Test 1 - Spring 2011 (100pts max) - ECE 251
Name:

NETWORK ANALYSIS II
April 19, 2011

I. (15 pts) Using the table of trigonometric identities below, compare the two wave-forms v_1 and v_2 and determine which one is leading the other. Determine by what angle.

$$\boxed{\begin{aligned} -\sin\omega t &= \sin(\omega t \pm 180^\circ) \\ -\cos\omega t &= \cos(\omega t \pm 180^\circ) \\ \mp\sin\omega t &= \cos(\omega t \pm 90^\circ) \\ \pm\cos\omega t &= \sin(\omega t \pm 90^\circ) \end{aligned}}$$

- $v_1 = -33sin(8t - 9^o)$ $= -33\cos(8t - 9^\circ - 90^\circ)$
- and, $v_2 = 12cos(8t - 1^o)$

$\boxed{V_2 = 12\angle{-1^\circ}}$

$V_1 = -33\cos(8t - 99^\circ)$

$= 33\cos(8t - 99^\circ + 180^\circ) = 33\cos(8t + 81^\circ)$

$\boxed{V_1 = 33\angle 81^\circ}$

$\rightarrow$ V_1 leads V_2 by 82°

II. (15 pts) Express the following complex number in both rectangular and polar forms.

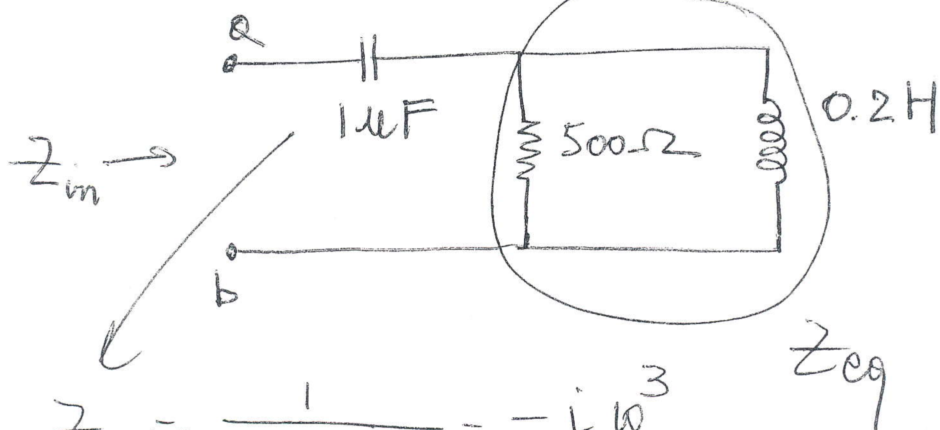
$\mathbf{Z} = (8 - j15)(4 + j16) - j$

$$Z = (8 - j15)(4 + j16) - j$$

$$= 32 + j128 - j60 + 240 - j$$

$$= 272 + j67 = 280.1 \angle 13.84^{\circ}$$

III. (20 pts) Consider the combination of impedances in the circuit below and determine the impedance Z_{in} seen between points a and b, if the circuit operates at the angular frequency $\omega = 10^3$ rad/s. Express the impedance Z_{in} both in rectangular and polar form.



$$Z_C = \frac{1}{j\,10^3\,10^{-6}} = -j\,10^3$$

$$Z_L = j\,10^3(0.2) = 200j$$

$$Z_{eq} = \frac{Z_L R}{R + Z_L} = \frac{500(200j)}{500 + 200j} = \frac{1000j}{5+2j}$$

$$Z_{eq} = \frac{1000j(5-2j)}{25+4} = \frac{2000 + 5000j}{29}$$

$$\rightarrow Z_{in} = -1000j + \frac{2000 + 5000j}{29}$$

$$Z_{in} = \frac{2000}{29} + \left(\frac{5000}{29} - 1000\right)j = 69 + j\,828$$

