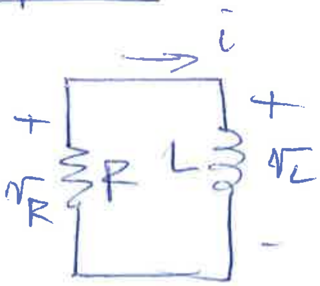
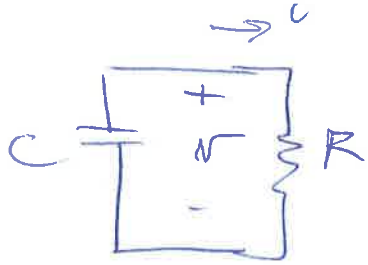
Review

Chapter 8 — Source free



$$\boxed{\frac{di}{dt} + \frac{R}{L}\,i = 0} \rightarrow \underline{\text{Sol}}: i(t) = I_0\, e^{-\frac{Rt}{L}}$$



$$\boxed{\frac{dv}{dt} + \frac{v}{RC} = 0}$$

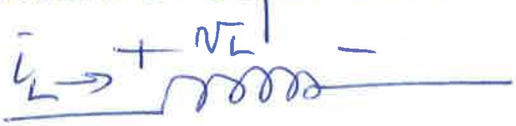
$$\underline{\text{Sol}}: v(t) = V_0\, e^{-\frac{t}{RC}}$$

Driven (forced) network

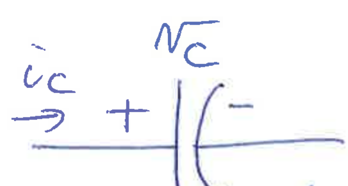
$$i(t) = \underset{\substack{\| \\ i(t\to+\infty)}}{I_f} + I_0\, e^{-\frac{Rt}{L}}$$

$$v(t) = \underset{\substack{\| \\ (v(t\to+\infty)}}{V_f} + V_0\, e^{-\frac{t}{RC}}$$

Constitutive equations



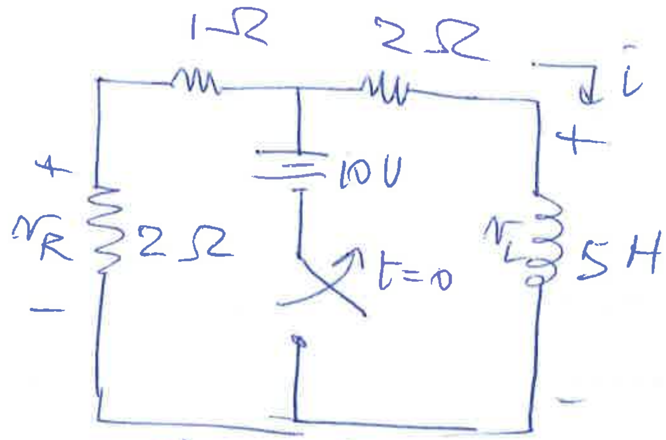
$$\boxed{v_L = L\,\frac{di_L}{dt}}$$



$$\boxed{i_C = C\,\frac{dv_C}{dt}}$$

Passive sign convention

(10)



(a) Write differential eq for v_R for $t>0$

(b) Solve characteristic equation

(c) v_R? Before opening the switch, just after opening switch, and at t = 1 s

(a) After opening switch.

$$L\frac{di}{dt} + \frac{R_{eq}}{\cancel{2}}\, i = 0 \qquad R_{eq} = 5\,\Omega$$

$$v_R = -2i$$

$$L\frac{d\left(-\frac{v_R}{2}\right)}{dt} + 5\left(-\frac{v_R}{2}\right) = 0$$

$$2.5\frac{dv_R}{dt} + 2.5\, v_R = 0 \qquad \frac{dv_R}{dt} + v_R = 0$$

$$v_R = Ae^{st}$$

(b) $Ase^{st} + Ae^{st} = 0$

$$s + 1 = 0$$

$$s = -1$$

$$\rightarrow v_R = Ae^{-t}$$

(c) $t = 0^-$ $\quad i(0^-) = 5\,A = i(0^+) \rightarrow v_R(0^+) = -2\,i(0^+) = -10$

