Electronics II, Final - Spring Quarter: June 6, 2012 (200pts max)
Name: SOLUTION

I. (30 pts): The voltage gain of an amplifier is described by the following transfer function:

$$A(s) = \frac{10^8 s^2}{(s+10)(s+10^3)(s+10^6)}$$

- Write A(s) in the generic from $A_M F_L(s) F_H(s)$ and give the expressions for A_M, $F_L(s)$ and $F_H(s)$.
- What is the midgap gain A_M of the amplifier (in dB)? What are the zeroes and poles of A(s) on the low and high frequency sides?
- Make a Bode plot of the magnitude of the transfer function A(s). Use the attached log-log plot. Make sure to indicate clearly the locations of all the zeroes and poles of the amplifier on the Bode plot.
- From your Bode plot, find the approximate values of the gain (in dB) at the angular frequencies ω = 1, 10, 10^2, 10^3, 10^4, and 10^5 in rad/s.

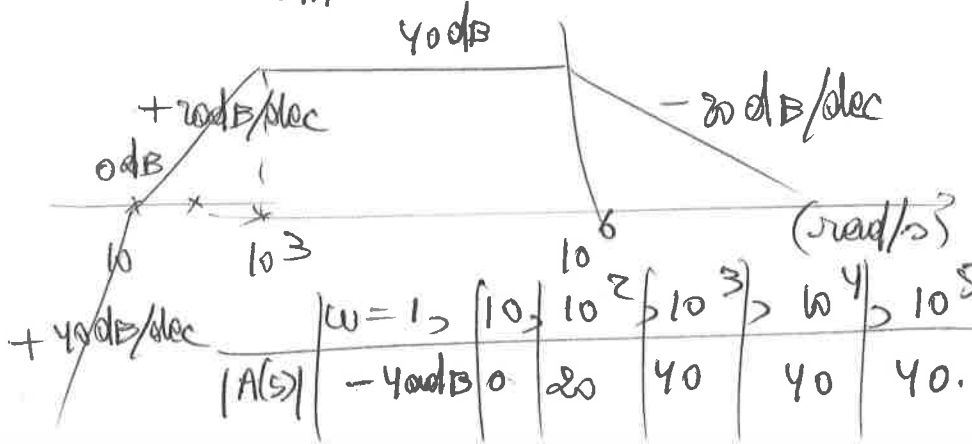
$$A(s) = \frac{10^8}{\left(1+\frac{10}{s}\right)} \frac{1}{\left(1+\frac{10^3}{s}\right)} \cdot \frac{1}{10^6} \frac{1}{\left(1+\frac{s}{10^6}\right)}$$

$$= \underbrace{10^2}_{A_M \,\to\, 40\text{dB}} \underbrace{\frac{1}{\left(1+\frac{10}{s}\right)\left(1+\frac{10^3}{s}\right)}}_{F_L(s)} \underbrace{\frac{1}{\left(1+\frac{s}{10^6}\right)}}_{F_H(s)}$$

