

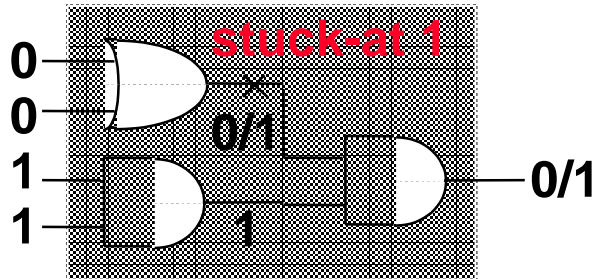
# Combinational Test Generation

- **Test Generation (TG) Methods**
  - (1) From truth table (2) Using Boolean equation (3) Using Boolean difference (4) From circuit structure
- **TG from Circuit Structure**
  - Common Concepts
  - Algorithms : D-Algorithm (Roth 1967), 9-V Algorithm (Cha 1978), PODEM (Goel 1981), FAN (Fujiwara 1983), Socrates (Schultz 1987)

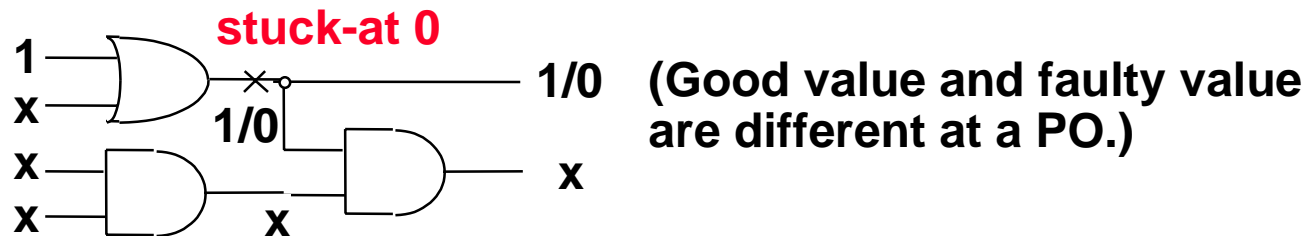
**(Source: NCTU 李崇仁 教授)**

# A Test Pattern

- A test pattern



- A test pattern with don't cares

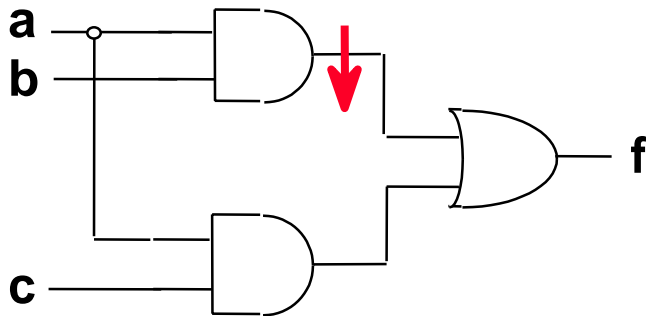


- Test generation: generates a test for a target fault.

# Test Generation Methods

## (From Truth Table)

Ex: How to generate tests for the stuck-at 0 fault (fault  $\alpha$ )?



abc	f	$f_\alpha$
000	0	0
001	0	0
010	0	0
011	0	0
100	0	0
101	1	1
✓ 110	1	0
111	1	1

Impractical !!

# Test Generation Methods

## (Using Boolean Equation)

Since  $f = ab+ac$ ,  $f_\alpha = ac \Rightarrow$

$T_\alpha$  = the set of all tests for fault  $\alpha$

$$= \text{ON\_set}(f) * \text{OFF\_set}(f_\alpha) + \text{OFF\_set}(f) * \text{ON\_set}(f_\alpha)$$

$$= \{(a,b,c) \mid (ab+ac)(ac)' + (ab+ac)'(ac) = 1\}$$

$$= \{(a,b,c) \mid abc'=1\}$$

$$= \{(110)\}.$$

High complexity !!

Since it needs to compute the faulty function for each fault.

- *ON\_set(f): All input combinations that make f have value 1.*
- *OFF\_set(f): All input combinations that make f have value 0.*

# Boolean Difference

- **Physical Meaning of Boolean Difference**

- For a logic function  $F(X)=F(x_1, \dots, x_i, \dots, x_n)$ , find all the input combinations that make the change of value in  $x_i$  also cause the change of value in  $F$ .

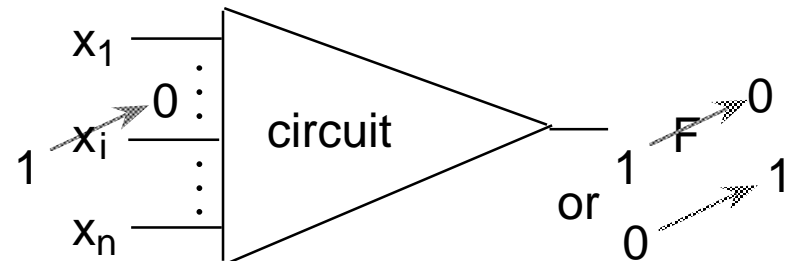
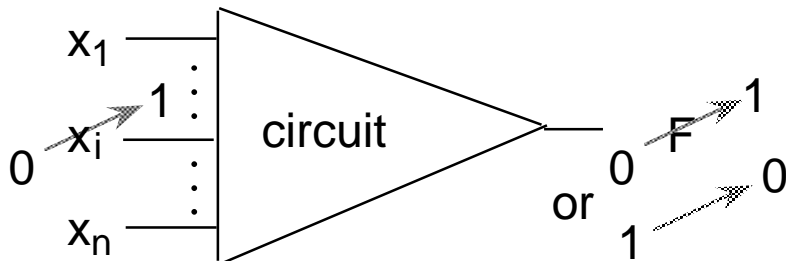
- **Logic Operation of Boolean Difference**

