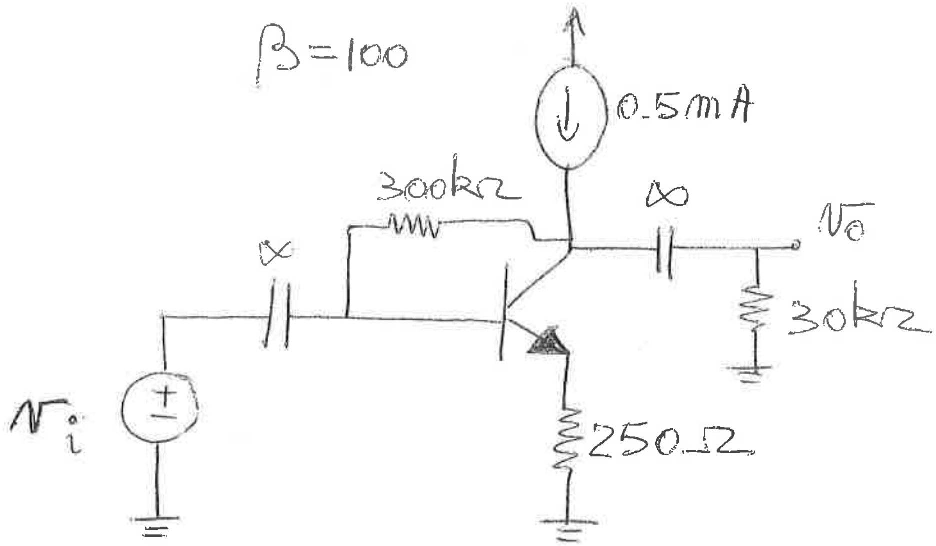
$$I_B = \frac{6.3-3}{100\,k\Omega} = 33\,\mu A \quad (+4)$$

$$\frac{I_E}{I_B} = \beta + 1 = \frac{3\,mA}{33\,\mu A} = 90.9$$

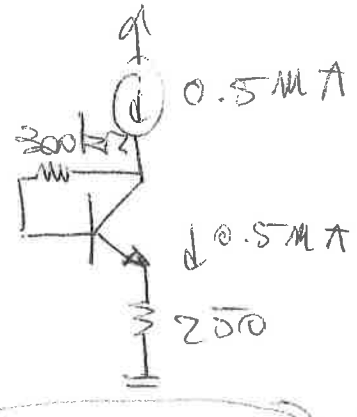
$$\rightarrow \boxed{\beta = 89.9} \quad (+4)$$

3. (30 pts) **The BJT as an amplifier**

- (a) In the circuit below, assume $V_{BE} = 0.7V$ and calculate the value of the collector current and the voltage at the collector.
- (b) Using the T-model of the BJT and neglecting the effect of r_0, draw the small signal equivalent circuit of the amplifier.
- (c) Using the results of part (b), calculate the voltage gain of the amplifier, v_0/v_i.



(a) DC



$I_C = 0.99 \times 0.5 mA = 0.495 mA$

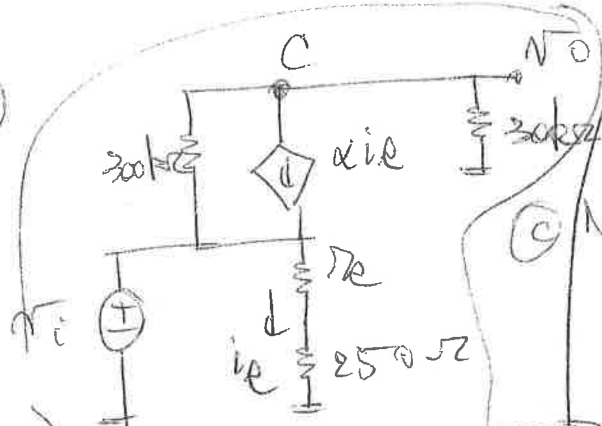
$V_C = I_E R_E + V_{BE} + R_B I_B$

$= 0.5 * 0.175 + 0.7$

$+ 0.005 \times 300$

$\boxed{V_C = 2.28V}$ +10

(b)



+10

$i_e = v_i/(r_e + R_E) = v_i/300$

$r_e = \frac{V_T}{I_E} = 50\Omega$

(c) Node eq. at C

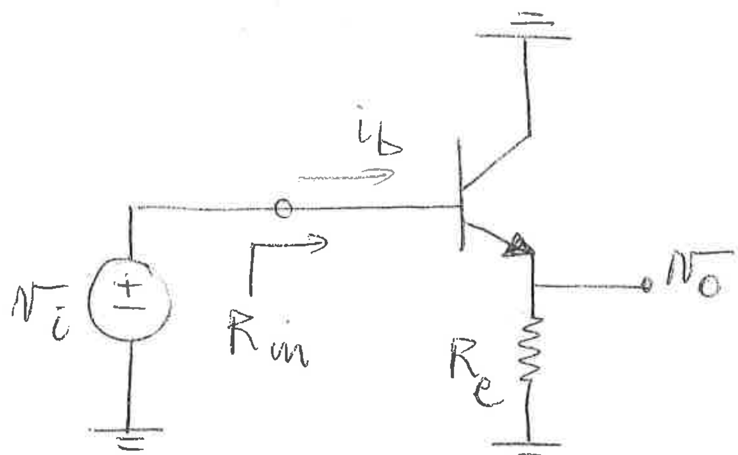
$\frac{v_o - v_i}{300k\Omega} + \alpha i_e + \frac{v_o}{30k} = 0$

$\boxed{\frac{v_o}{v_i} = -90V/V}$

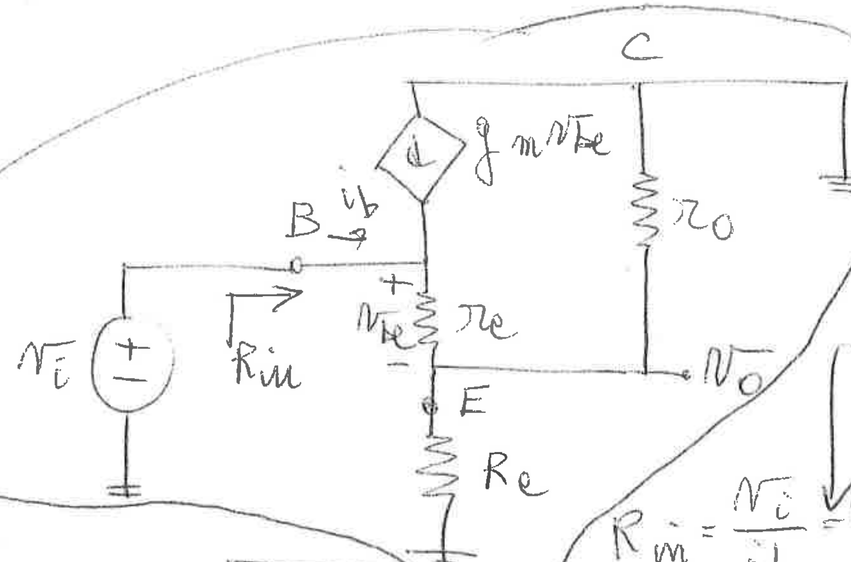
+10

4. (30 pts) **The BJT as an amplifier**

- (a) Is the amplifier below a (1) common-emitter, (2) common-base, or (c) common-collector configuration (circle the correct answer)?
- (b) Including the effects of r_0 and using the T-model of the transistor, draw the small-signal equivalent circuit of the amplifier. Derive the analytical expression for the small-signal voltage gain $A_v = v_0/v_i$ of the amplifier.
- (c) Derive the expression for the input resistance R_{in}.