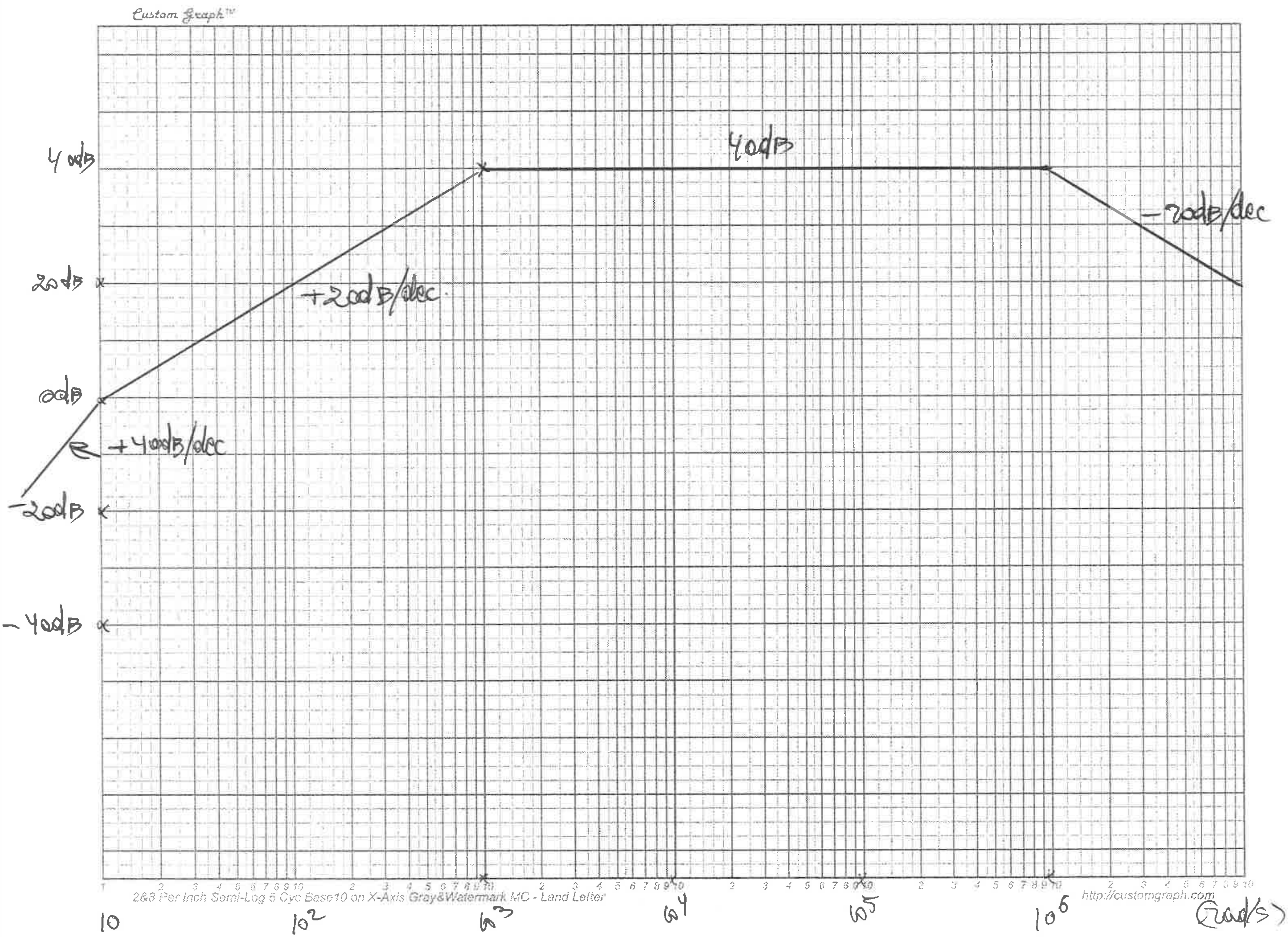
zeros $\omega_{Z1L} = 0 = \omega_{Z2L}$

poles $\omega_{P1L} = 10$ rad/s; $\omega_{P2L} = 10^3$ rad/s

$\omega_{Z1H} = \infty$ rad/s; $\omega_{P1H} = 10^6$ rad/s

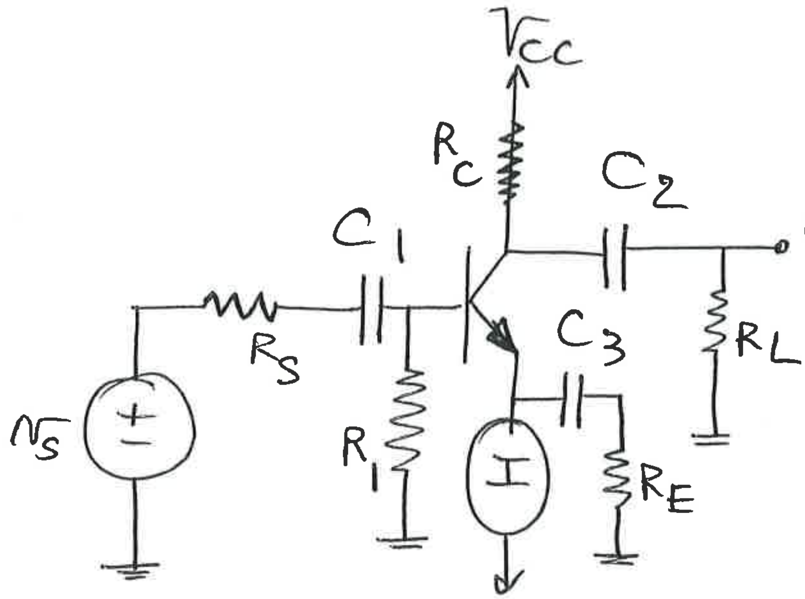


ω	1	10	10^2	10^3	10^4	10^5
|A(s)|	-40dB	0	20	40	40	40.