- The Boolean difference of  $F(X)$  w.r.t. input  $x_i$  is

$$\frac{dF(X)}{dx_i} = F_i(0) \oplus F_i(1) = \overline{F_i(0)} \cdot F_i(1) + F_i(0) \cdot \overline{F_i(1)},$$

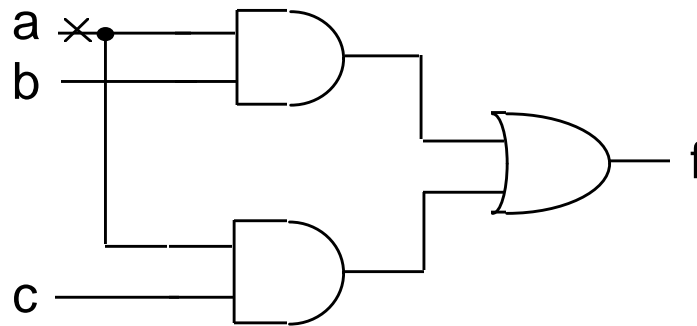
where  $F_i(0) = F(x_1, \dots, 0, \dots, x_n)$  and  $F_i(1) = F(x_1, \dots, 1, \dots, x_n)$ .

- **Relationship between TG and Boolean Difference**



# Applying Boolean Difference to Test Generation (1/2)

Case 1: Faults are present at PIs.



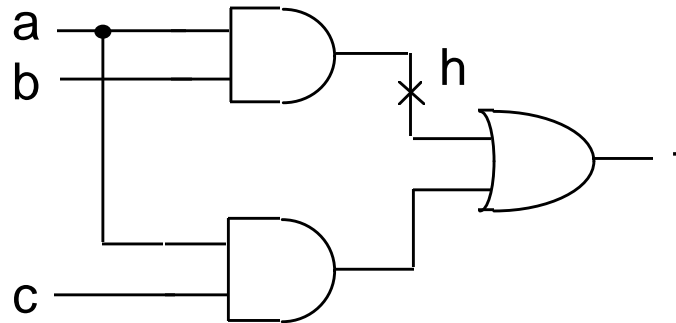
$$f = ab+ac \Rightarrow \frac{df}{da} = f_a(0) \oplus f_a(1) = 1 \bullet (b+c) + 0 = b+c$$

The set of all tests for line  $a$  s-a-1 is  $\{(a,b,c) \mid a' \bullet (b+c)=1\} = \{(01x), (0x1)\}$ .

The set of all tests for line  $a$  s-a-0 is  $\{(a,b,c) \mid a \bullet (b+c)=1\} = \{(11x), (1x1)\}$ .

# Applying Boolean Difference to Test Generation (2/2)

Case 2: Faults are present at internal lines.



$$f = h+ac, h = ab \Rightarrow \frac{df}{dh} = f_h(0) \oplus f_h(1) = \bar{a}\bar{c} \cdot 1 + ac \cdot \bar{1} = \bar{a} + \bar{c}$$

The set of all tests for line  $h$  s-a-1 is

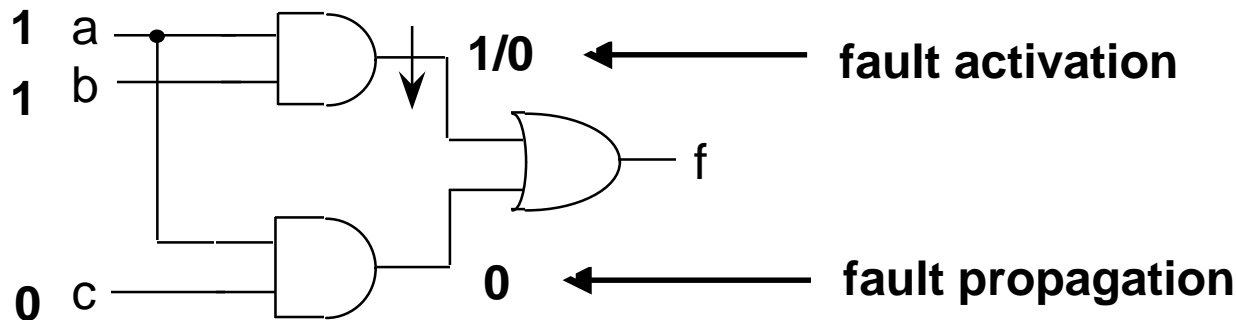
$$\{ (a,b,c) | h' \cdot (a'+c') = 1 \} = \{ (a,b,c) | (a'+b') \cdot (a'+c') = 1 \} = \{ (0xx), (x00) \}.$$

The set of all tests for line  $h$  s-a-0 is

$$\{ (a,b,c) | h \cdot (a'+c') = 1 \} = \{ (110) \}.$$

# Test Generation Methods

## (From Circuit Structure)



- Two basic goals:

Fault activation (FA)

Fault propagation (FP)

=> Line justification (LJ)

where **1/0** means that the good value is 1 and the faulty value is 0 and is denoted as **D**. Similarly, **0/1** is denoted as **D'**.

