Feedback Effect on Amplifier with 2 poles

$\boxed{\text{pole mixing}}$

$$A(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{PH_1}}\right)\left(1+\frac{s}{\omega_{PH_2}}\right)}$$

$$A_f(s) = \frac{A(s)}{1+\beta_f A(s)}$$

$$A_f(s) = \frac{\dfrac{A_0}{\left(1+\frac{s}{\omega_{PH_1}}\right)\left(1+\frac{s}{\omega_{PH_2}}\right)}}{1+\beta_f \dfrac{A_0}{\left(1+\frac{s}{\omega_{PH_1}}\right)\left(1+\frac{s}{\omega_{PH_2}}\right)}}$$

$$A_f(s) = \frac{A_0}{\left[1+\frac{s}{\omega_{PH_1}}\right]\left[1+\frac{s}{\omega_{PH_2}}\right]+\beta_f A_0}$$

$$A_f(s) = \frac{A_0}{1+\beta_f A_0 + s\left[\frac{1}{\omega_{PH_1}}+\frac{1}{\omega_{PH_2}}\right]+s^2\left(\frac{1}{\omega_{PH_1}\omega_{PH_2}}\right)}$$

$$A_f(s) = \frac{\dfrac{A_0}{1+\beta_f A_0}}{\left[1+\dfrac{s}{1+\beta_f A_0}\left[\dfrac{1}{\omega_{PH_1}}+\dfrac{1}{\omega_{PH_2}}\right]+\dfrac{s^2}{\omega_{PH_1}\omega_{PH_2}(1+\beta_f A_0)}\right]}$$

$$A_f(s) = \frac{A_{f_0}}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}}$$

Define $\omega_0^2 = \omega_{PH_1}\omega_{PH_2}(1+B_f A_0)$

or $\boxed{\omega_0 = \sqrt{\omega_{PH_1}\omega_{PH_2}(1+B_f A_0)}}$

Q is defined such that

$$\frac{1}{\omega_0 Q} = \frac{1}{[1+B_f A]}\left[\frac{1}{\omega_{PH_1}} + \frac{1}{\omega_{PH_2}}\right]$$

$$Q = \frac{\overbrace{(1+B_f A)\,\omega_{PH_1}\omega_{PH_2}}^{\omega_0^2}}{\omega_0\left[\omega_{PH_1} + \omega_{PH_2}\right]}$$

$$\boxed{Q \triangleq \frac{\omega_0}{\omega_{PH_1} + \omega_{PH_2}}}$$

$$\Rightarrow \quad A_f(s) = \frac{\overbrace{A_{f_0}}^{\left(\frac{A_0}{1+B_f A_0}\right)}}{\boxed{1 + \frac{s}{\omega_0 Q}} + \frac{s^2}{\omega_0^2}}$$

pole splitting

Proximity of the other pole

location of new poles of amplifier with feedback

$$1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2} = 0 \quad \text{or} \quad s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0.$$

$$s = \frac{-\frac{\omega_0}{Q} \pm \sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4 \cdot \omega_0^2}}{2}$$

<u>pole mixing</u>

$$s = \frac{-\omega_0}{2Q} \pm \left(\frac{\omega_0}{2Q}\right)\sqrt{1 - 4Q^2}$$

$$Q = \frac{\sqrt{\omega_{PH_1}\,\omega_{PH_2}(1+\beta_f A_0)}}{\omega_{PH_1} + \omega_{PH_2}}$$

with no feedback, Q reaches a minimum

$$\boxed{Q_{min} = \frac{\sqrt{\omega_{PH_1}\,\omega_{PH_2}}}{\omega_{PH_1} + \omega_{PH_2}}}$$

$$s = -\omega_{PH_1}, -\omega_{PH_2}. \quad \text{(original poles)}$$

If $\beta_f > 0$. $\quad \beta_f A_0 > 0$. , $Q \uparrow$

until $1 - 4Q^2 = 0 \rightarrow Q = 0.5$

$$\boxed{s = -\frac{\omega_0}{2Q} = -\left[\frac{\omega_{PH_1} + \omega_{PH_2}}{2}\right]}$$