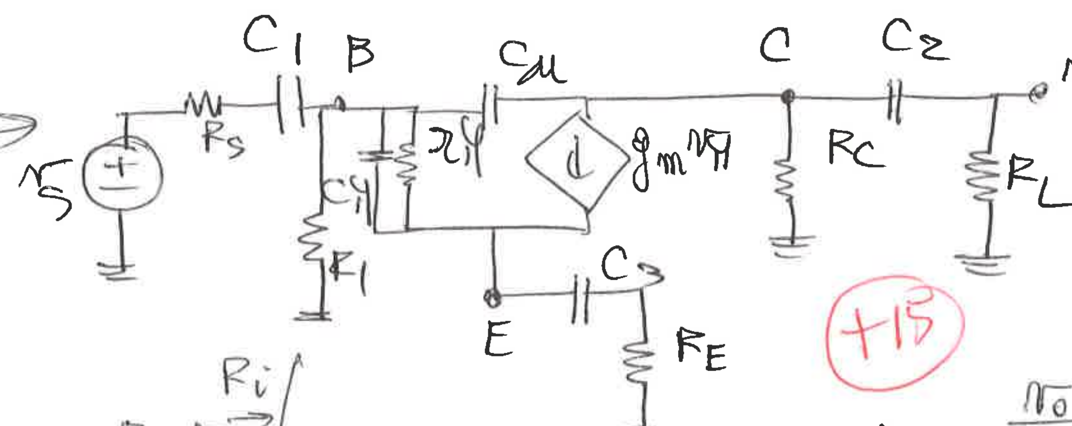


Custom Graph™
40dB
40dB
+20dB/dec.
−20dB/dec
20dB
0dB
+40dB/dec
−20dB
−40dB
10
10^2
10^3
10^4
10^5
10^6
(rad/s)
2&3 Per Inch Semi-Log 5 Cyc Base10 on X-Axis Gray&Watermark MC - Land Letter
http://customgraph.com

II. (30 pts): For the amplifier shown below

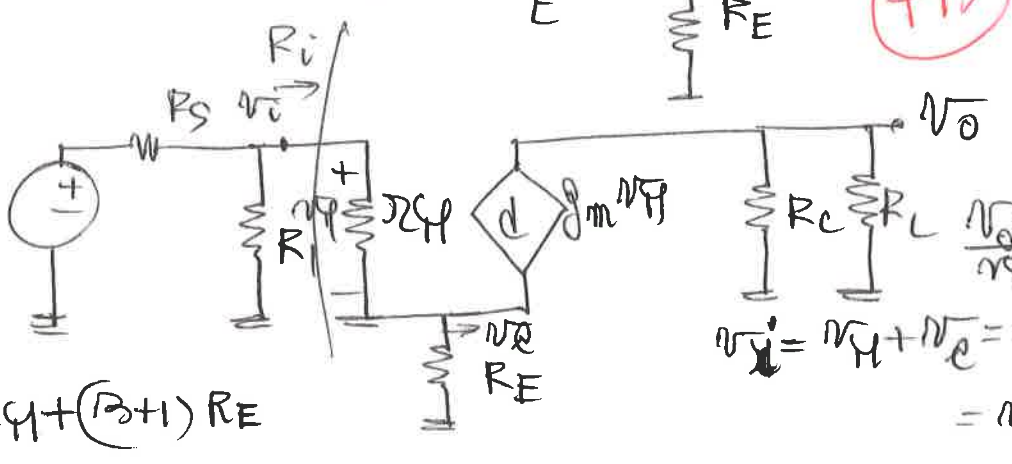


- What type of configuration is the amplifier (circle correct answer): (a) common emitter, (b) common base, (c) or common collector? +5
- Draw the small ac equivalent circuit of the network (hybrid - π model) and derive an analytical expression for the midband voltage gain of the amplifier. Neglect the effects of the r_x and r_0 when drawing the small AC equivalent circuit.
- Use the Gray-Searles procedure to derive an analytical expression of the low frequency pole associated to the coupling capacitor C_1.



+15

A_m



$$\frac{v_o}{v_s} = \frac{v_o}{v_\pi} \cdot \frac{v_\pi}{v_i} \cdot \frac{v_i}{v_s}$$

$$\frac{v_o}{v_\pi} = -g_m (R_C // R_L)$$

$$v_i = v_\pi + v_e = r_\pi i_\pi + (g_m v_\pi + i_\pi) R_E$$
$$= v_\pi + (g_m r_\pi + 1) \frac{v_\pi R_E}{r_\pi}$$