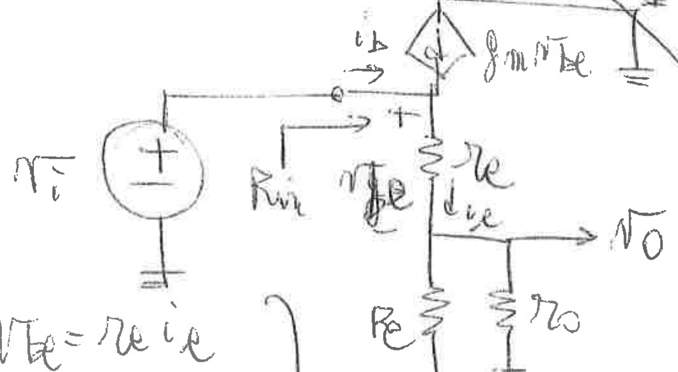
+16



$i_e = (\beta+1) i_b$

$R_{in} = \frac{v_i}{i_b} = (\beta+1)\frac{v_i}{i_e}$ +7

$$\boxed{R_{in} = (\beta+1)\left[r_e + (R_e // r_o)\right]}$$



$v_i = v_{be} + v_o = v_o\left[1 + \frac{v_{be}}{v_o}\right]$

$v_{be} = r_e i_e$

$v_o = (R_e // r_o) i_e$

$$\boxed{\frac{v_o}{v_i} = \frac{1}{1+\frac{v_{be}}{v_o}} = \frac{1}{1+\frac{r_e}{(r_e//R_o)}}}$$ +7

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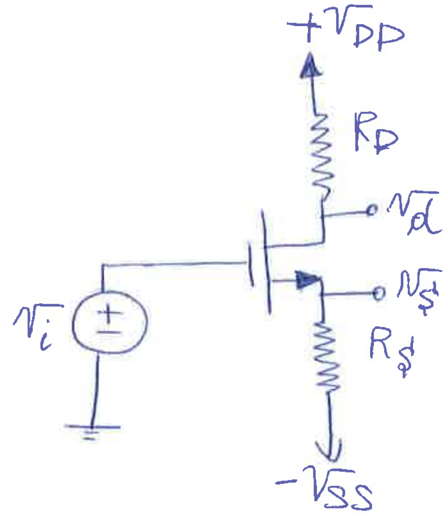
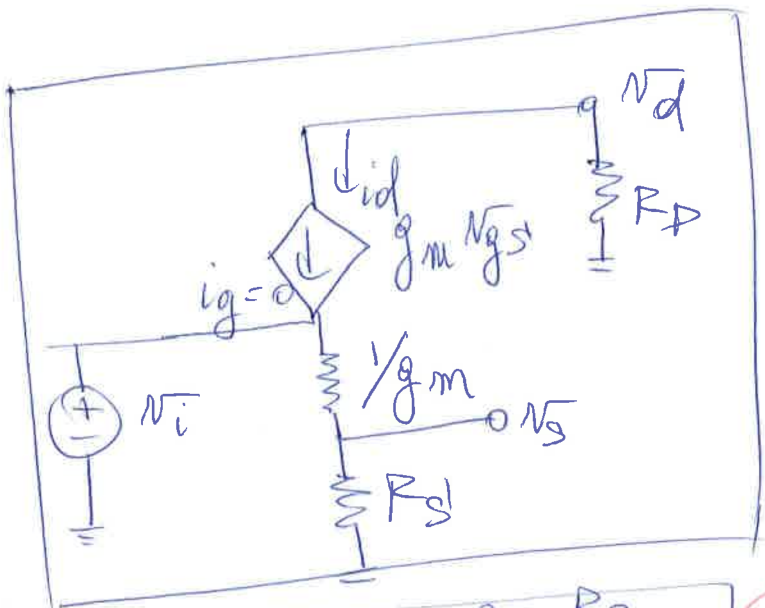
8. (30 pts) For the NMOS amplifier below, replace the transistor with its T equivalent circuit while neglecting the effect of r_0. Calculate the voltage gains v_s/v_i and v_d/v_i.





+10

$$\boxed{\frac{v_s}{v_i} = \frac{R_s}{R_s + \frac{1}{g_m}} = \frac{g_m R_s}{[1+g_m R_s]}}$$ +5

$$v_d = -g_m v_{gs} R_D$$ +5

$$v_{gs} = \frac{1/g_m \, v_i}{1/g_m + R_s} = \frac{v_i}{1+g_m R_s}$$ +5

So $$\boxed{\frac{v_d}{v_i} = \frac{-g_m R_D}{1+g_m R_s}}$$ +5

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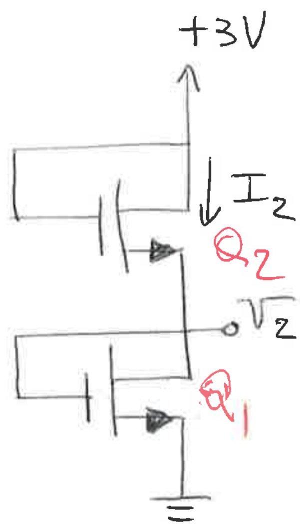
V. For the circuit shown below, both transistors are characterized by the following parameters $\mu_n C_{ox} = 20\ \mu A/V^2$, $V_t = 1$V, $\lambda = 0$, L= 10 μm, and $W = 30\mu m$. Find the labeled current and voltage.