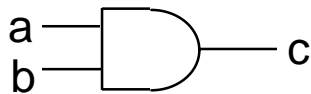
**D** and **D'** are called fault effects (FE).



# Common Concepts for Structural TG

- The FA problem  $\Rightarrow$  a LJ problem.
- The FP problem  $\Rightarrow$ 
  - (1) Select a FP path to a PO  $\Rightarrow$  decisions.
  - (2) Once the path is selected  $\Rightarrow$  a set of LJ problems.
- The LJ problems  $\Rightarrow$  decisions or implications .

ex:

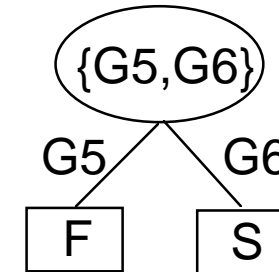
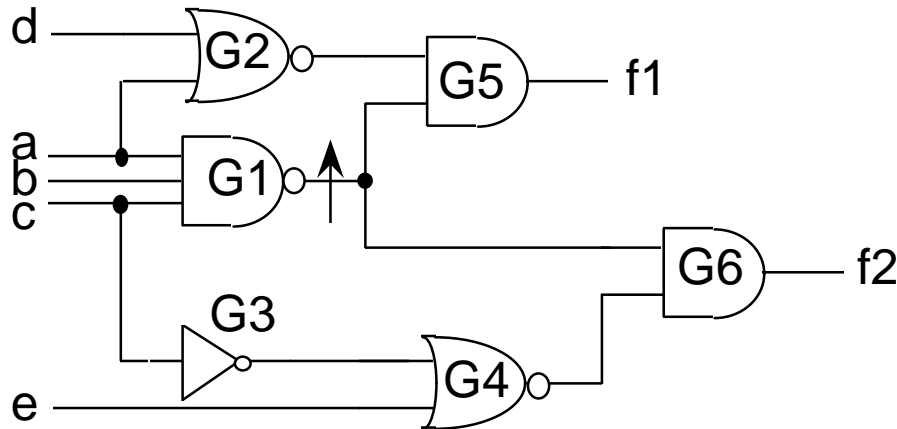


To justify  $c=1 \Rightarrow a=1$  and  $b=1$ . (implication)

To justify  $c=0 \Rightarrow a=0$  or  $b=0$ . (need make decisions)

- **Incorrect decision  $\Rightarrow$  Backtracking  $\Rightarrow$  Another decision.**
- Once the fault effect is propagated to a PO and all line values to be justified are justified, the test is generated. Otherwise, the decision process must be continued repeatedly until all possible decisions have been tried.

# Ex: Decisions When Fault Propagation



The corresponding decision tree

FA  $\Rightarrow a=1, b=1, c=1 \Rightarrow G1 = D', G3=0$ ; FP  $\Rightarrow$  through G5 or G6.

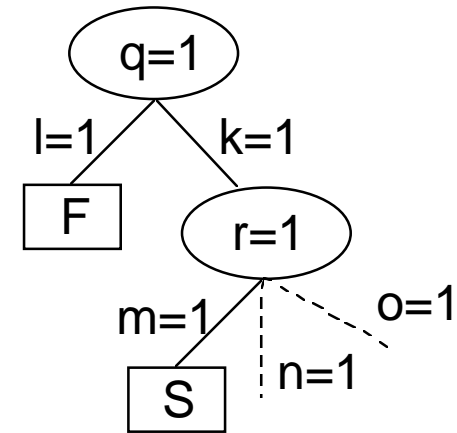
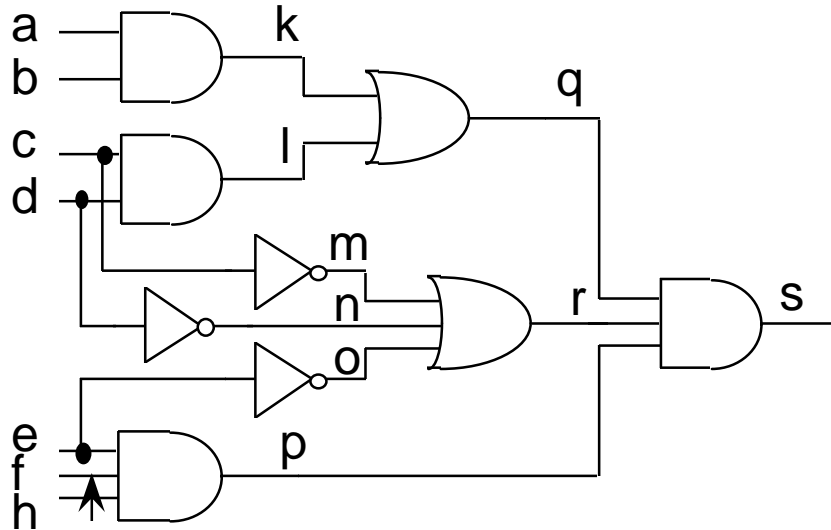
Decision: through G5  $\Rightarrow G2=1 \Rightarrow d=0, a=0. \Rightarrow$  inconsistency  $\Rightarrow$  backtracking!!

