

I. (25 pts): HP and LP filter?

as $\omega \rightarrow 0$, L is \rightarrow short $\rightarrow \frac{V_o}{V_i}$ finite
 as $\omega \rightarrow \infty$, $L \rightarrow \infty$ (open) $\rightarrow V_o = 0$
 \Rightarrow LP

• For the first order filter shown below, explain qualitatively (i.e., using the fact that an inductor behaves as a short at DC and as an open circuit at high frequency) if this is a low-pass or high-pass filter.

• Depending on your previous answer, put the transfer function of the filter $T(s) = V_o(s)/V_i(s)$ in the form

$$T(s) = \frac{K}{1 + \frac{s}{\omega_0}} \quad (1)$$

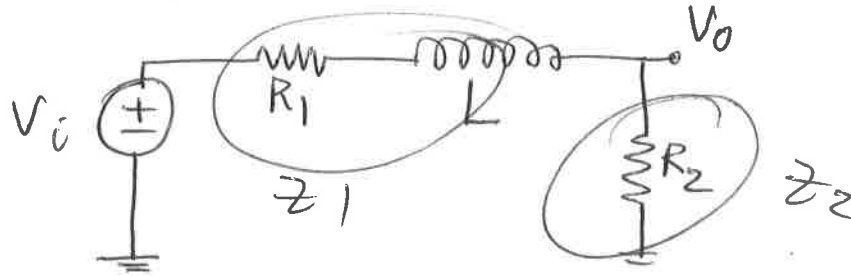
or

$$T(s) = \frac{K}{1 + \frac{\omega_0}{s}} \quad (2)$$

• Find the value of K by either calculating $T(s=0)$ or $T(s = \infty)$ depending of your previous answer.

• Find an analytical expression for ω_0 , the 3dB frequency of the filter.

Remember that the impedance of an inductor is $Z(s) = sL$



$$\frac{V_o}{V_i} = T(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2}{R_2 + R_1 + sL} = \frac{R_2}{R_1 + R_2} \left[1 + \frac{sL}{R_1 + R_2} \right]^{-1}$$