Q_1 Q_2 identical and operating in saturation with equal drain currents

$\rightarrow V_{GS_1} = V_{GS2}$

$\Rightarrow V_2 = 3 - V_2 \Rightarrow V_2 = \underline{+1.5V}$

$I_2 = \frac{1}{2} \times 20 \times \frac{30}{10}(1.5-1)^2 = \underline{7.5 \mu A}$

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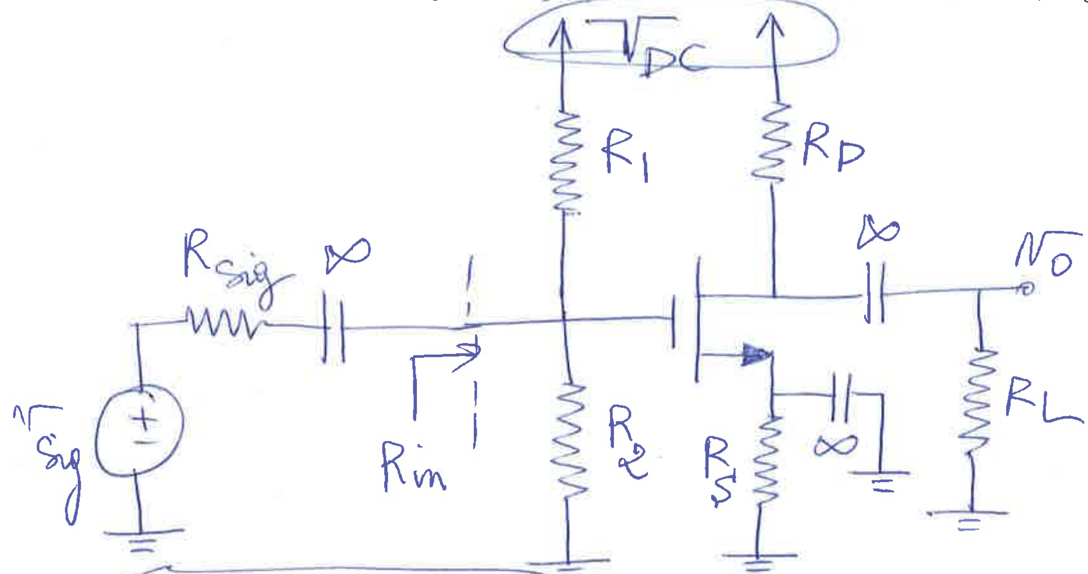
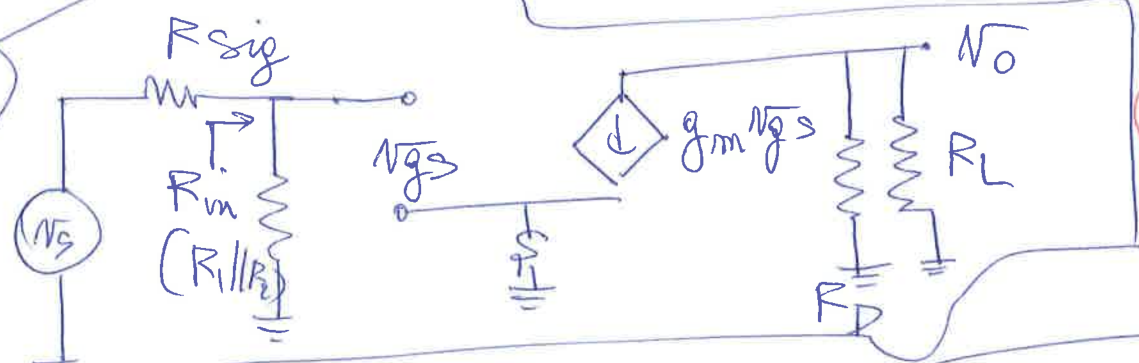
7. (30 pts) For the amplifier circuit shown below:

- Which type of amplifier configuration is it? common source, common gate, or common drain. Circle your answer. +5
- Use the hybrid-π equivalent AC circuit for the transistor and draw the small signal equivalent circuit for the amplifier.
- Give an analytical expression for the input resistance R_{in}, as shown on the diagram.
- Give an analytical expression for the overall voltage gain v_0/v_{sig}.



b



+10

c $\boxed{R_{in} = (R_1 // R_2)}$ +5

d +10

$$v_o = -g_m v_{gs} (R_D // R_L)$$

$$v_{gs} = \frac{(R_1 // R_2)}{R_{sig} + (R_1 // R_2)} v_{sig}$$

$$\frac{v_o}{v_{sig}} = -g_m \frac{(R_D // R_L)(R_1 // R_2)}{\left[R_{sig} + (R_1 // R_2)\right]}$$

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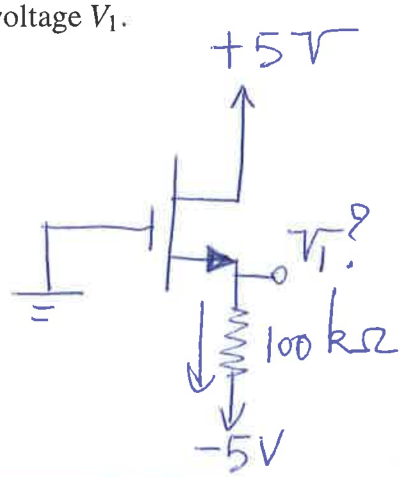
6. (20 pts) The transistor in the circuit below has a $V_t = 1V$, $k_n'(\frac{W}{L}) = 0.4mA/V^2$ and $\lambda = 0$.

Find the labeled node voltage V_1.



$V_D = 5$

$V_G = 0$

V_S ?

$V_{DS} \geq V_{GS} - V_t$

$V_D > V_G - V_t$

$5 > 0 - 1$ ✓

→ Sat

$$\frac{V_1 + 5}{100} = I = \frac{1}{2} 0.4 (5 - \underbrace{100 I - 1}_{V_1})^2 \quad +5$$

$I = 0.0\cancel{45}$ / 0.036 mA +5

only acceptable solution

because $V_{GS} < V_t$ → cut-off +5

$$\boxed{V_1 = -5 + 100 I = -1.4V} \quad +5$$

or $I = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_t)^2$

$= \frac{1}{2} k' \frac{W}{L} (0 - V_1 - V_t)^2$

$= \frac{1}{2} k' \frac{W}{L} (-(-5 + 100 I) - 1)^2$

$I = 0.2 (4 - 100 I)^2 \rightarrow I_1, I_2$ above

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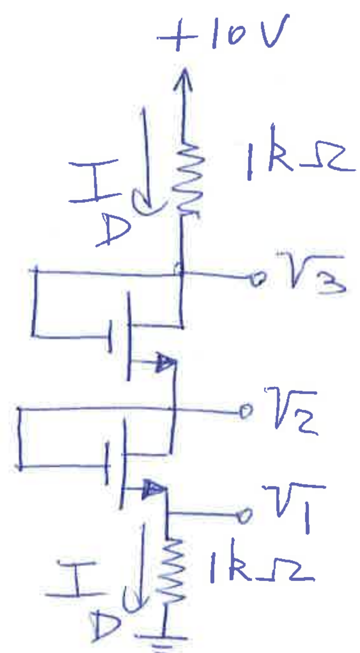
5. (20 pts) The NMOS transistors in the circuit below have $V_t = 1V$, $k_n'(\frac{W}{L}) = 2mA/V^2$ and $\lambda = 0$.