Decision: through G6  $\Rightarrow G4=1 \Rightarrow e=0. \Rightarrow$  done!!

The resulting test is 111x0.

**D-frontier:** The set of all gates whose output value is currently x but have one or more fault signals on their inputs. Ex: Initially, the D-frontier of this example is  $\{G5, G6\}$ .

# Ex: Decisions When Line Justification



The corresponding decision tree

FA  $\Rightarrow h=D'$ ; FP  $\Rightarrow e=1, f=1 (\Rightarrow o=0)$ ; FP  $\Rightarrow q=1, r=1$ .

To justify  $q=1 \Rightarrow l=1$  or  $k=1$ .

Decision:  $l=1 \Rightarrow c=1, d=1 \Rightarrow m=0, n=0 \Rightarrow r=0. \Rightarrow$  inconsistency  $\Rightarrow$  backtracking!!

Decision:  $k=1 \Rightarrow a=1, b=1$ .

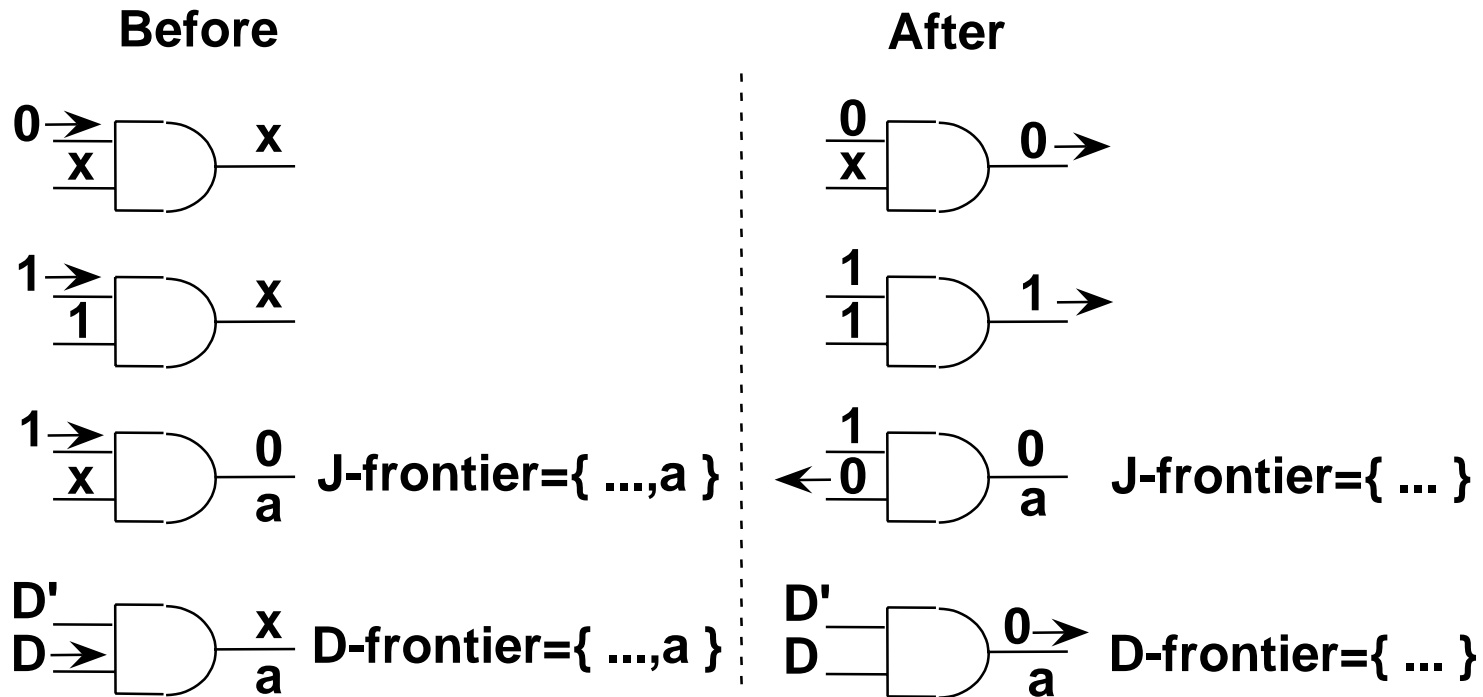
To justify  $r=1 \Rightarrow m=0$  or  $n=0 (\Rightarrow c=1$  or  $d=1)$ .  $\Rightarrow$  done!!

**J-frontier:** The set of all gates whose output value is known but is not implied by its input values. Ex: Initially, the J-frontier of the example is  $\{q=1, r=1\}$ .