$$= K \left[1 + \frac{s}{\omega_0} \right]^{-1}$$

$$K = \frac{R_2}{R_1 + R_2} \quad \omega_0 = \frac{R_1 + R_2}{L}$$

II. (25 pts): The voltage gain of an amplifier is described by the following transfer function

$$A(s) = \frac{10^5 s}{(s+10)(s+500)} \quad (3)$$

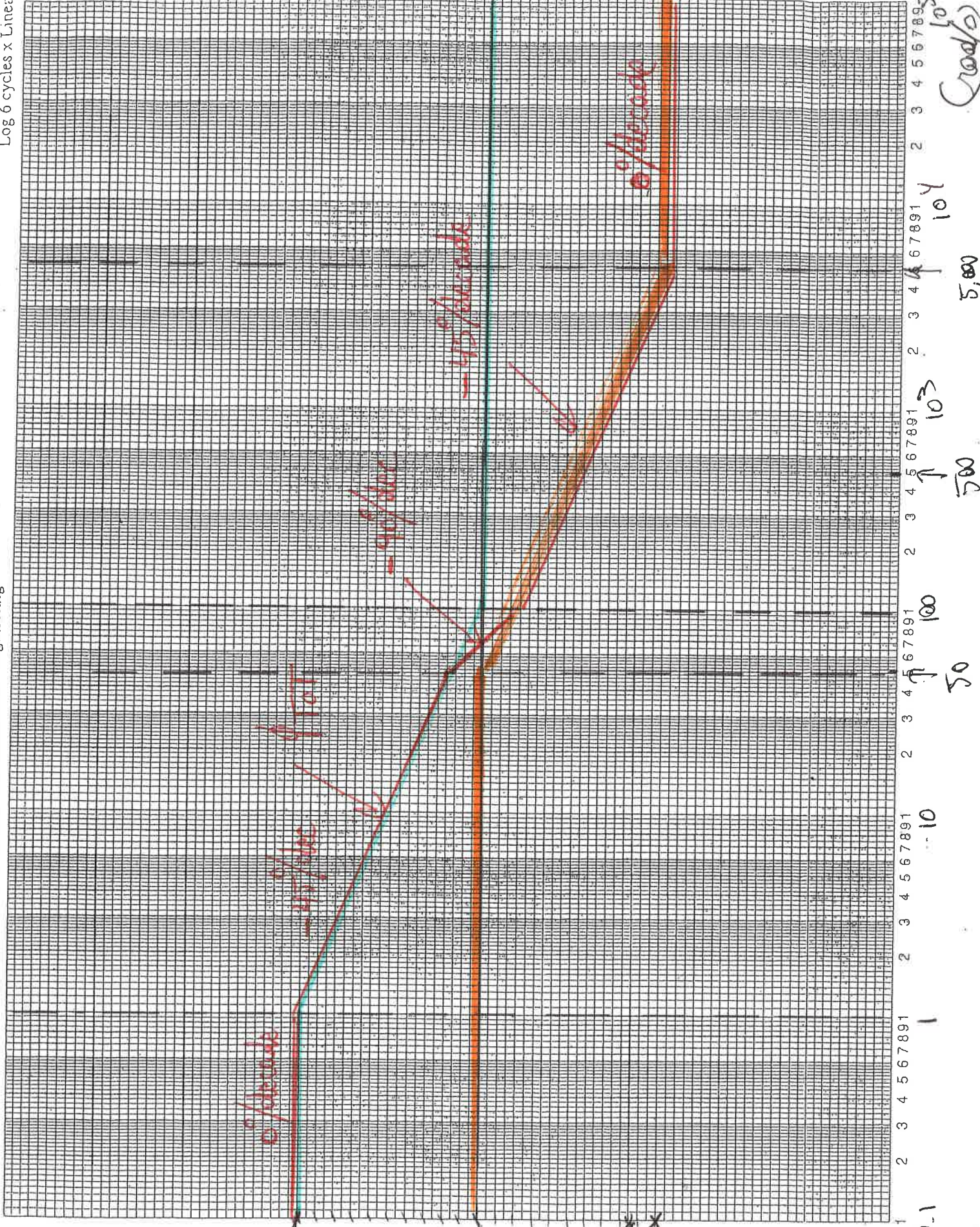
(a) Write $A(s)$ in the generic form $A_M F_L(s) F_H(s)$ and give the explicit expressions for A_M , $F_L(s)$, and $F_H(s)$.

(b) Make a Bode plot of the phase $A(s)$ using the attached diagram. Make sure to clearly indicate all the locations of the zeroes and poles and the rate of decay ($\pm 45^\circ$ or $\pm 90^\circ/\text{decade}, \dots$) over the various ranges of frequencies. Also indicate with vertical lines the locations where the breaks in the phase occurs (at one-tenth and ten times the frequencies of the zeroes and poles).

$$\begin{aligned}
 A(s) &= 10^5 \frac{1}{\left(1 + \frac{10}{s}\right)} \frac{1}{500} \frac{1}{\left(1 + \frac{s}{500}\right)} \\
 &= \frac{10^5 s}{500} F_L(s) F_H(s) \\
 A_M &= 200 \quad \left(1 + \frac{10}{s}\right) \quad \left(1 + \frac{s}{500}\right)
 \end{aligned}$$

Log 6x0 ps

AutocAD



0.1

1

50

100

500

1000

5000

10000

III. (20 pts): The voltage gains of two different amplifiers are given by the following two transfer functions:

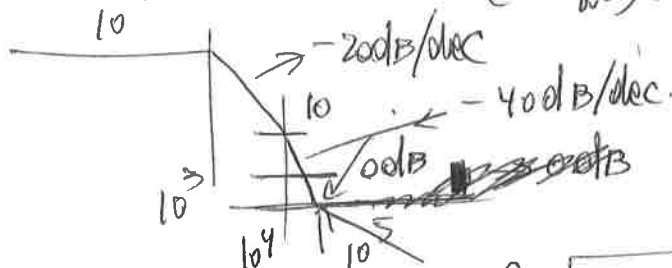
$$T_1(s) = \frac{10^4(s+10^5)}{(s+10^3)(s+10^4)} \quad (4)$$

$$T_2(s) = \frac{10^5s(s+10^4)}{(s+10)(s+10^3)} \quad (5)$$

- Write $T_1(s)$ and $T_2(s)$ in the generic form $A_M F_L(s) F_H(s)$ and give the expression for A_M , $F_L(s)$, and $F_H(s)$.
- At what value(s) of the angular frequency does the magnitude of the transfer functions $T_1(s)$ and $T_2(s)$ reach unity (or 0 dB)?

