

Quiz on filters
Tuesday May 22

10

NAME: *Solution*

A high-pass second order filter has a high-frequency gain equal to 20. The maximum of the transfer function is equal to 400. Find the Q-factor of the filter

$$T(s) = \frac{a_2 s^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$s \rightarrow \infty \quad |T(s)| = a_2 = 20.$$

$$\text{Max} \quad T_{\text{max}} = \frac{a_2 Q}{\sqrt{1 - \frac{1}{4Q^2}}} = 400.$$

$$\rightarrow \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} = 20$$

$$Q^2 = 400 \left(1 - \frac{1}{4Q^2}\right) = \frac{400}{4Q^2} (4Q^2 - 1)$$

$$Q^4 = 400Q^2 - 100 \quad \leftarrow Q^2 = \frac{100}{Q^2} (4Q^2 - 1)$$

$$Q^2 = y$$

$$y^2 - 400y + 100 = 0$$

$$\rightarrow y = \frac{400 \pm \sqrt{(400)^2 - 4 \cdot 100}}{2}$$

$$y_+ = \frac{400 + \approx 400}{2} = 400.$$

$$\rightarrow \boxed{Q \approx 20}$$