$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2 = 0$$

$$s = \frac{-\omega_0}{2Q} \pm \frac{\omega_0}{2Q}\sqrt{1-4Q^2} = -\frac{(\omega_{PH1}+\omega_{PH2})}{2} \pm \frac{\omega_0}{2Q} j\sqrt{4Q^2-1}$$

$$\omega_0 = \sqrt{\omega_{PH1}\,\omega_{PH2}\,(1+\beta f A_0)}$$

$$Q = \frac{\omega_0}{[\omega_{PH1}+\omega_{PH2}]}$$

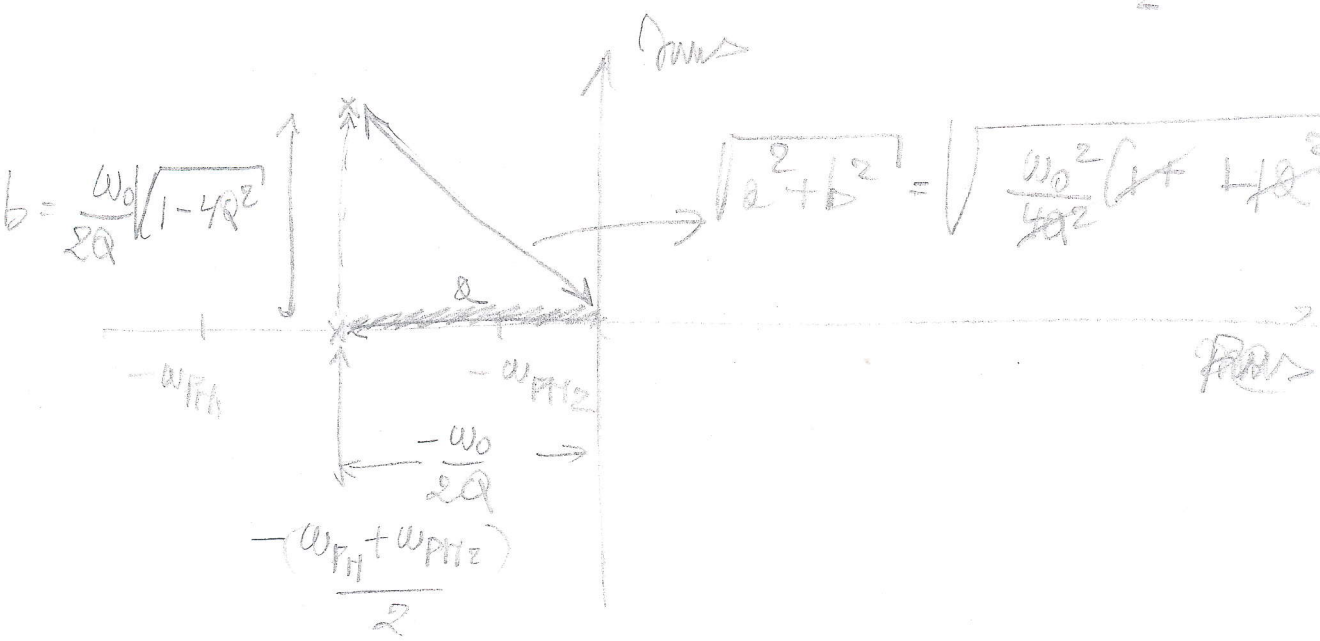
if $Q > 0.5$

$\beta f = 0$

$$s = -\frac{(\omega_{PH1}+\omega_{PH2})}{2} \pm \left(\frac{\omega_{PH1}+\omega_{PH2}}{2}\right)\sqrt{1-\frac{4\,\omega_1^2\omega_2^2}{[\omega_1+\omega_2]^2}}$$

$$s = -\frac{1}{2}(\omega_1+\omega_2) \pm \left(\frac{\omega_1+\omega_2}{2}\right)\sqrt{\frac{(\omega_1-\omega_2)^2}{(\omega_1+\omega_2)^2}}$$

$$= -\frac{1}{2}(\omega_1+\omega_2) \pm \frac{(\omega_1-\omega_2)}{2} = \begin{matrix} -\omega_1 \\ -\omega_2 \end{matrix}$$



$$\sqrt{a^2+b^2} = \sqrt{\frac{\omega_0^2}{\cancel{4Q^2}}(\cancel{1}+\cancel{4Q^2}-\cancel{1})} = \omega_0$$