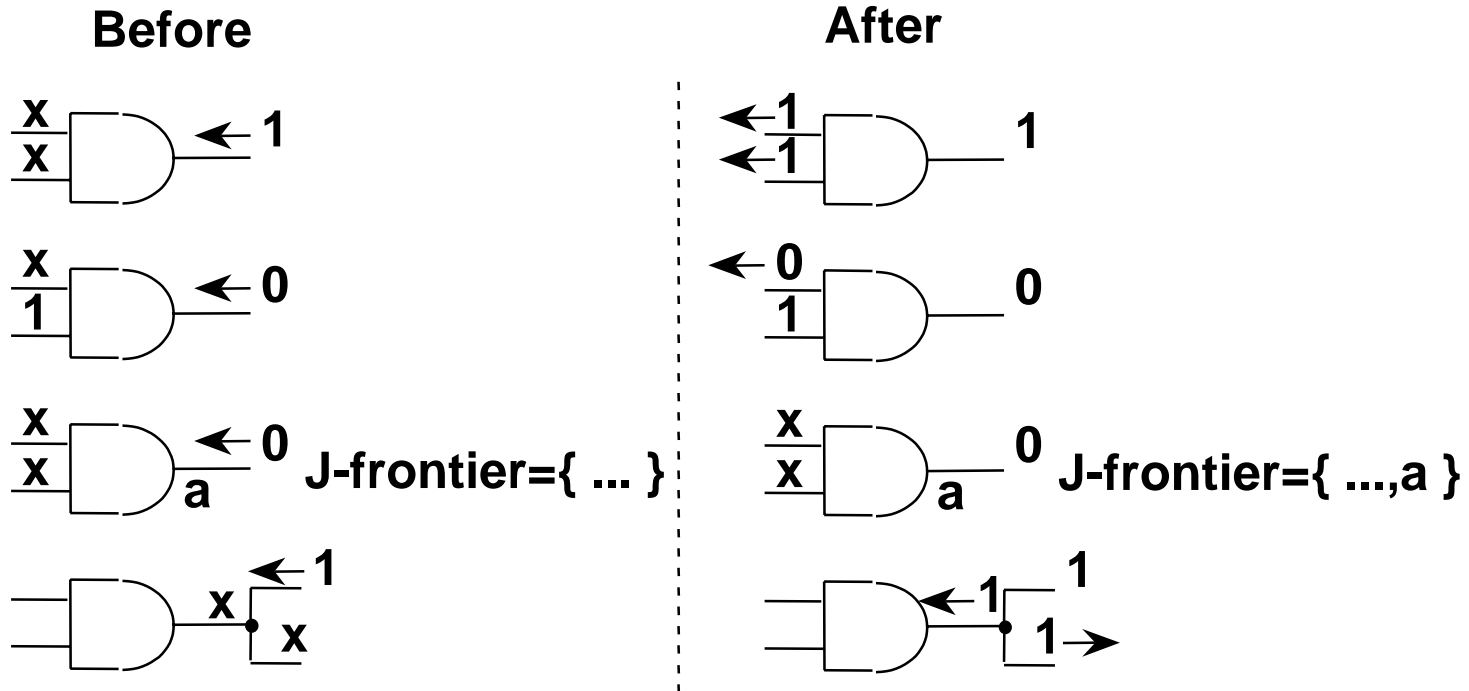
# Implications

- **Implication: computation of the values that can be **uniquely** determined.**
  - **Local implication: propagation of values from one line to its immediate successors or predecessors.**
  - **Global implication: the propagation involving a larger area of the circuit and reconvergent fanout.**
- **Maximum implication principle: perform as many implications as possible.**
- **Maximum implications help us to either reduce the number of problems that need decisions or to reach an inconsistency sooner.**

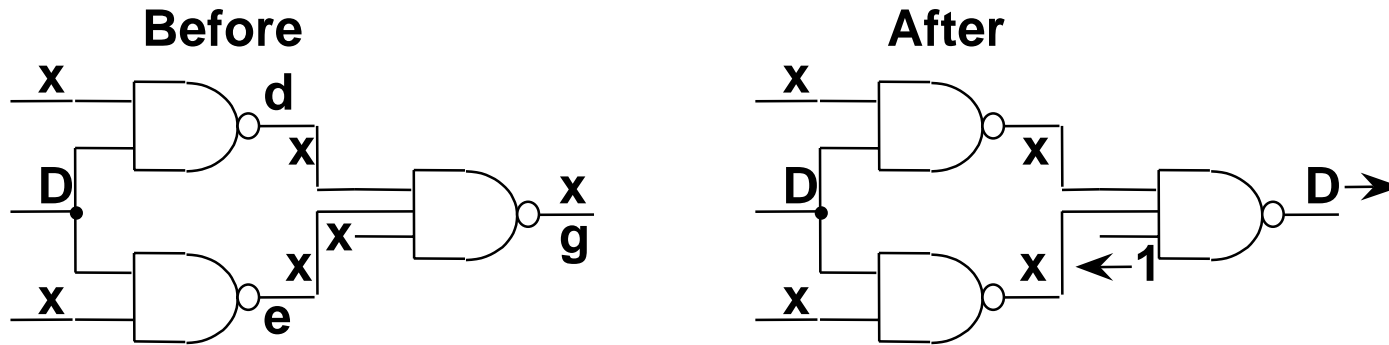
# Local Implications (Forward)