$$\rightarrow A = -10$$

$$v_R(0^-) = \frac{2}{3}10 = 6.667\,V \qquad v_R(1) = -10e^{-1} = -3.679\,V$$

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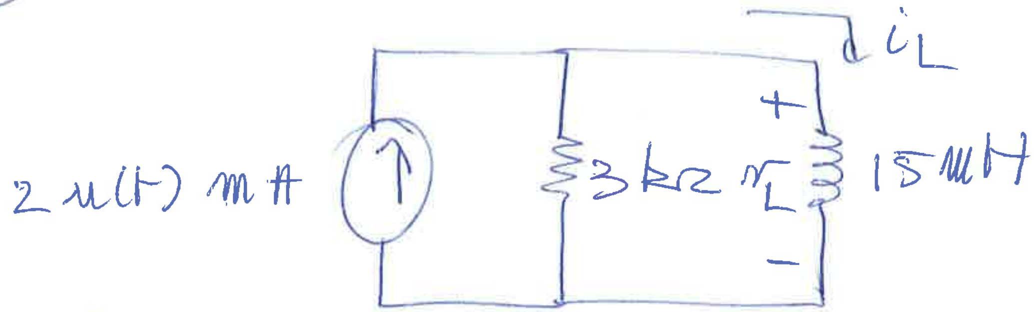


(a) Find $i_L(t)$

(b) Use $i_L(t)$ to get $v_L(t)$.

$$i_L(t) = i_f + Ae^{-\frac{t}{\tau}} = i_f + Ae^{-2000t}$$

$$\tau = \frac{L}{R} = \frac{15\,mH}{3\,k\Omega} = 5\,10^{-6}$$

$$\frac{1}{\tau} = 2\,10^{5}$$

$t \to \infty$ $\quad i_f = 2\,mA$

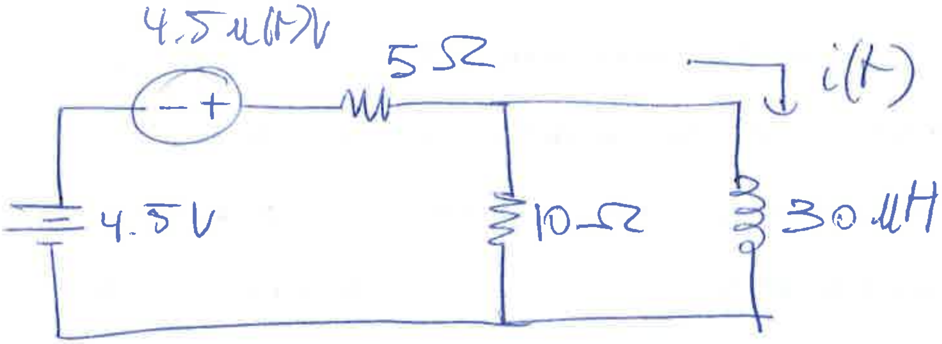
$t = 0^-$ $\quad i_L(0^-) = 0 = i_L(0^+) \longrightarrow A = -2\,mA$

$$\boxed{i_L(t) = \left[2 - 2e^{-2\,10^{5}t}\right]u(t)\ mA}$$

<u>t>0</u>

$$v_L = L\frac{di_L}{dt} = 15\,10^{-3}\left(-2\,10^{-3}\right)\left(-2\,10^{5}\right)e^{-2\,10^{5}t}$$

$$\boxed{v_L(t) = 6e^{-2\,10^{5}t}\ V}$$



(a) $i(t)\ \forall t$ (b) $i(t)$ at $t = 1.5\,\mu s$.

Natural response $\tau = \frac{L}{R_{TH}} = \frac{30\,10^{-6}}{(5//10)} = 9\,10^{-6}\,s$

$$i(t) = i_f + A\,e^{-\frac{10^6}{9}t}$$

$$i_f = \frac{9}{5}\,A.$$

$$i(t) = \frac{9}{5} + A\,e^{-\frac{10^6}{9}t}$$

At $t=0$ $\quad i(0^-) = i(0^+) = \frac{4.5}{5} \longrightarrow A = -\frac{4.5}{5} = 0.9$

So $\quad i(t) = \frac{9}{5} - 0.9\,e^{-\frac{10^6}{9}t}\ A$

(b) at $t = 1.5\,\mu s$

$$\boxed{i(t) = 1.038\,A}$$

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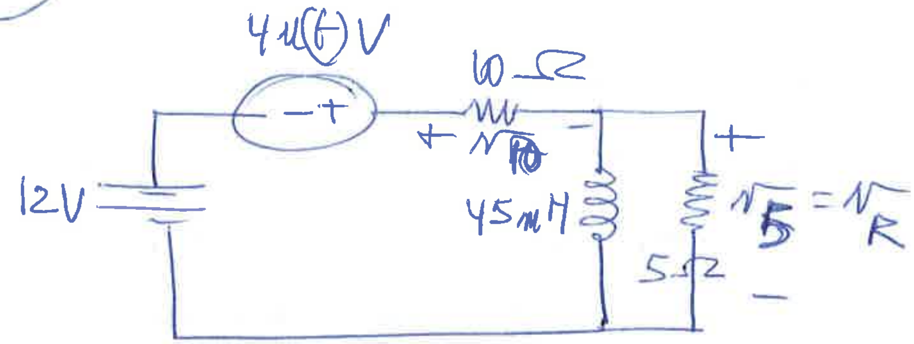