Find the labeled node voltages, V_1, V_2, and V_3.



$$\boxed{\frac{10-V_3}{1} = \frac{V_1}{1} = I \quad (1)}$$ +4

$$\boxed{I_{D_1} = \frac{V_1}{1} = \frac{1}{2}\cdot 2(V_3-V_2-1)^2 \quad (2)}$$ +4

$$\to V_1 = (V_3-V_2-1)^2$$

$$I_{D_2} = \frac{V_1}{1} = \frac{1}{2}\cdot 2(V_2-V_1-1)^2 \quad (3)$$ +4

$$\boxed{(2)(3) \to V_3-V_2-1 = V_2-V_1-1 \to V_1 = 2V_2-V_3 \quad (4)}$$

$$(1)(4) \to 2V_2 - V_3 = 10 - V_3 \to \boxed{V_2 = 5V}$$ +4

$$\to V_1 = (4-V_1)^2 \to \boxed{V_1^2 - 9V_1 + 16 = 0}$$

+4 $\to \boxed{V_1 = 6.\cancel{55} \text{ or } 2.45V}$

from (1) 2.45mA

$$\boxed{I_D = \cancel{2.4mA}} \to \boxed{V_3 = 10 - 2.45mA(1k\Omega) = 7.55V}$$

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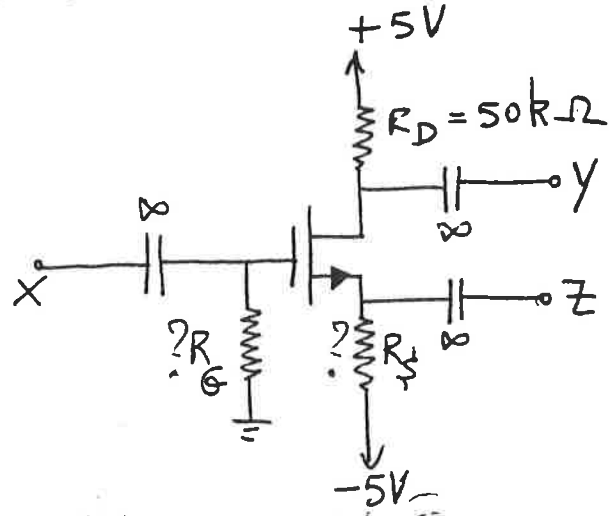
VI. The MOSFET in the circuit below has $V_t = 1$V, K=0.4mA/V^2, and $V_A = 40V$. (a) Assuming $R_D = 50k\Omega$, find R_S and R_G so that $I_D = 0.1mA$. (b) Find the values of g_m and r_o at the bias point. (c) If terminal Y is grounded, find the voltage gain from X to Z with Z open-circuited. (d) If terminal X is grounded and terminal Z is connected to a current source delivering a current of 10μA and having a resistance of 100 k Ω, find the voltage signal at Y. Neglect r_o in that case.

amplitude ... a.c.



R_G undetermined but must be high

Choose $R_G = 10\,M\Omega$ (+2)

$V_D = 0$

(+2) $R_D = \frac{5-0}{0.1} = 50\,k\Omega$

$0.1 = 0.4\,(V_{GS}-1)^2$

$V_{GS} = 1.5\,V$ (+2)

$\Rightarrow V_S = -1.5\,V$ (+2)

$R_S = \frac{-1.5-(-5)}{0.1} = 35k\Omega$ (+2)

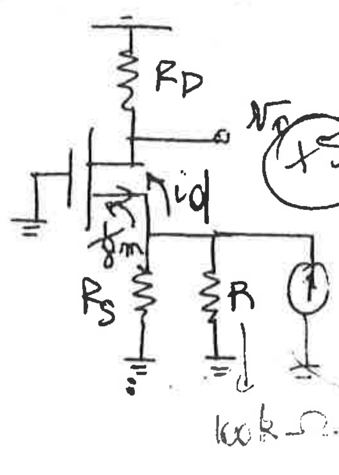
$g_m = 2K(V_{GS} - V_t) = 0.4\ mA/V$ (+2)

$r_o = \frac{V_A}{I_D} = 400k\Omega$ (+2)

(c)

$$A_v = \frac{(R_S // r_o)}{(R_S // r_o) + \frac{1}{g_m}} = 0.928\ V/V \quad (+2)$$

(d)



(+5)

$$i_d = i_i\,\frac{(R//R_S)}{(R//R_S) + 1/g_m} \quad (+2)$$

$$= 10\,\frac{(100//35)}{(100//35) + \frac{1}{0.4}} = 9.12\,\mu A.$$

i_i 10μA $\quad v_o = R_D\, i_d = 9.12 \times 50 = 456\,mV$ (+2)

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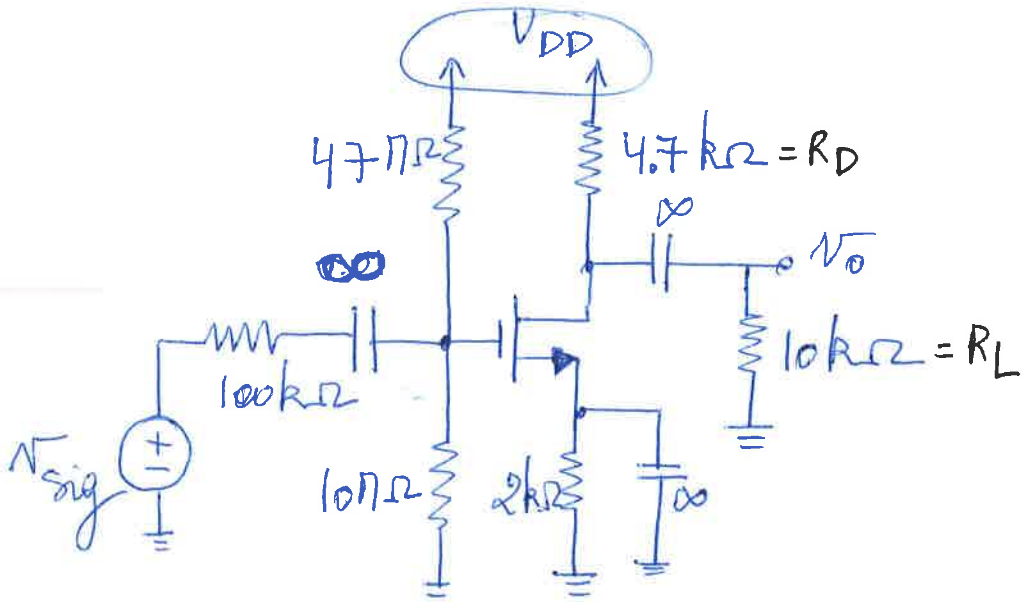
8. (25 pts) The amplifier circuit shown below is biased in the saturation mode of operation with a value of the transconductance $g_m = 1$ mA/V. Assume $\lambda = 0$.