$$T_1(s) = \frac{10^4 \cdot 10^5 \left(1 + \frac{\omega}{10^5}\right)}{10^3 \left(1 + \frac{\omega}{10^3}\right) \omega^4 \left(1 + \frac{\omega}{10^4}\right)} = \underbrace{10^2}_{A_M} F_H(s)$$

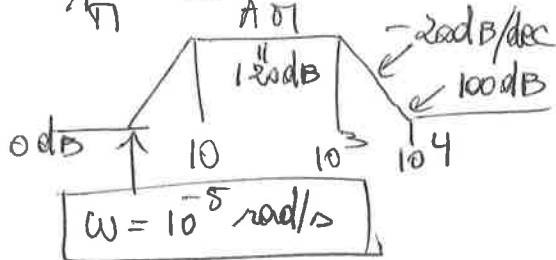
$$F_L(s) = 1 \quad F_H(s) = \frac{1 + \frac{\omega}{10^5}}{\left(1 + \frac{\omega}{10^3}\right) \left(1 + \frac{\omega}{10^4}\right)}$$



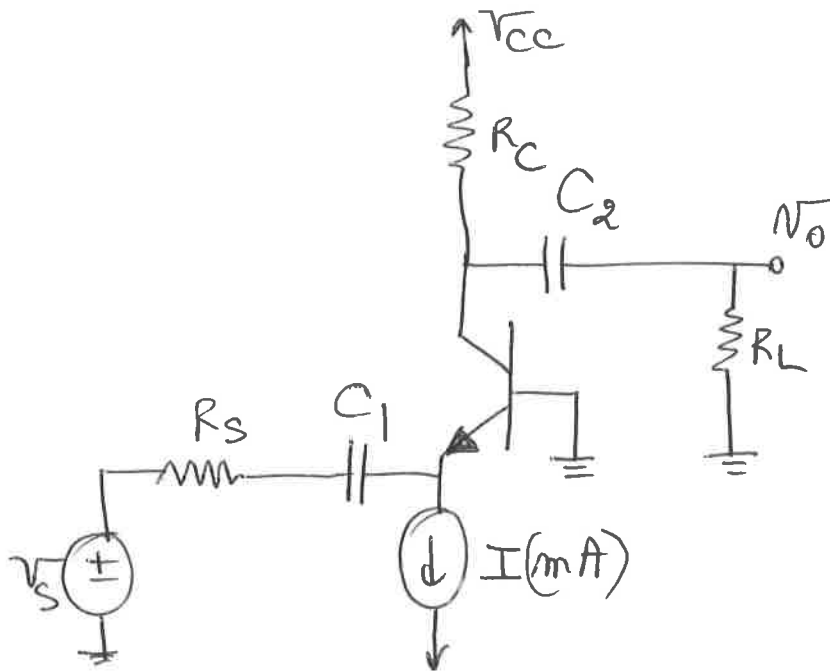
$$20 \log_{10} |T_1(s)| = 0 \text{ dB for } 10^4 < \omega < 10^5$$

$$T_2(s) = \omega^5 \frac{1}{\left(1 + \frac{\omega}{10}\right)} \frac{10^4 \left(1 + \frac{\omega}{10^4}\right)}{10^3 \left(1 + \frac{\omega}{10^3}\right)}$$