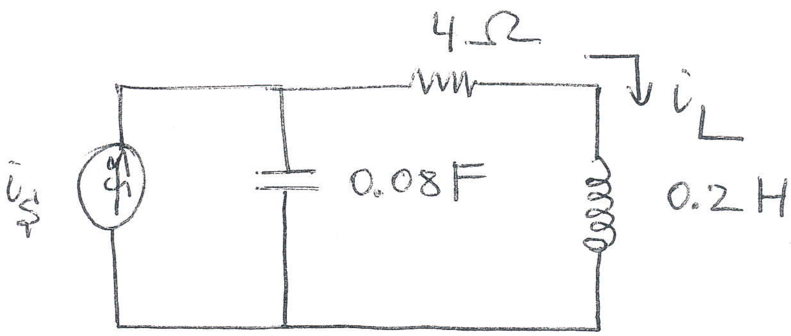
$$Z_{in} = \sqrt{69^2 + (828)^2}\ \tan^{-1}\left(-\frac{828}{69}\right)$$

$$= 831\angle -85.24^\circ$$

IV. (25 pts) We look at the steady state response of the circuit below. If the current i_L is given by the phasor

$$i_L = 20e^{j(10t+25^\circ)} \tag{1}$$

Express the source current $i_s(t)$ as a function of time.



$$v_L = L\frac{di_L}{dt} = 0.2\frac{d}{dt}\left[20e^{j(10t+25^\circ)}\right]$$

$$v_L = j40\,e^{j(\omega t+25^\circ)}$$

$$v_R = 80\,e^{j(\omega t+25^\circ)}$$

$$v_S = (80+j40)\,e^{j(\omega t+25^\circ)}$$

$$i_C = 0.08(80+j40)\,j10\,e^{j(\omega t+25^\circ)} = (-32+j64)\,e^{j(\omega t+25^\circ)}$$

$$\rightarrow i_S = (-12+j64)\,e^{j(\omega t+25^\circ)}$$

$$\boxed{i_S = 65.12\,e^{j(10t+125.62^\circ)}\ A}$$

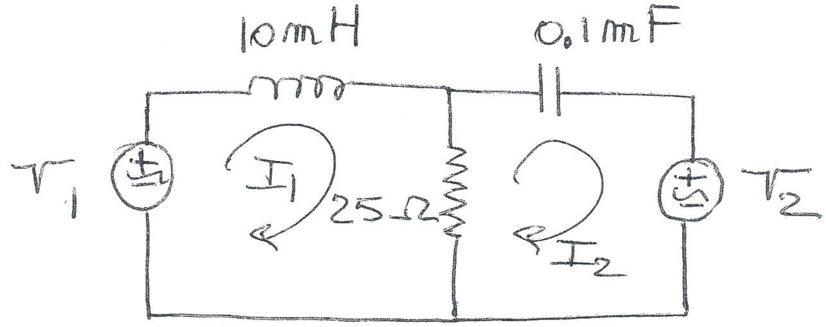
V. (25 pts) Use mesh analysis to write the two equations satisfied by the circulating phasor currents I_1 and I_2 in the circuit below if

$$v_1 = 20cos(1000t)V$$

and

$$v_2 = 20sin(1000t)V$$

DO NOT SOLVE THESE EQUATIONS.



$V_1 = 20\angle 0^\circ$ $\quad v_2 = 20\cos(1000t - 90^\circ)$

$\rightarrow V_2 = -j20$ or $20\angle -90^\circ$

$0.01H \rightarrow j10\Omega$

$0.1mF \rightarrow -j10\Omega$

$$\begin{cases} -20\angle 0^\circ + j10\, I_1 + 25(I_1 - I_2) = 0 \\ 25(I_2 - I_1) - j10\, I_2 + V_2 = 0 \end{cases}$$

$$\boxed{\begin{array}{l} (25 + j10) I_1 - 25 I_2 = 20\angle 0^\circ \\ -25 I_1 + (25 - j10) I_2 = 20\angle 90^\circ \end{array}}$$