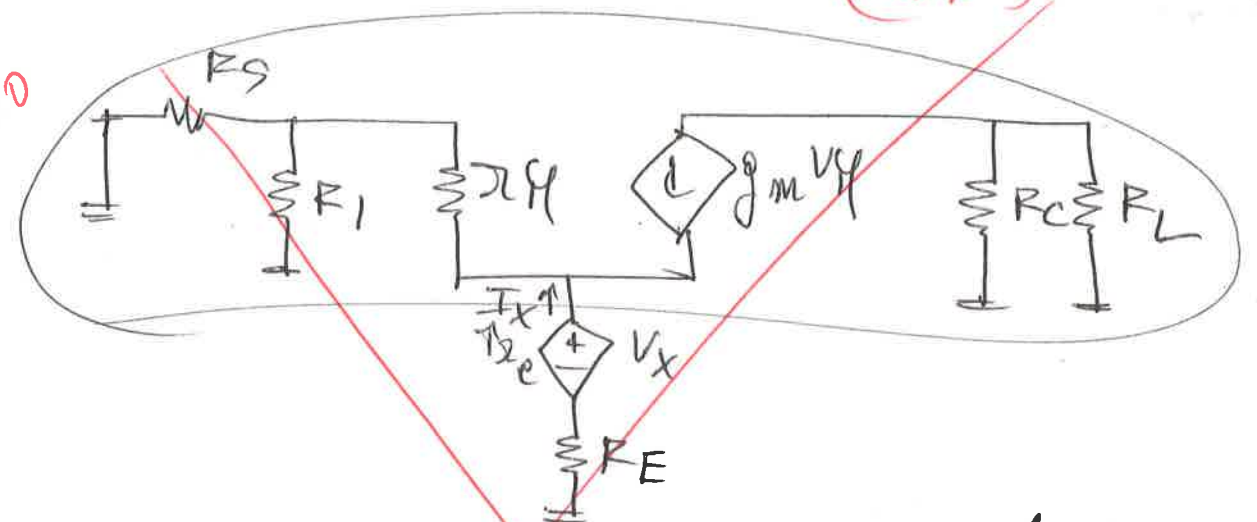
$R_i = r_\pi + (\beta+1) R_E$

$$A_v = \frac{v_o}{v_s} = -\frac{g_m(R_C//R_L)}{\left[1+(\beta+1)\frac{R_E}{r_\pi}\right]} \cdot \left\{\frac{[r_\pi+(\beta+1)R_E]//R_1}{R_S+[r_\pi+(\beta+1)R_E]//R_1}\right\}$$

$$\frac{(R_{in}//R_1)}{(R_{in}//R_1)+R_S}$$

Gray-Searle

+10

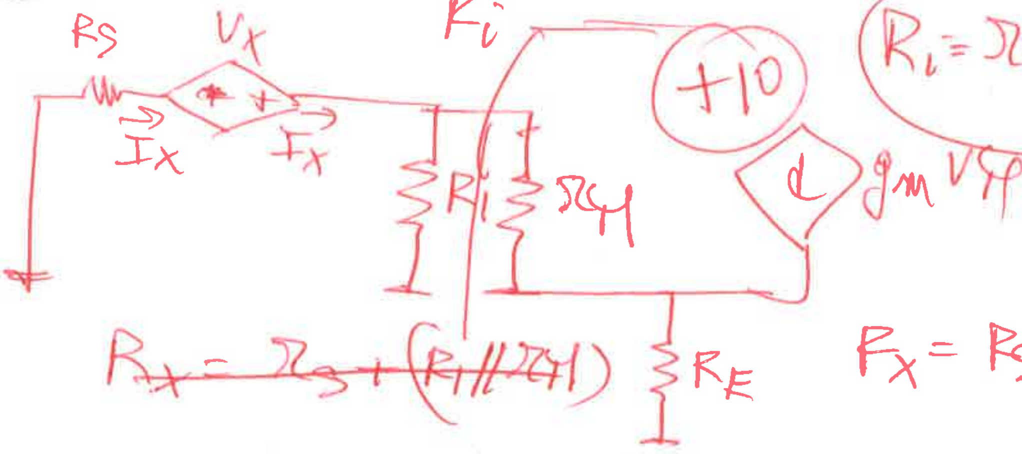


$$r_e = \frac{V_x}{I_x} = \frac{-I_\pi\left[r_\pi + (R_1//R_S)\right]}{-I_\pi - g_m v_\pi} = \frac{r_\pi + (R_1//R_S)}{1+\beta}$$

$$\rightarrow R_{C_3} = \frac{V_x}{I_x} = (r_e + R_E)$$

$$\rightarrow \boxed{\omega_{C_3} = \frac{1}{R_{C_3}C_3}}$$

(c)



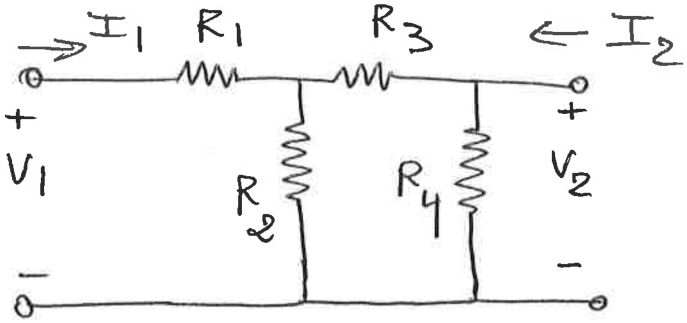
+10

$$R_i = r_\pi + (\beta+1)R_E$$

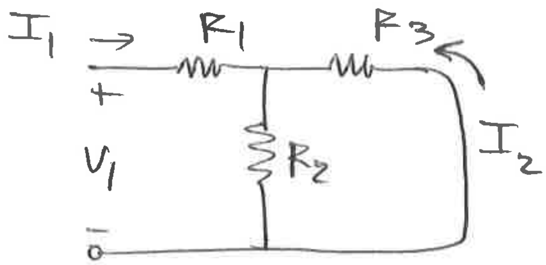
$$\cancel{R_x = r_s + (R_1//r_\pi)}$$

$$R_x = R_S + (r_\pi//R_i)$$

III. (20 pts): Find the analytical expressions for the h_{21} and g_{12} parameters of the following feedback network. Make sure to specify the units of each parameter.