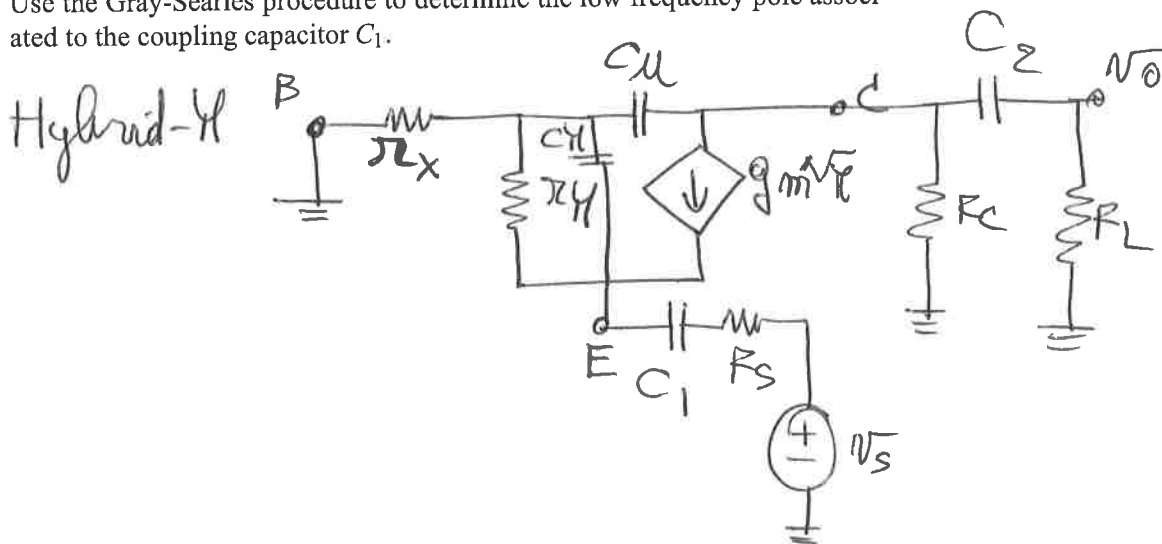
$$= \underbrace{10^6}_{A_M} F_L(s) F_H(s) \quad \left(\frac{1 + \frac{\omega}{10^4}}{1 + \frac{\omega}{10^3}} \right)$$



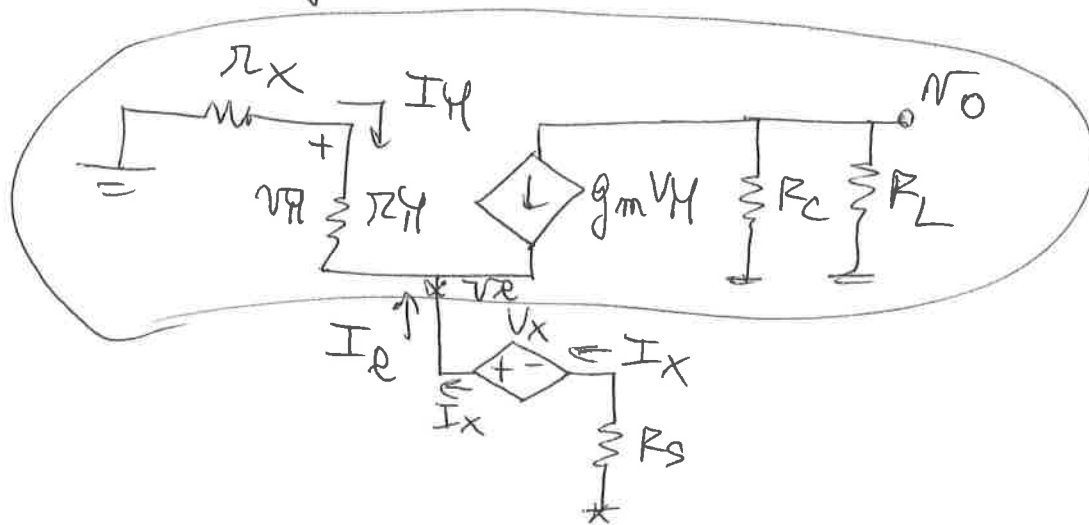
IV. (25 pts): For the amplifier shown below



- What type of configuration is the amplifier (circle correct answer): (a) common emitter, **(b) common base**, (c) or common collector.
- Draw the small ac equivalent circuit of the network including the coupling, bypass, and internal capacitances C_π and C_μ of the BJT. Neglect the effects of r_o when drawing the small AC equivalent circuit, but include r_x .
- Use the Gray-Searles procedure to determine the low frequency pole associated to the coupling capacitor C_1 .



low frequency



$$\frac{v_x}{I_x} = R_s + r_e$$

$$r_e = \frac{v_e}{I_e}$$

$$v_e = -I_{\pi}(r_x + r_{\pi})$$

$$I_e = -I_{\pi} - g_m v_{\pi} = -I_{\pi} \left(1 + \frac{g_m r_{\pi}}{\beta} \right)$$

$$\frac{v_e}{I_e} = \frac{r_x + r_{\pi}}{\beta + 1}$$

$$\rightarrow \boxed{\frac{v_x}{I_x} = R_s + \frac{r_x + r_{\pi}}{\beta + 1}}$$