(a) Find $v_R(t)$ $\forall t$

(b) $v_R(t)$ at $t = 2ms$

(a) $\tau = \frac{L}{R_{eq}} = \frac{45 \, 10^{-3}}{(5//10)} = 0.0135s$

$v_R(t) = v_f + v_n$

$v_f = 0$ since inductor acts as a short

$v_R(t) = A e^{-74.07t}$

$t = 0^-$ $i_L(0^-) = \frac{12}{10} = 1.2A = i_L(0^+)$

at $t = 0^+$ KVL $12 + 4 = v_R(0^+) + 10\left[1.2 + \frac{v_R(0^+)}{5}\right]$

$\rightarrow v_R(0^+) = \frac{4}{3} V$

$\rightarrow \boxed{v_R(t) = \frac{4}{3} e^{-74.07t}}$

(b) at $t = 2ms$ $\boxed{v_R(2ms) = 1.15V}$

(69) The switch in circuit below has been open for a long time. (a) Find $i_L(t)$ for $t<0$. (b) $i_L(t)$ for $t>0$

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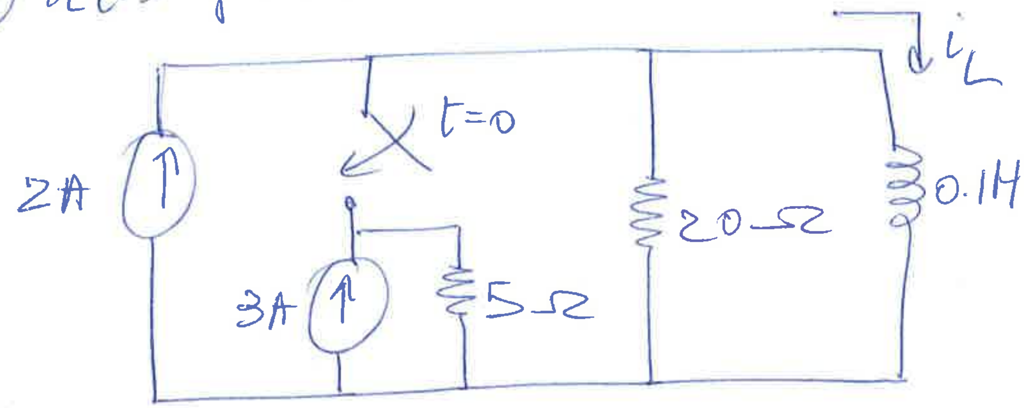
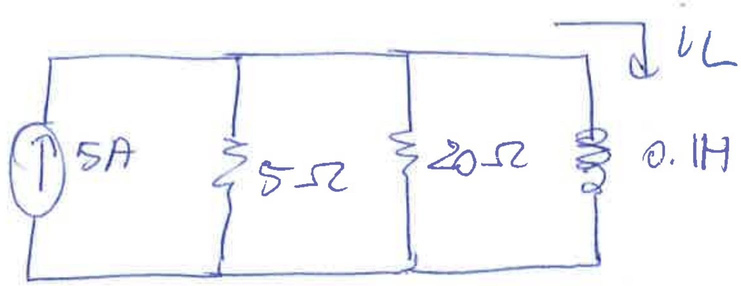


$t<0$ $\quad$ ~~$i_L(t)$~~ $= i_L(0^-) = 2A$

$t>0$



$$i_L(t) = i_f + A e^{-\frac{t}{\tau}} \qquad \tau = \frac{L}{(5 // 20)} = \frac{L}{4}$$