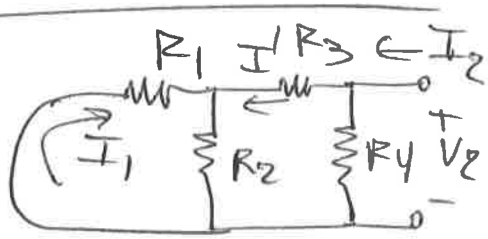
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$



$$I_2 = -\frac{R_2}{R_2+R_3} I_1$$

$$\boxed{h_{21} = \frac{-R_2}{R_2+R_3}}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$



$$I' = \frac{R_4}{\left[R_4+R_3+(R_1 \| R_2)\right]} I_2$$

$$I_1 = \frac{-R_2}{R_1+R_2} I'$$

$$I_1 = -\frac{R_2R_4}{(R_1+R_2)} \frac{1}{\left[R_3+R_4+(R_1 \| R_2)\right]} I_2$$

$$\boxed{g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{-R_2R_4}{(R_1+R_2)\left[R_3+R_4+(R_1 \| R_2)\right]}}$$

Basic current Amplifier A Ideal $R_i = 0$

$R_o = \infty$

IV. (20 pts): A shunt-series feedback amplifier uses a basic amplifier with a gain of 500A/A and an input and output resistances R_i and R_o equal to 0.5 $k\Omega$ and 100 $k\Omega$, respectively. The feedback factor β_f is 0.2A/A. Find the gain A_f, the input resistance, $R_{i,f}$ and output resistance $R_{o,f}$ of the amplifier with feedback.

$$A_f = \frac{A}{1 + B_f A} = \frac{500}{1 + 0.2(500)} = \frac{500}{101}$$

$$A_f \simeq 5$$

$$R_{if} = R_i / (1 + B_f A) = \frac{0.5 k\Omega}{101} \simeq 5\Omega$$

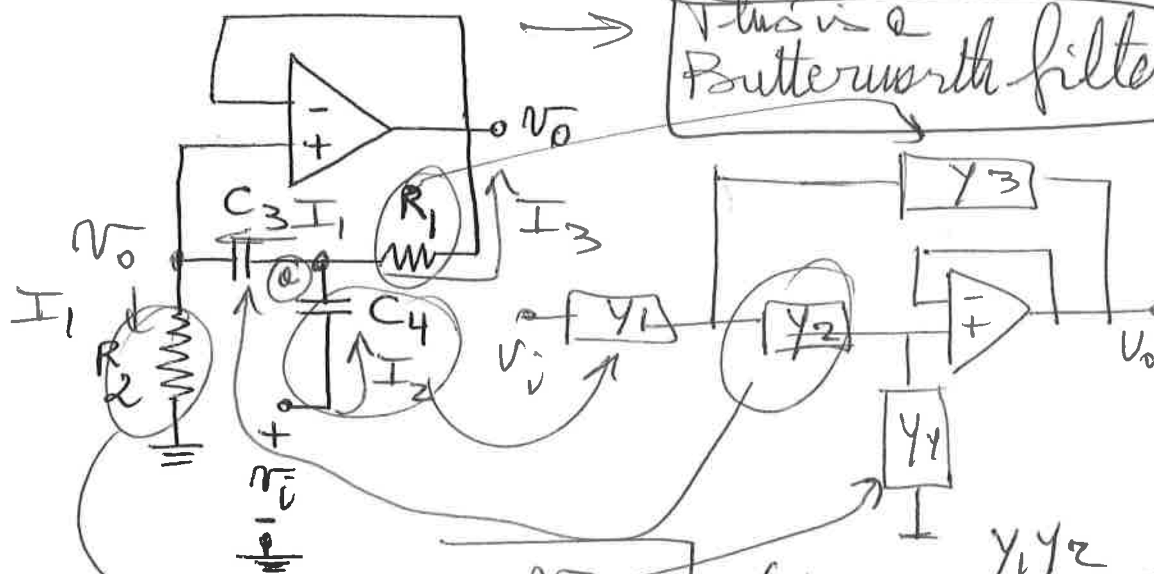
$$R_{of} = R_o (1 + B_f A) = 100 k\Omega \; 101 \simeq 10 M\Omega$$

V. (30 pts): Derive the transfer function T(s) = V_0/V_i for the filter below. Assume the op-amp is ideal.

(1) Explain qualitatively why this filter is a high-pass filter.

(2) Show that T(s) is of the form $\frac{a_2 s^2}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2}$ and determine the expressions of a_2 , ω_0 and Q as a function of the components of the circuit (resistors and capacitors). other solution