- Use the hybrid-π model of the transistor and draw the small signal equivalent circuit for the amplifier.
- Calculate the voltage gain v_O/v_{sig}.



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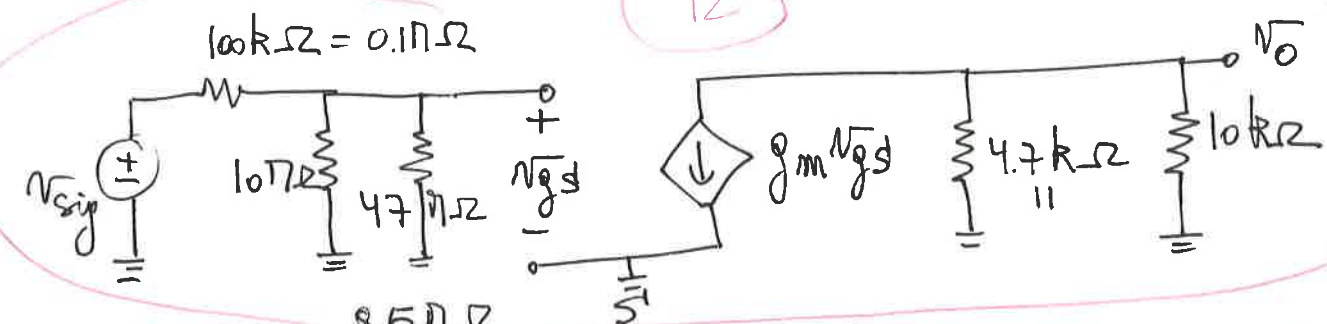

$$v_{gs} = \left[\frac{\overset{8.5M\Omega}{(47\|10)}}{(47\|10)+0.1}\right] v_{sig}$$

$$\frac{v_o}{v_{sig}} = -g_m (R_D\|R_L)\cdot\left\{\frac{(47\|10)}{(47\|10)+0.1}\right\}$$

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$$\frac{v_o}{v_{sig}} = -1(4.7k\|10k)\cdot\left[\frac{8.25}{8.25+0.1}\right]$$

$$\boxed{\frac{v_o}{v_{sig}} = -3.16\,V/V}$$

+1

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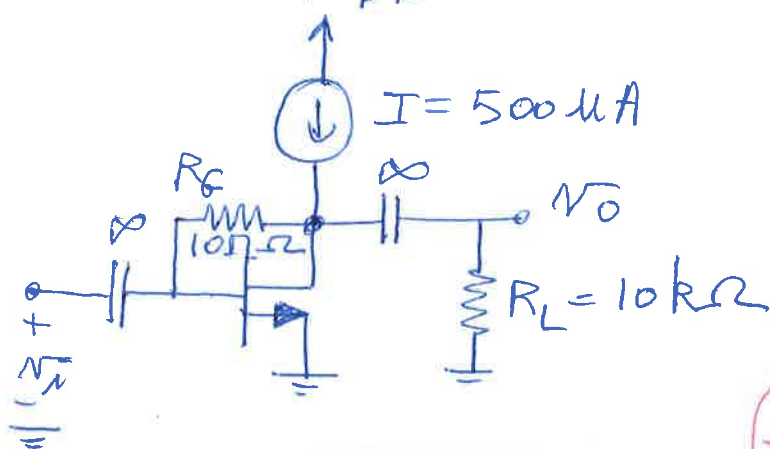
6. (25 pts) In the circuit below, the NMOS has a $|V_t|$ of = 0.9 V and a V_A of 50 V and operates with $V_D = 2V$.

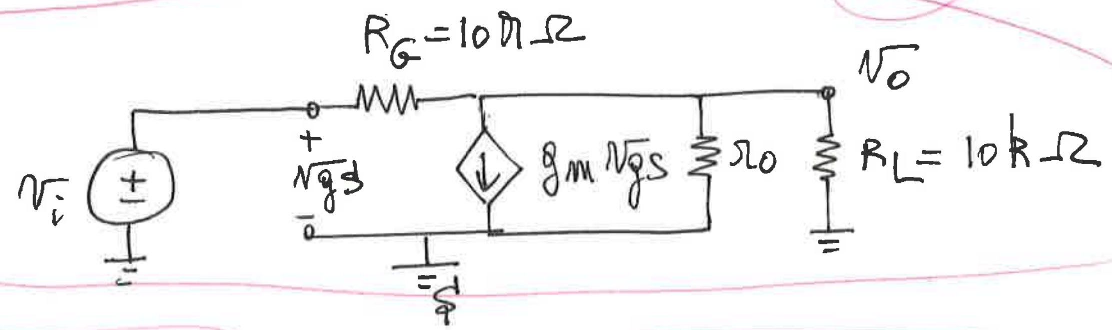
(a) • Using the hybrid-π model of the MOS transistor, draw the small signal equivalent circuit of the amplifier. Include the effect of r_0.

(b) • What do the DC value V_D becomes if the current I is changed to 1 mA? Neglect the effects of V_A in the expression of I_D.



+13

(a)



(b) $I_{D_1} = \frac{k_n'}{2} \frac{W}{L} (V_{GS_1} - V_t)^2$ For $I_{D_1} = 500\,\mu A$

+12

$I_{D_2} = \frac{k_n'}{2} \frac{W}{L} (V_{GS_2} - V_t)^2$ For $I_{D_2} = 1\,mA$

$\rightarrow \frac{I_{D_1}}{I_{D_2}} = \frac{(V_{GS_1} - V_t)^2}{(V_{GS_2} - V_t)^2} \rightarrow V_{GS_2} = V_t + \sqrt{2}\,(V_{GS_1} - V_t)$

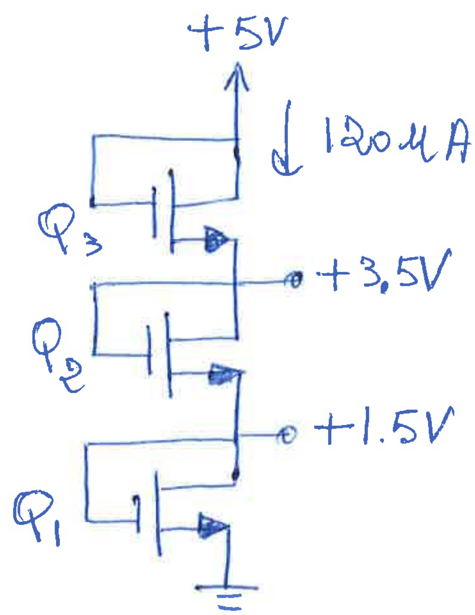
$V_{GS_2} = 0.9 + \sqrt{2}(2 - 0.9) = \underline{\underline{2.5V}}$

$V_{D_1} = V_{GS_1}$

$V_{D_2} = V_{GS_2}$

5. (25 pts) The NMOS transistors in the circuit below have $V_t = 1V$, $\mu_n C_{ox} = 120\mu A/V^2$, $\lambda = 0$, and $L_1, L_2, L_3 = 1\ \mu m$.