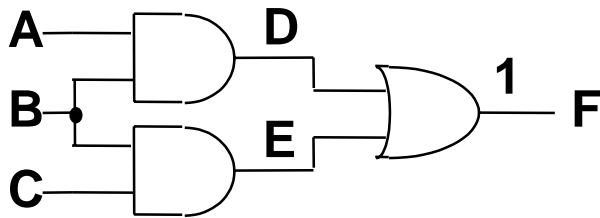
# Local Implications (Backward)



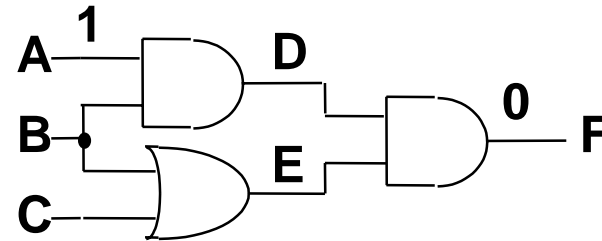
# Global Implications



(1) **Future unique D-drive.**



(2) **F=1 implies B=1.**  
**(Static learning)**

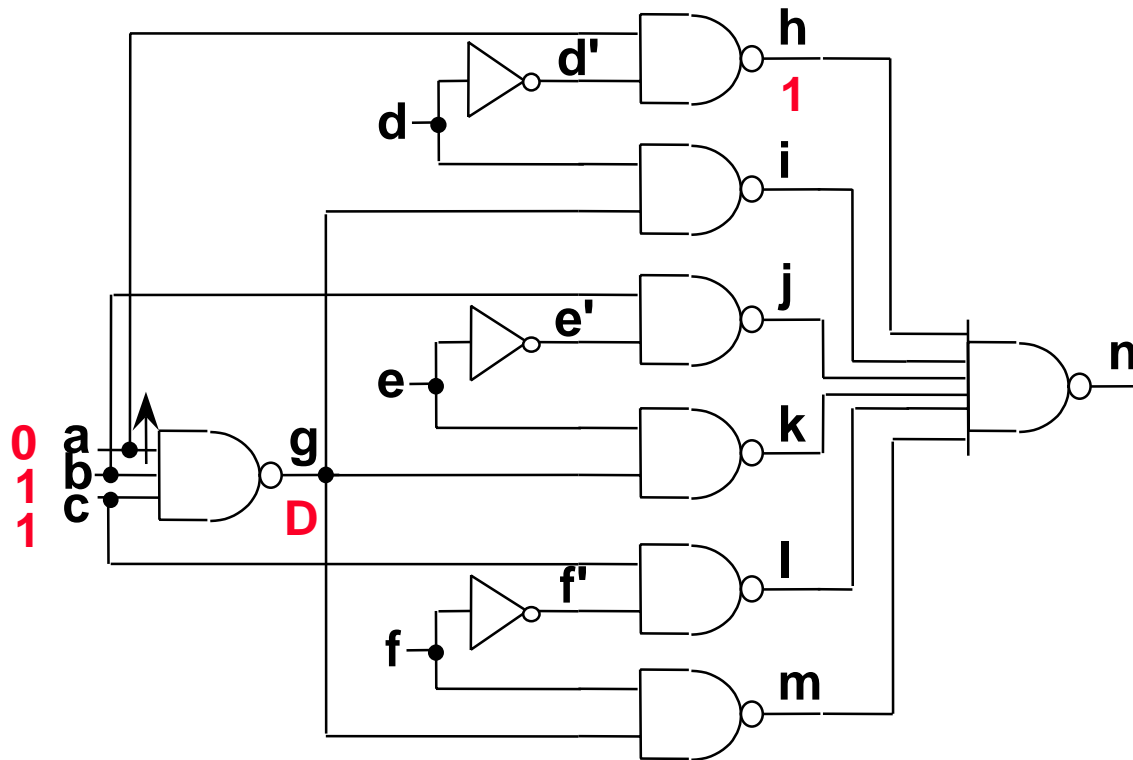


(3) **F=0 implies B=0 when A=1.**  
**(Dynamic learning)**

(2), (3) are based on contraposition law:  $(A \Rightarrow B) \Leftrightarrow (!B \Rightarrow !A)$ .

# D-Algorithm: Example

- Logic values = {0, 1, D, D', x}.