$\boxed{\text{This is a Butterworth filter}}$



$$T(s) = \frac{Y_1Y_2}{Y_1Y_2+Y_4(Y_1+Y_2+Y_3)}$$

$v_+ = v_- = v_o \rightarrow I_1 = \frac{v_o}{R_2}$

$$v_a - v_o = \frac{I_1}{sC_3}$$

$$v_a = v_o + \frac{I_1}{sC_3} = v_o + \frac{v_o}{sR_2C_3} = \left(1+\frac{1}{sR_2C_3}\right)v_o$$

$$I_2 = sC_4(v_i - v_a)$$

$$I_2 = I_1 + I_3$$

$$sC_4(v_i - v_a) = sC_3(v_a - v_o) + \frac{v_a - v_o}{R_1}$$

$$sC_4 v_i = s(C_3+C_4)v_a + \frac{v_a}{R_1} - \frac{v_o}{R_1} - sC_3 v_o$$

$$sC_4 v_i = \left(s(C_3+C_4)+\frac{1}{R_1}\right)\left(1+\frac{1}{sR_2C_3}\right)v_o - \frac{v_o}{R_1} - sC_3 v_o$$

$$v_o \left\{ \left(\frac{1 + sR_1(C_3+C_4)}{R_1} \right) \left(\frac{1 + sR_2C_3}{sR_2C_3} \right) - \frac{1}{R_1}(1 + sR_1C_3) \right\}$$

$$= sC_4 v_i$$

$$\frac{v_o}{v_i} = \frac{s^2 R_1 R_2 C_3 C_4}{\left(1 + sR_1(C_3+C_4)\right)\left[1 + sR_2C_3\right) - sR_2C_3(1 + sR_1C_3)}$$

$$T(s) = \frac{s^2 R_1 R_2 C_3 C_4}{1 + \cancel{s^2 R_1R_2C_3(C_3+C_4)} + s^2R_1R_2C_3(\cancel{C_3}+C_4) \\ sR_1(C_3+C_4) + \cancel{sR_2}C_3 - \cancel{sR_2}C_3 - \cancel{s^2R_1R_2C_3^2}}$$

$$T(s) = \frac{s^2 R_1 R_2 C_3 C_4}{1 + s^2 R_1R_2C_3C_4 + sR_1(C_3+C_4)}$$

$$T(s) = \frac{s^2}{s^2 + s\frac{R_1}{R_1R_2}\frac{(C_3+C_4)}{C_3C_4} + \frac{1}{R_1R_2C_3C_4}}$$

$$= \frac{a_2 s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

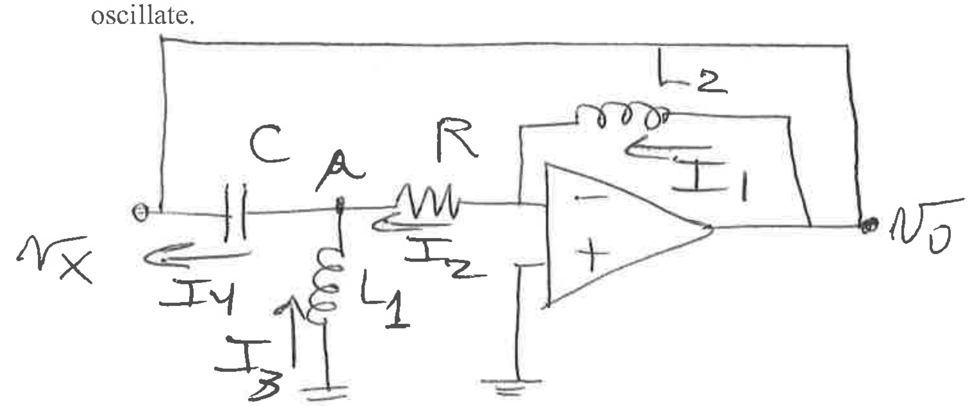
$$\boxed{Q = \sqrt{\frac{R_2}{R_1}}\sqrt{\frac{C_3C_4}{C_3+C_4}}}$$

$$a_2 = 1 \qquad \omega_0 = \frac{1}{\sqrt{R_1R_2C_3C_4}} \qquad \frac{\omega_0}{Q} = \frac{1}{R_2(C_3 \| C_4)} \rightarrow Q = \frac{R_2(C_3\|C_4)\,\omega_0}{\cancel{\sqrt{R_1R_2C_3C_4}}}$$

VI. (30 pts): Consider the circuit below used to build an oscillator. The op-amp is assumed to be ideal, i.e, it has an infinite impedance resistance looking into the positive and negative terminals of the op-amp. Relate first v_x to v_o using the fact that for the ideal op-amp $v_+ = v_-$. Then, set $v_x = v_o$. You will end up with an equation of the form

$$v_o(a + jb) = 0. \tag{2}$$

- The only way that equation can be satisfied for a non-zero v_o is if both a and b are equal to zero. The condition b = 0 will give you the frequency of oscillation of the circuit in terms of the passive elements in the network. Write the explicit analytical expression for the frequency of oscillation.
- The condition a = 0 will give you the relation betweem the passive elements in the network for the oscillations to be observed. Write down the explicit relation which must be satisfied by the passive components for the circuit to oscillate.