Find the required values of the gate widths W_1 of Q_1, W_2 of Q_2, W_3 of Q_3 to obtain the voltage and current values indicated on the figure.



$$I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

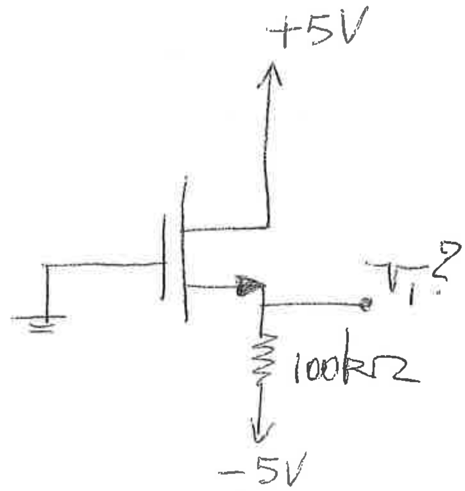
$$k_n' = \mu_n C_{ox}$$

$$V_{GS_1} = 1.5V \rightarrow 120\mu A = \frac{1}{2} \cdot 120 \cdot \frac{W_1}{L_1} (1.5-1)^2 \Rightarrow \underline{\underline{W_1 = 8\ \mu m}}$$

$$V_{GS_2} = 3.5 - 1.5 = 2V \Rightarrow 120 = \frac{1}{2}\, 120\, \frac{W_2}{L_2} (2-1)^2 \Rightarrow \underline{\underline{W_2 = 2\ \mu m}}$$

$$V_{GS3} = 1.5V \rightarrow W_3 = W_1 = \underline{\underline{8\ \mu m}}$$

5. (20 pts) In the circuit below, the enhancement-NMOS transistor has a threshold voltage $V_t = 1V$. Futhermore $k_n'(W/L) = 0.4$ mA/V^2 and $\lambda = 0$. Find the labeled voltage V_1.



$V_D = 5V$

$V_G = 0$

V_S ? unknown ?

$V_{DS} \gtrless V_{GS} - V_t$

$V_D > V_G - V_t$

5 ✓ 0 − 1

→ Saturation (+5)

$\frac{V_1 + 5}{100} = I = \frac{1}{2}(0.4)(5 - V_1)^2$ (+5)

$= \frac{1}{2}(0.4)(5 - 100I - 1)^2$

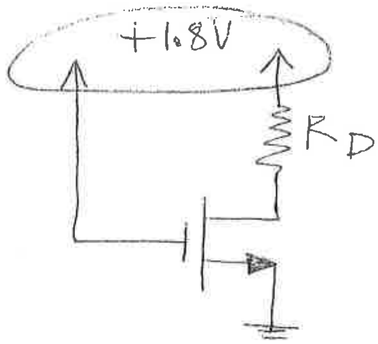
→ $I = \cancel{0.045}$ / $\boxed{0.036\ mA}$ (+5)

$V_{GS} < V_t$ cut off

$\boxed{V_1 = -5 + 100 I = -1.4V}$ (+5)

6. (20 pts) The transistor below has a threshold voltage $V_t = 0.5V$. Futhermore $k_n' = 0.4\ mA/V^2$ and $\lambda = 0$. Show that the transistor operates at the edge of saturation if the following condition is satisfied

$$\frac{W}{L} R_D = 1.5k\Omega$$



$k_n' = 0.4\ mA/V^2$

$V_t = 0.5V$

$\lambda = 0$

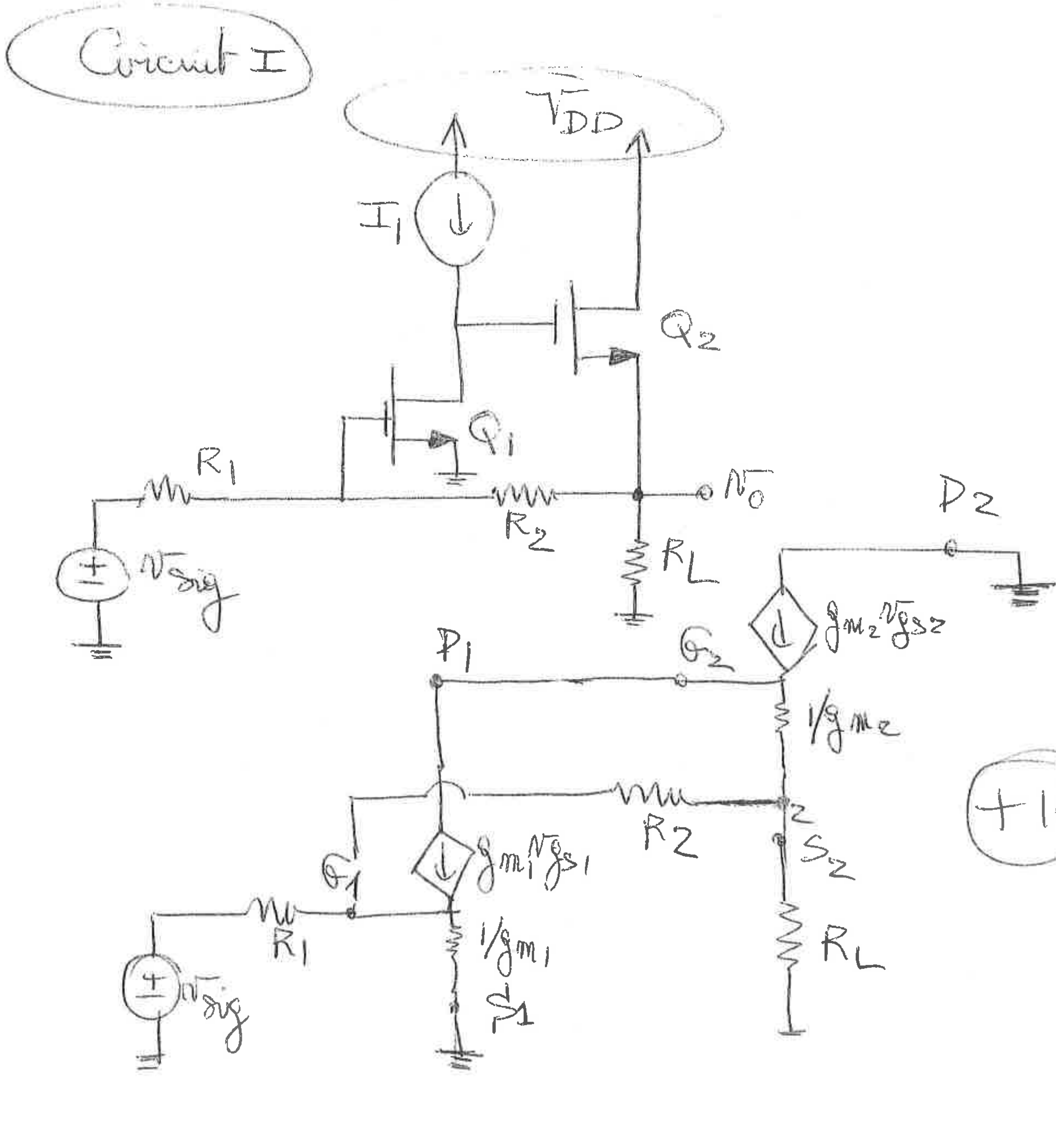
Saturation boundary $V_{OD} = 5V = R_D I_D$ +5