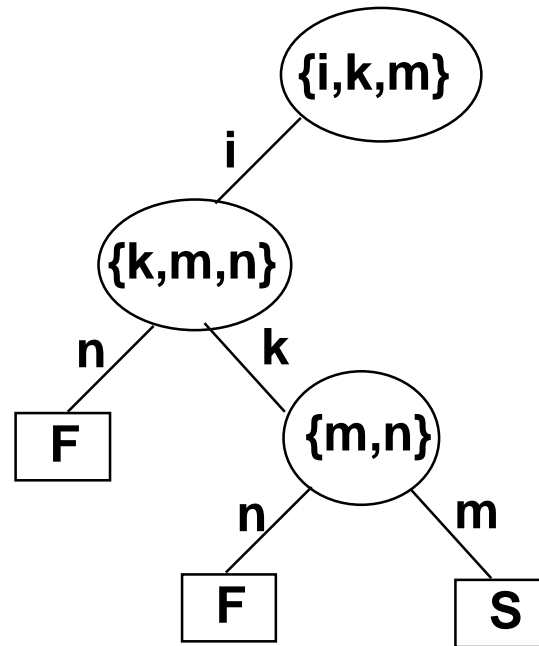


# D-Algorithm: Value Computation

Decision	Implication	Comments
	<b>a=0</b> <b>h=1</b> <b>b=1</b> <b>c=1</b> <b>g=D</b>	<b>Active the fault</b>  <b>Unique D-drive</b>
<b>d=1</b>	<b>i=D</b> <b>d?0</b>	<b>Propagate via i</b>
<b>j=1</b> <b>k=1</b> <b>l=1</b> <b>m=1</b>	    <b>n=D</b> <b>e?0</b> <b>e=1</b> <b>k=D</b>	<b>Propagate via n</b>          <b>Contradiction</b>

<b>e=1</b>	<b>k=D</b> <b>e?0</b> <b>j=1</b>	<b>Propagate via k</b>
<b>l=1</b> <b>m=1</b>	   <b>n=D</b> <b>f?0</b> <b>f=1</b> <b>m=D</b>	<b>Propagate via n</b>      <b>Contradiction</b>
<b>f=1</b>	   <b>m=D</b> <b>f?0</b> <b>l=1</b> <b>n=D</b>	<b>Propagate via m</b>

# D-Algorithm: Decision Tree

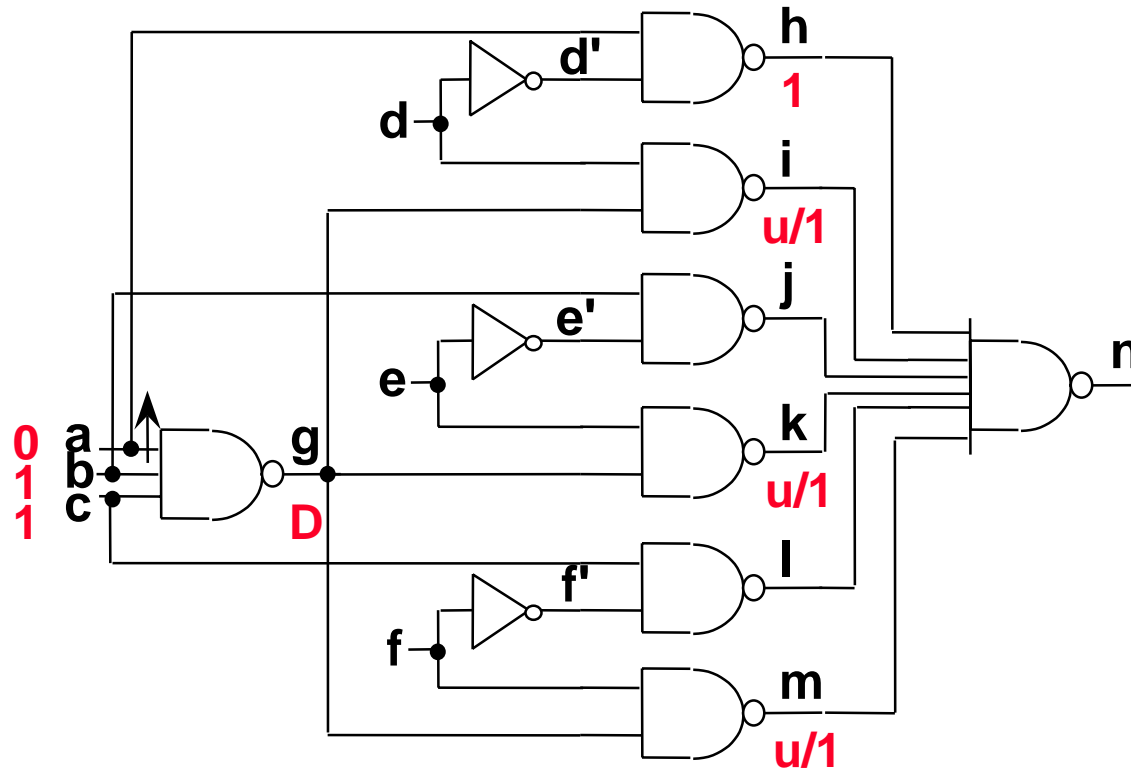


Two times of backtracking!!

- **Decision node:** the associated D-frontier.
- **branch:** the decision taken, i.e., the gate selected from the D-frontier.
- The D-algorithm first tried to propagate the fault solely through  $i$ , then through both  $i$  and  $k$ , and eventually succeeded when all three paths were simultaneously sensitized.