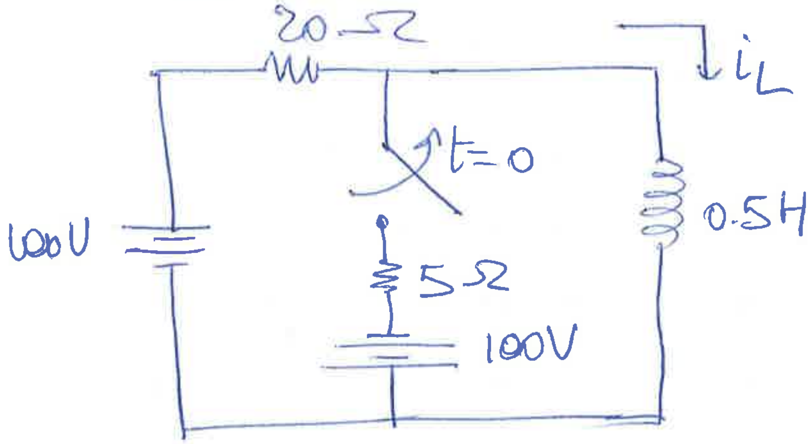
$$i_L(t) = \underset{\substack{\| \\ 5A}}{i_f} + A e^{-40t}$$

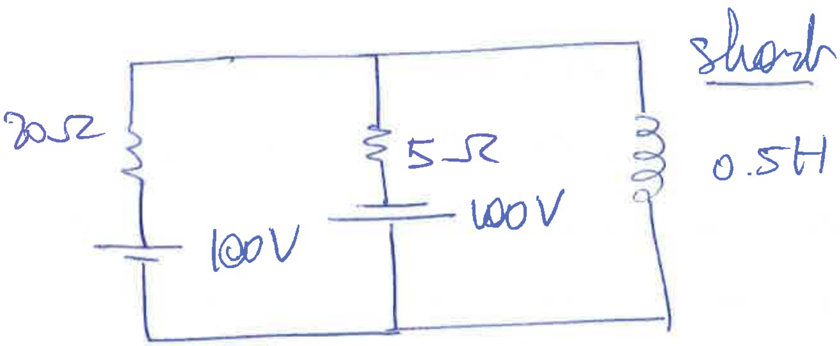
$\longrightarrow A = -3$ since $\boxed{i_L(0^+) = i_L(0^-) = 2A}$

$$\Longrightarrow \boxed{i_L(t) = 5 - 3e^{-40t} \ A}$$

72 a) i_L for $t<0$

b) $i_L(0^+)$ c) $i_L(\infty)$; d) $i_L(t)\ \forall t>0$





$$i_L(0^-) = \frac{100}{20} - \frac{100}{5} = -15A \quad t<0$$

$$i_L(0^+) = -15A$$

$$i_L(\infty) = 5A \qquad \tau = \frac{0.5}{20} = \frac{1}{40}$$

$$i_L(t) = i_L(\infty) + A e^{-\frac{t}{\tau}}$$

$$i_L(t) = 5A + A e^{-40t}$$

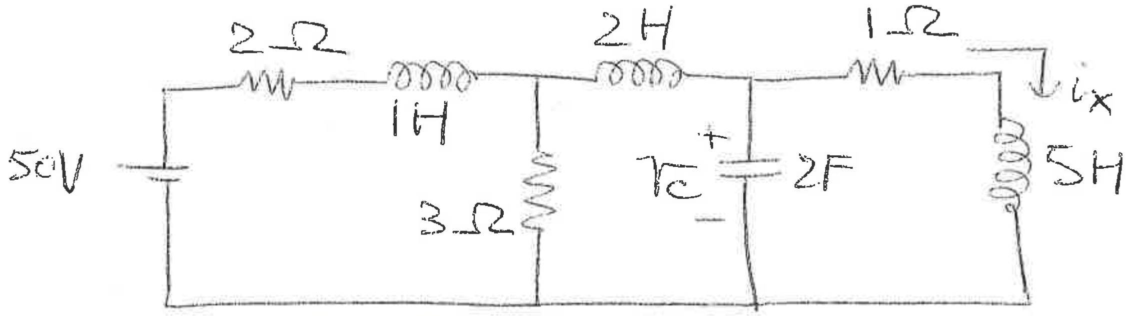
$$i_L(0^+) = -15 \rightarrow A = -20$$

$$\boxed{i_L(t) = (5 - 20e^{-40t})\ A}$$

Test 3 - Fall 2010 (100pts max) - M. Cahay
Name: Solution

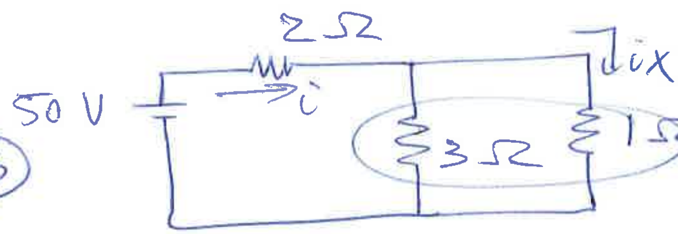
NETWORK ANALYSIS I
November 12, 2010

I. (25 pts) The circuit below has been connected a long time ago. Calculate the current i_x and the potential V_c across the capacitor.