$$0.5 = \frac{1}{2} k_n' \frac{W}{L} (1.8 - 0.5)^2 R_D \quad +10$$

$$\rightarrow \boxed{\frac{W}{L} R_D = 1.48\ k\Omega} \quad +5$$

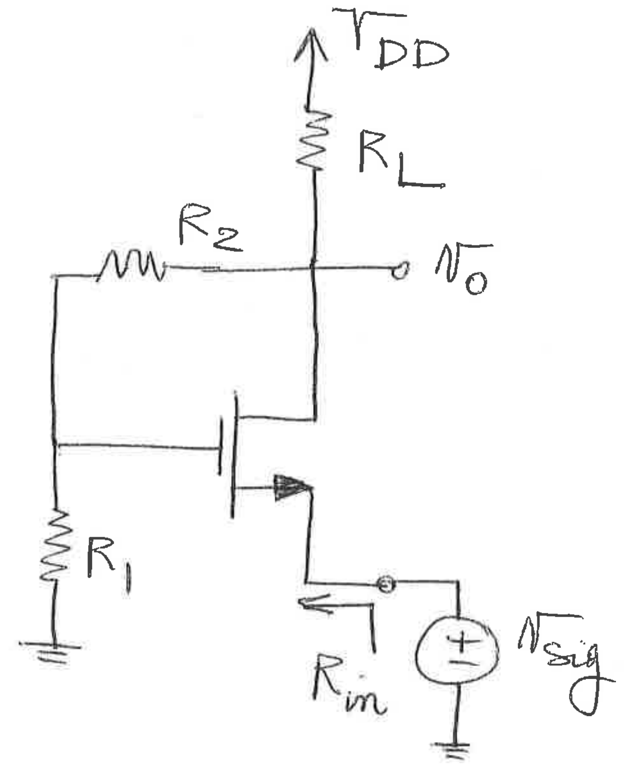
7. (30 pts) For the two amplifiers shown below, draw their small signal equivalent circuit.

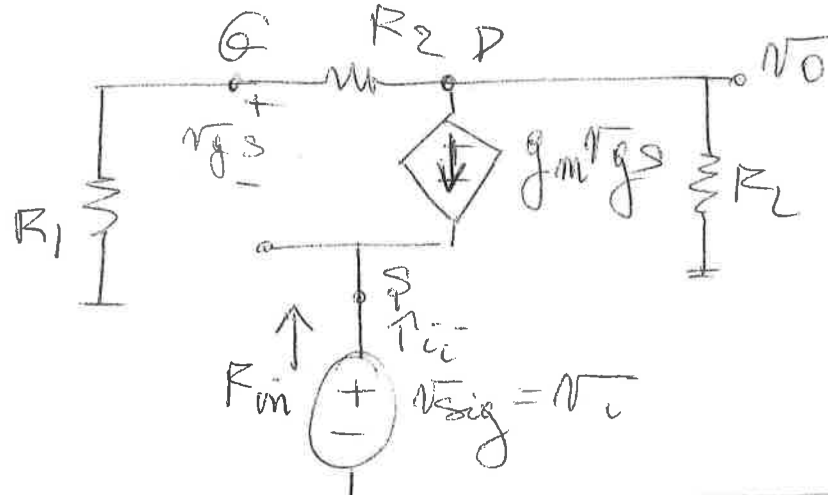
- Use the T-model for both transistors in circuit I and neglect the effect of r_0.
- For circuit II, using the hybrid-π model of the transistor and neglect the effect of r_0. Use your small signal equivalent circuit to derive an analytical expression of the input resistance R_{in}.



+10

Circuit II





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@ D $\quad \frac{v_{gs}+v_i}{R_1} + g_m v_{gs} + \frac{v_o}{R_L} = 0$

$$v_{gs} = v_o\left(\frac{R_1}{R_1+R_2}\right) - v_i$$

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$$\rightarrow \frac{g_m v_o}{R_1+R_2} + \cancel{\frac{v_i}{R_1}} + g_m v_i + \frac{v_o}{R_L} + \frac{v_o R_1}{R_1+R_2} = 0$$

$$\rightarrow v_o\left[\frac{1+g_m R_1}{R_1+R_2} + \frac{1}{R_L}\right] = g_m v_i$$

$$\frac{v_o}{v_i} = g_m \Big/ \left[\frac{g_m R_1+1}{R_1+R_2} + \frac{1}{R_L}\right]$$

$$R_i = \frac{v_{sig}}{i_i}$$

$$-g_m v_{gs} = \left(\frac{1}{R_L} + \frac{1}{R_1+R_2}\right) v_o$$

$$i_i = -g_m v_{gs}$$

$$i_i = \left(\frac{1}{R_L} + \frac{1}{R_1+R_2}\right) v_o$$

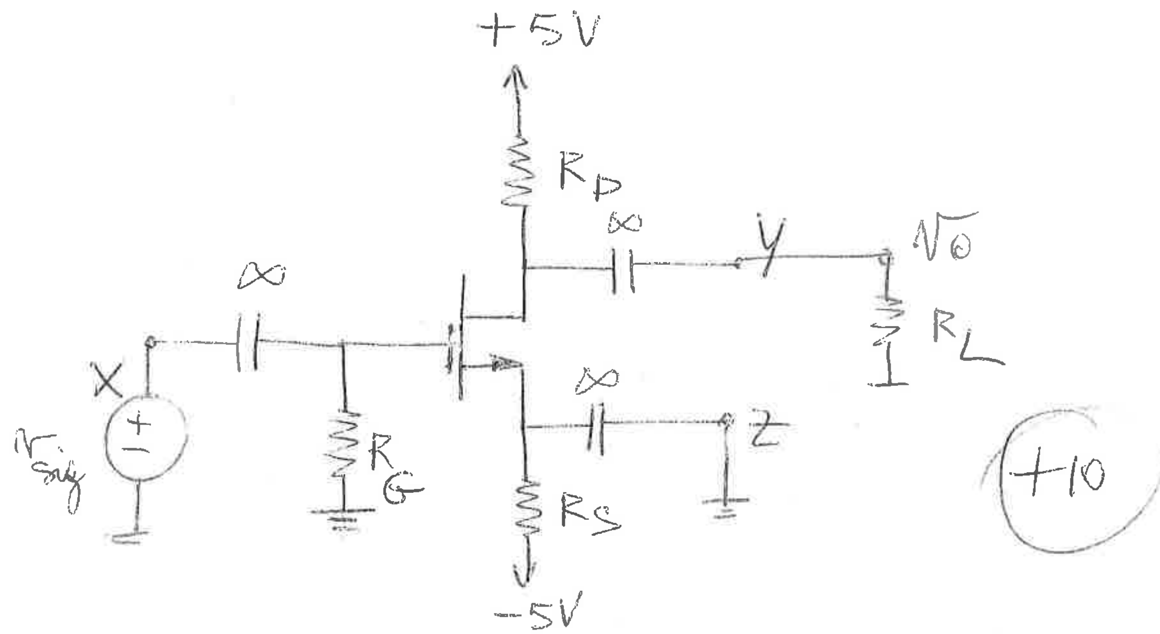
$$\frac{i_i}{v_i} = \frac{v_o}{v_i}\left(\frac{1}{R_1+R_2} + \frac{1}{R_L}\right)$$