$$v_o = sL_2 I_1$$

$$I_1 = I_2 = \frac{v_o}{sL_2}$$

$$v_a = -RI_2 = -v_o \frac{R}{sL_2}$$

$$I_3 = \frac{0 - v_a}{sL_1} = \frac{v_o R}{s^2 L_1 L_2}$$

$$I_y = I_1 + I_2$$

$$= \frac{V_o}{sL_2} + \frac{V_o R}{s^2 L_1 L_2}$$

$$= \frac{V_o}{sL_2}\left(1 + \frac{R}{sL_1}\right)$$

$$V_x = V_a - \frac{1}{sC} I_4$$

$$V_o - V_x = -\frac{V_o R}{sL_2} - \frac{1}{sC}\frac{V_o}{sL_2}\left(1 + \frac{R}{sL_1}\right)$$

$$V_o\left[1 + \frac{R}{sL_2} + \frac{1}{s^2 L_2 C}\left(1 + \frac{R}{sL_1}\right)\right] = 0$$

$$1 - j\frac{R}{\omega L_2} - \frac{1}{\omega^2 L_2 C} + \frac{R}{s^3 L_1 L_2 C} = 0$$

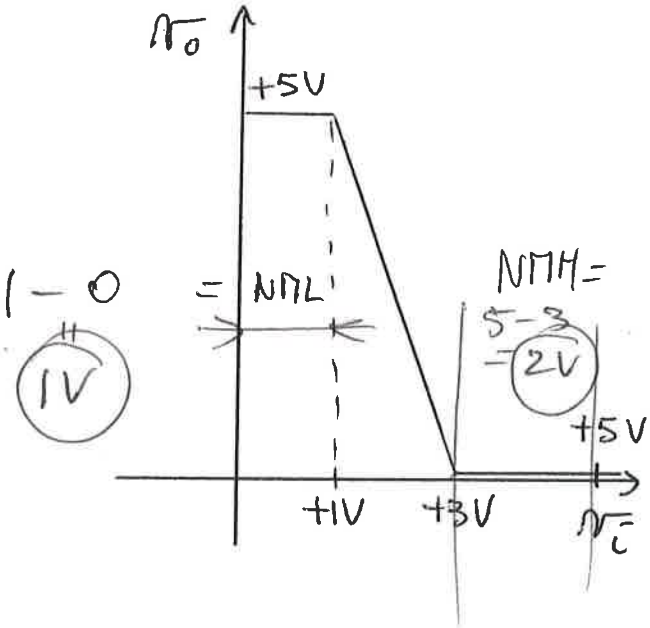
$$\left(1 - \frac{1}{\omega^2 L_2 C}\right) - j\frac{R}{\omega L_2} - \frac{R}{j\omega\, \omega^2 L_1 L_2 C} = 0$$

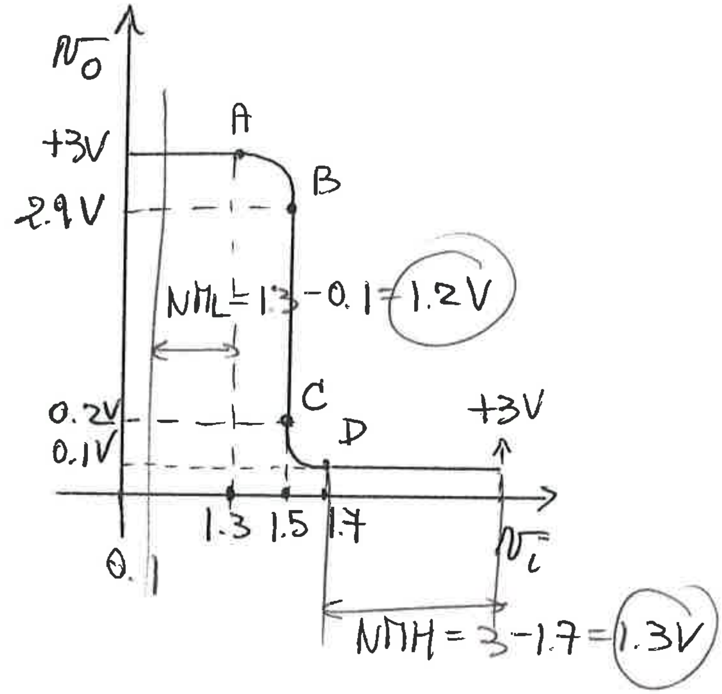
$$\omega = \frac{1}{\sqrt{L_2 C}} \qquad -j\frac{R}{\omega L_2} + j\frac{R}{\omega^3 L_1 L_2 C} = 0$$