long time $\left.\begin{array}{l} L = \text{short} \\ C = \text{open} \end{array}\right]$ (+5)

(+5)



$\rightarrow R_{eq} = \frac{3}{4}\,\Omega$

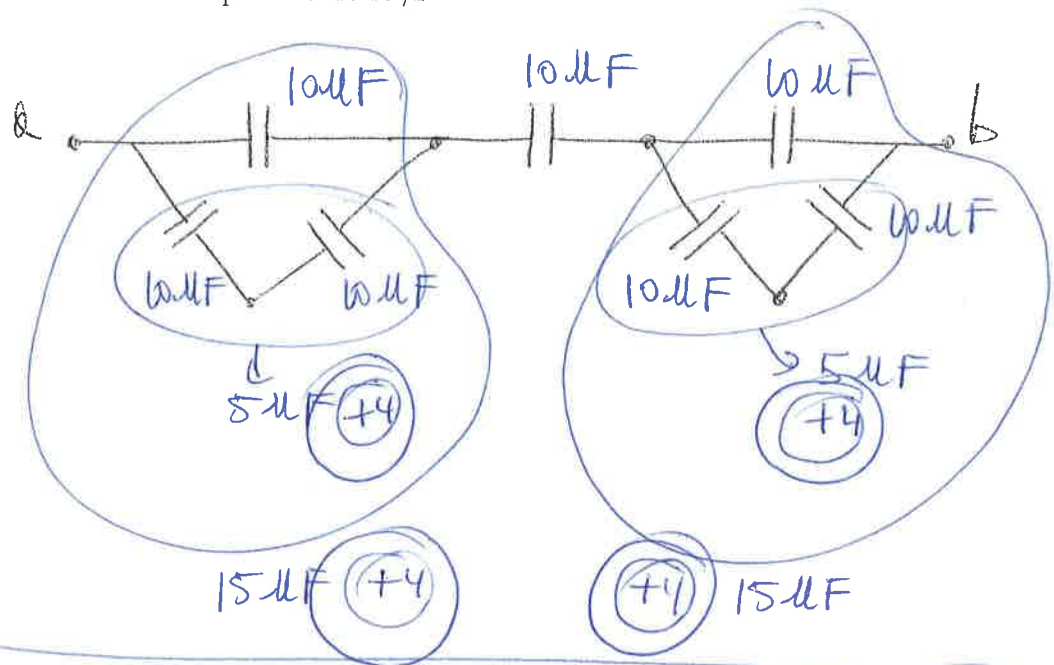
$$\boxed{i = \frac{50}{2.75} = 18.2\text{ A}}$$ (+5)

Current divider

$$\boxed{i_x = \frac{3}{4}\, i = 13.6\text{ A}}$$ (+5)

$$\boxed{V_c = 1\Omega \cdot i_x = 13.6\text{ V}}$$ (+5)

II. (25 pts) Determine the equivalent capacitance between points a and b for the network below if all capacitors are 10 μF.



$$\frac{1}{C_{eq}} = \frac{1}{15\,\mu F} + \frac{1}{15\,\mu F} + \frac{1}{10\,\mu F} = \frac{2}{15\,\mu F} + \frac{1}{10\,\mu F}$$ +7

$$C_{eq} = \frac{150\,\mu F}{20+15} = \frac{150}{35}\,\mu F = 4.286\,\mu F$$

+2

III. (25 pts) A voltage V(t) = 3 V for t < 0, and $3e^{-t/5}$ V, for t ≥ 0, is applied to a 300 μF capacitor.

(a) Compute the difference in energy stored on capacitor between time t=0s and t=2ms.

(b) Determine the current flowing through the capacitor at t = 1.2s.

a) $$w_C(t=2ms) - w_C(0) = \int_0^{t=2ms} p\,dt = \frac{1}{2}C\left[v^2(t=2ms) - v^2(0)\right]$$

$$= -1.080\,\mu J$$

+15

b) $$i_C = C\frac{dv_C}{dt} = 300\,10^{-6}\left(\frac{-3}{5}e^{-\frac{t}{5}}\right)\Big|_{t=1.2s}$$

+10

$$\Rightarrow i_C = -\frac{900}{5}10^{-6}e^{-\frac{1.2}{5}} = -141.593\mu A$$

IV. (25 pts) Starting with the time dependent signal for the inductor current i_L shown below, sketch the voltage v_L across the inductor as a function of time in the time interval $0 < t < 60$ ms.

$L = 0.2H$

$v_L = L\frac{di}{dt}$