Second-Order Filter Functions

<u>Generic form</u>

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + \frac{\omega_0^2}{Q} + \omega_0^2}$$

Magnitude of $T(s)$ for Low pass and order filter when

$$a_1 = 0$$
$$a_2 = 0$$
$$a_0 \neq 0$$

Then

$$T(s) = \frac{a_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$s = j\omega$

$$\Rightarrow T(j\omega) = \frac{a_0}{\left[-\omega^2 + \omega_0^2 + \frac{\omega_0}{Q} j\omega\right]}$$

$$\boxed{|T(j\omega)| = \frac{a_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega_0 \omega}{Q}\right)^2}}}$$

Let's calculate magnitude of $T(s)$ at $s = j\omega$

$$|T(j\omega)| = \frac{a_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega_0 \omega}{Q}\right)^2}}$$

<u>Bode Plot</u> $|T(j\omega)|$ versus ω

If denominator reaches a minimum, $|T(j\omega)|$ will reach a maximum.

Let us calculate the frequency at which this occurs. We rewrite

$$|T(s=j\omega)| = \frac{a_0}{\sqrt{g(\omega)}}$$

$g(\omega)$ reaches a minimum when $\frac{d}{d\omega} g(\omega) = 0$

i.e,

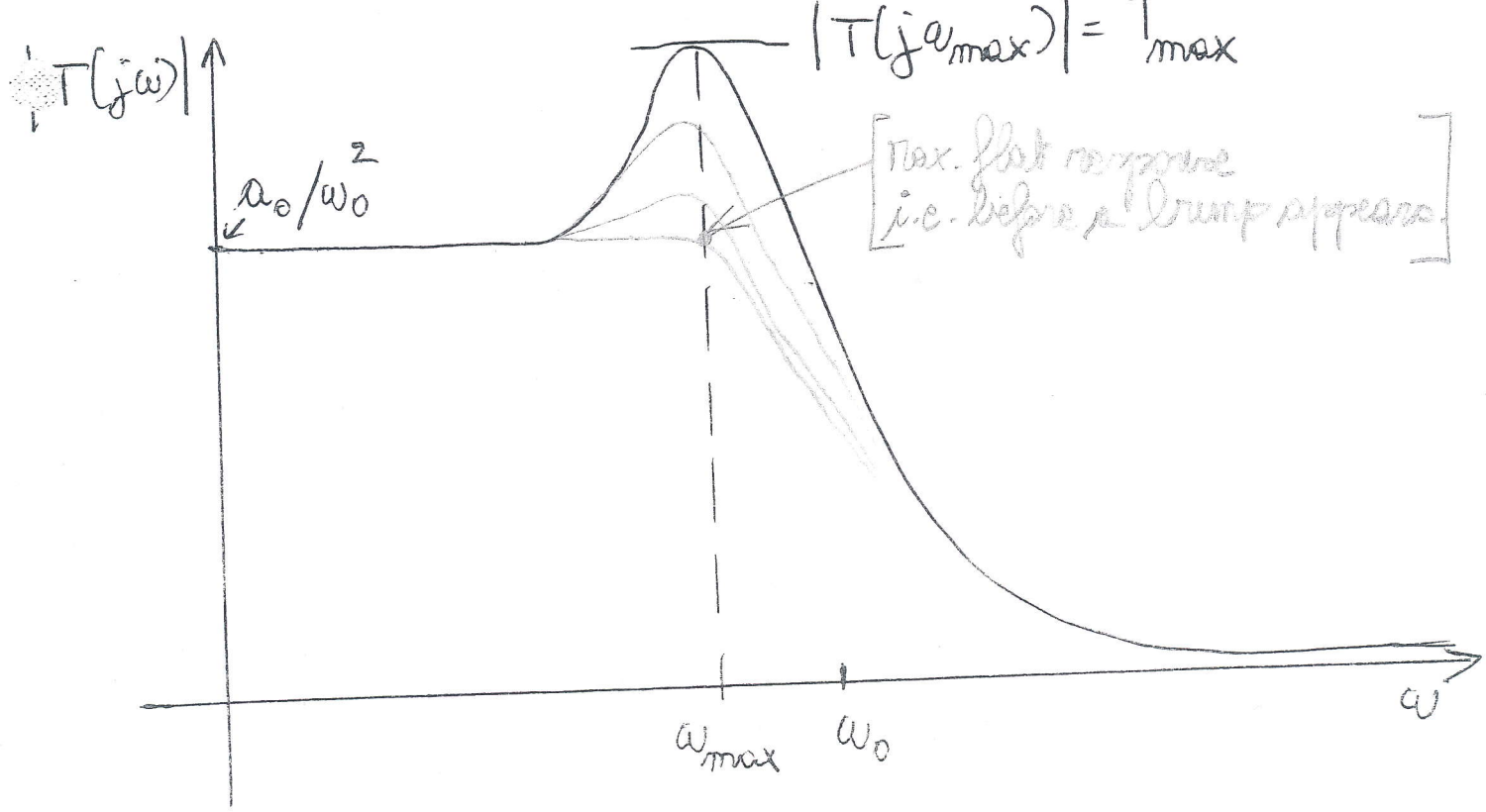
$$2(\omega_0^2 - \omega^2)(-2\omega) + 2\left(\frac{\omega_0 \omega}{Q}\right)\left(\frac{\omega_0}{Q}\right) = 0$$