$$\frac{i_i}{v_i} = \frac{g_m}{\left[\frac{g_m R_1 + 1}{R_1+R_2} + \frac{1}{R_L}\right]} \cdot \frac{1}{\left[\frac{1}{R_1+R_2} + \frac{1}{R_L}\right]}$$

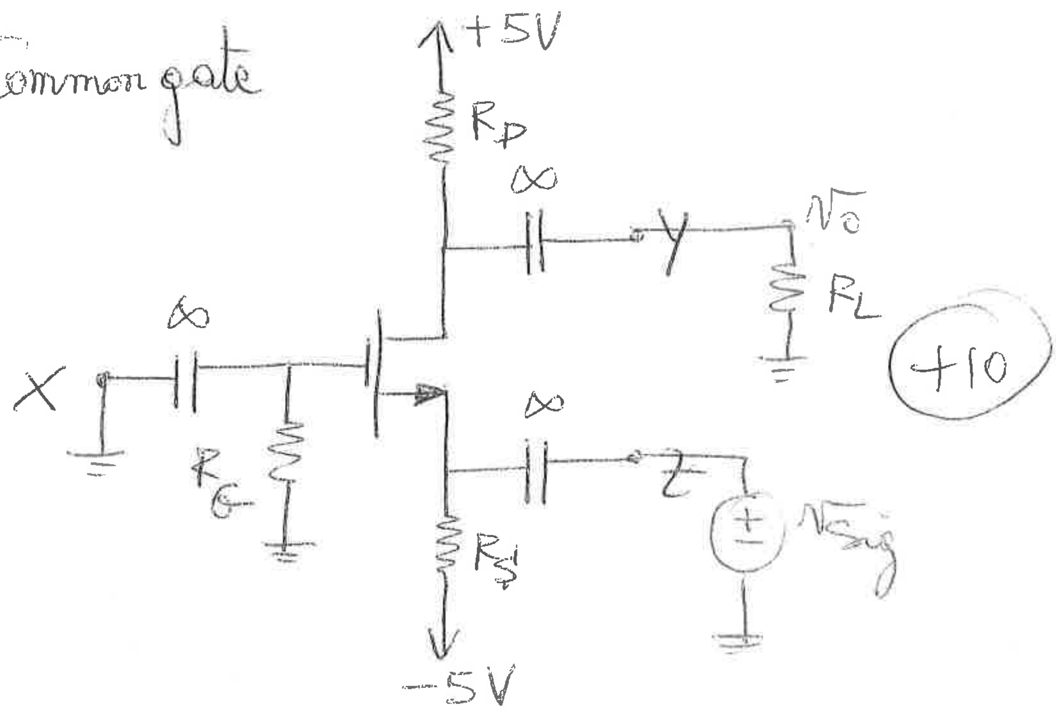
$$\longrightarrow \boxed{R_{in} = \frac{v_i}{i_i} = \frac{\left[\frac{g_m R_1 + 1}{R_1+R_2} + \frac{1}{R_L}\right]}{g_m} \cdot \left[\frac{1}{R_1+R_2} + \frac{1}{R_L}\right]}$$

8. (30 pts) For the amplifier shown below with 3 terminals X,Y, and Z, show how to connect to which terminal a signal source v_{sig}, load resistance R_L, and AC ground to build a common-source, common-gate, and common-drain amplifier. Draw the three separate circuits (10 points each).

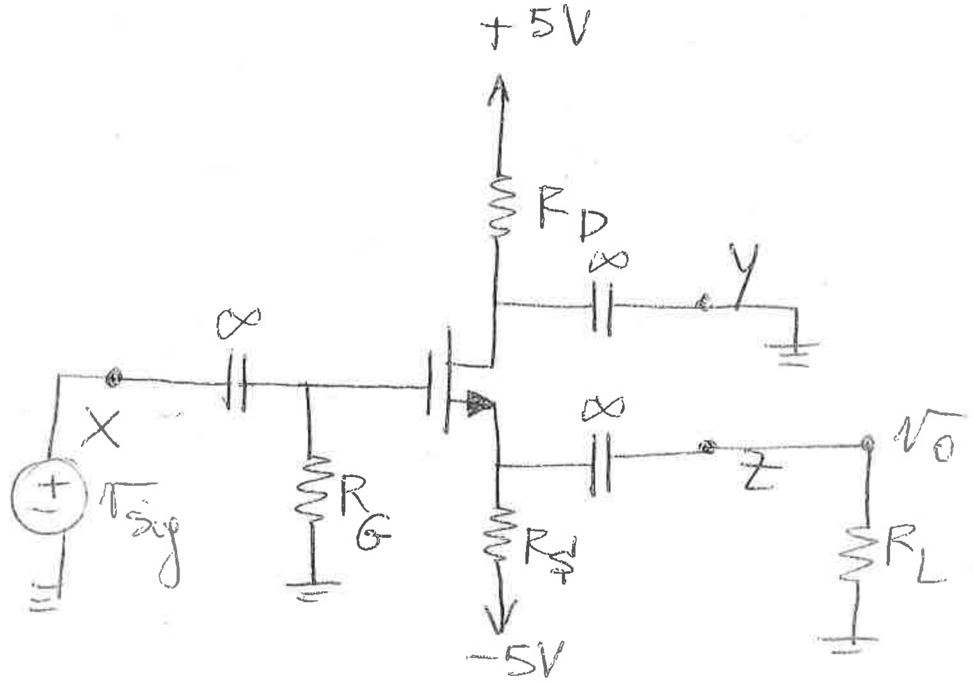
Common Source



Common gate



Common Drain



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