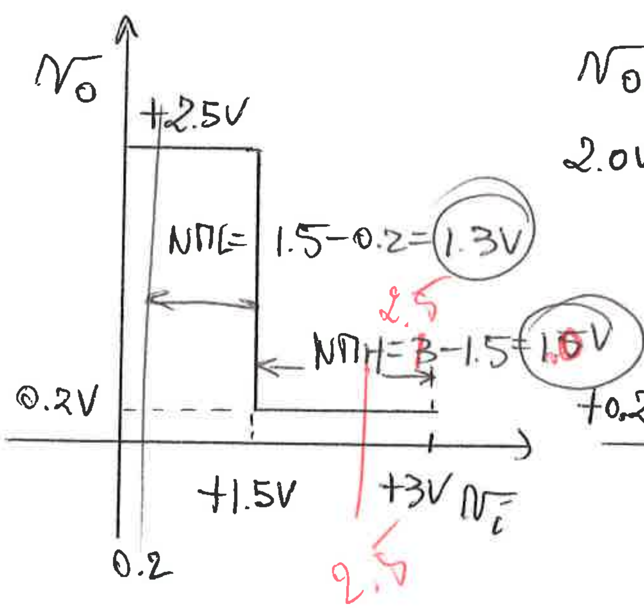
$$\frac{\cancel{R}}{\cancel{\omega} L_2} = \frac{\cancel{R}}{\omega^{\cancel{3}2} L_1 L_2 C}$$

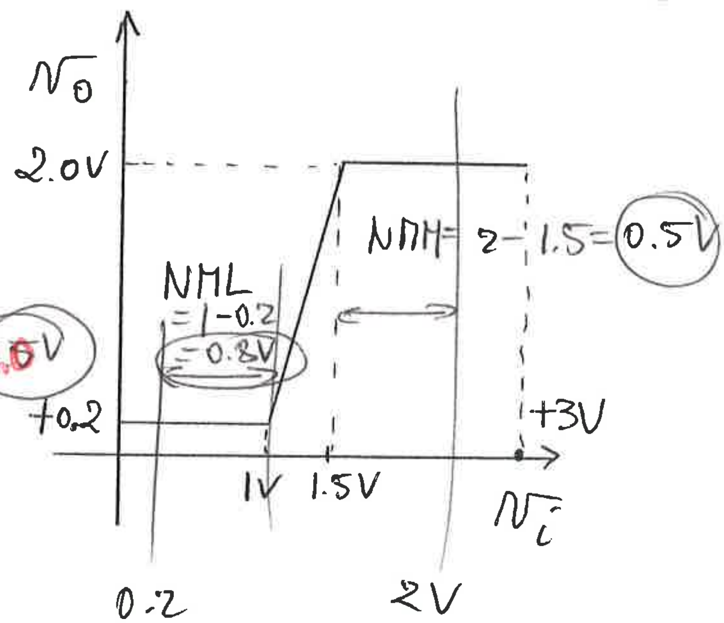
$$\frac{1}{L_2} = \frac{\cancel{L_2} C}{L_1 \cancel{L_2} C} \rightarrow L_1 = L_2$$

VII. (20 pts): For each of the inverter transfer characteristics below, calculate the noise margins for both the low (NML) and high state (NMH) of the inverter. Indicate how you calculate these values from each graph. The low state is defined as low voltage range for the input and the high state is defined as the high voltage range of the output.

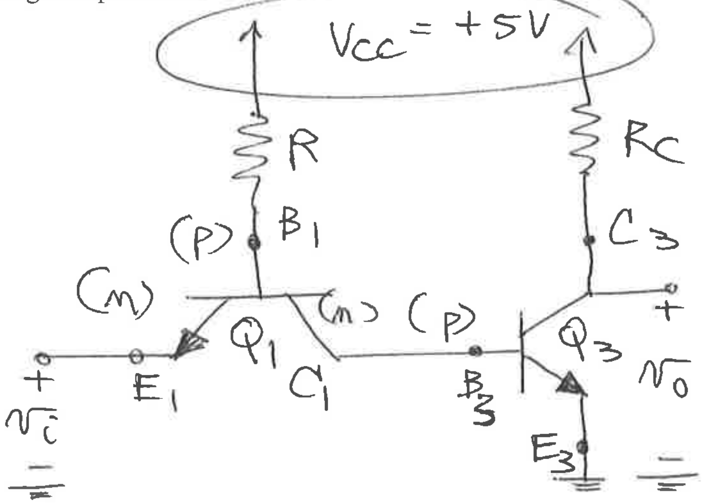








VIII. (20 pts): In the circuit shown below, assume that the forward bias across the EB junction of Q1 and Q3 is 0.7 V when they start conducting and answer the following ten questions.



1. When $v_i = 0.2$ V, the transistor Q1 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.
2. When $v_i = 0.2$ V, v_O will be equal to 5 V.
3. When $v_i = 0.2$ V, the transistor Q3 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.
4. When v_i = 5V, the transistor Q1 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.
5. When v_i = 5V, the transistor Q3 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.
6. When $v_i = 2.5$V, the pn Base-Emitter junction of Q1 is forward biased, TRUE or FALSE.
7. When $v_i = 2.5$V, Q1 and Q3 are both in the cutoff mode, TRUE or FALSE.
8. When $v_i = 5$V, $V_{B1,C1} = V_{B3,E3} = 0.8$ V and . $V_{C3,E3}$ 0.1 V, TRUE or FALSE.
9. When $v_i = 0.2$V, the current I_R mostly flows into the emitter of Q1, TRUE or FALSE.
10. When $v_i = 0.4$V, $V_{B1,C1} = V_{B3,E3} =$ ... 0.55 V.