Rearranging we get

$$2(\omega_0^2 - \omega^2) = \omega_0^2/Q^2$$

$$\Rightarrow \omega_{max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

Bode Plot (Magnitude)



$$\omega_{max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

Since $Q > 0$, $\omega_{max} < \omega_0$

T_{max} ?

$$T_{max} = \frac{a_0}{\sqrt{\left(\omega_0^2 - \omega_{max}^2\right)^2 + \left(\frac{a_0\,\omega_{max}}{Q}\right)^2}}$$

i.e.,

$$T_{max} = \frac{a_0}{\sqrt{\left[\omega_0^2 - \omega_0^2\left(1 - \frac{1}{2Q^2}\right)\right]^2 + \left(\frac{a_0}{Q}\right)^2 \omega_0^2\left(1 - \frac{1}{2Q^2}\right)}}$$

So

$$T_{max} = \frac{\alpha_0}{\sqrt{\omega_0^4\left[\frac{1}{2Q^2}\right]^2 + \frac{\omega_0^4}{Q^2}\left(1-\frac{1}{2Q^2}\right)}}$$

$$T_{max} = \frac{\alpha_0}{\omega_0^2\sqrt{\frac{1}{4Q^4} + \frac{1}{Q^2} - \frac{1}{2Q^4}}}$$

$$T_{max} = \frac{\alpha_0}{\omega_0^2\sqrt{\frac{1}{Q^2} - \frac{1}{4Q^4}}}$$

$$T_{max} = \frac{\alpha_0 Q}{\omega_0^2\sqrt{1-\frac{1}{4Q^2}}}$$

$$T_{max} = T(j\omega=0)\,\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}}$$

$T_{max} < T(0)$ as long as

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} < 1$$

So, there is no bump in $|T(j\omega)|$ until

$$\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} = 1$$

i.e.,

$$Q^2 = 1 - \frac{1}{4Q^2}$$

$$\rightarrow Q^2 + \frac{1}{4Q^2} = 1$$

This occurs when $Q = \frac{1}{\sqrt{2}} = 0.707$

This value of Q corresponds to what is called the maximally flat transfer function

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For $Q > \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.707$

There will be a bump (boss) appearing in the Bode plot of the magnitude of $|T(s)|$