# 9-V Algorithm: Example

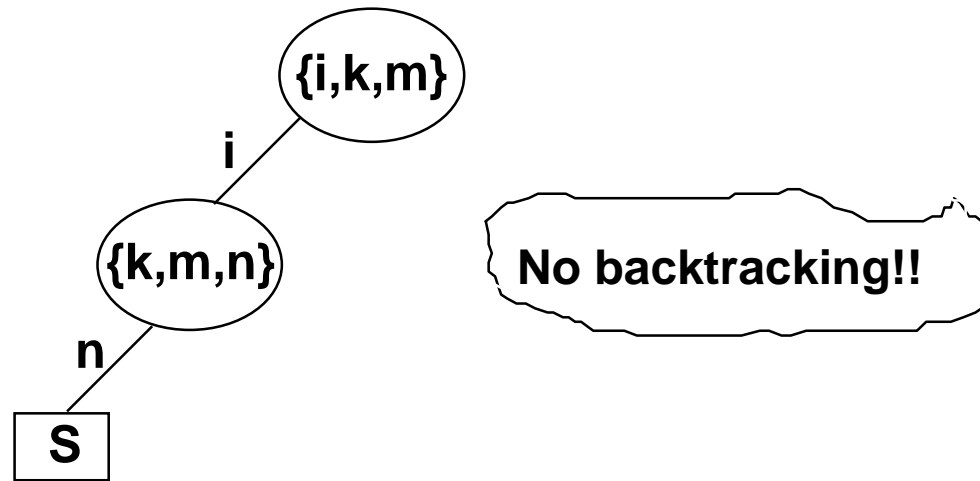
- Logic values =  $\{0/0, 0/1, 0/u, 1/0, 1/1, 1/u, u/0, u/1, u/u\}$ , where  $0/u=\{0,D'\}$ ,  $1/u=\{D,1\}$ ,  $u/0=\{0,D\}$ ,  $u/1=\{D',1\}$ ,  $u/u=\{0,1,D,D'\}$ .
- Thus reduces the amount of search done for multiple path sensitization in D-algorithm.



# 9-V Algorithm: Value Computation

Decision	Implication	Comments			
	<b>a=0</b> <b>h=1</b> <b>b=1</b> <b>c=1</b> <b>g=D</b> <b>i=u/1</b> <b>k=u/1</b> <b>m=u/1</b>	<b>Activate the fault</b>  <b>Unique D-drive</b>	<b>l=u/1</b> <b>j=u/1</b>	<b>n=D</b> <b>f?u/0</b> <b>f=1</b> <b>f?0</b> <b>e?u/0</b> <b>e=1</b> <b>e?0</b> <b>k=D</b> <b>m=D</b>	<b>Propagate via n</b>
<b>d=1</b>	<b>i=D</b> <b>d?0</b> <b>n=1/u</b>	<b>Propagate via i</b>			

# 9-V Algorithm: Decision Tree



- The main difference between the D-algorithm and 9-V algorithm is: Whenever there are  $k$  possible paths for fault propagation, the D-algorithm may eventually try all the  $2^k - 1$  combinations of paths. However, since the 9-V algorithm tries only one path at a time without precluding simultaneous fault propagation on the other  $k-1$  paths, it will enumerate at most  $k$  ways of fault propagation.

# PODEM (Path-Oriented Decision Making)

- We have seen that the problems of FA and FP lead to sets of LJ problems. The LJ problems can be solved via value assignments.
- In D-algorithm, the value assignments are allowed on any internal lines. => backtracking could occur at any line.
- However, PODEM allows value assignments only on PIs.  
=> backtracking can occur only at the PIs.
  - It treats a value  $V_k$  to be justified for line  $k$  as an objective  $(k, V_k)$ .
  - A backtracing procedure maps the objective into a PI assignment that is likely to contribute to achieving the objective.
  - Why called PODEM (Path-Oriented DEcision Making) ?

# A Simple Backtracing Procedure

Objective =  $(k, V_k)$ .

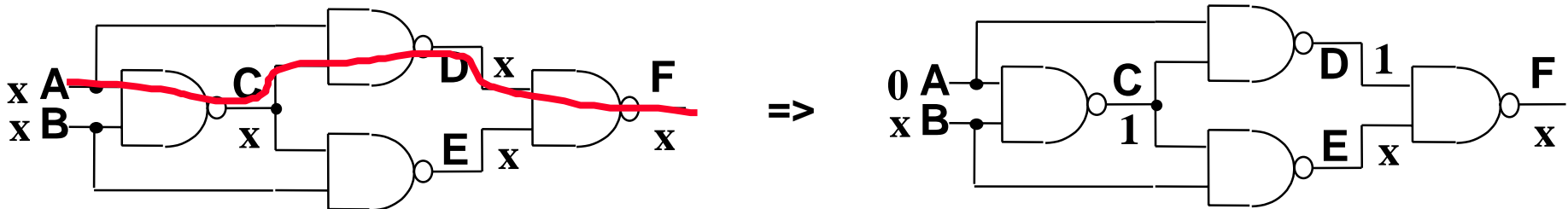
Step 1. Find a  $x$ -path from line  $k$  to a PI, say  $A$ .

Step 2. Count the inversion parity of the path.

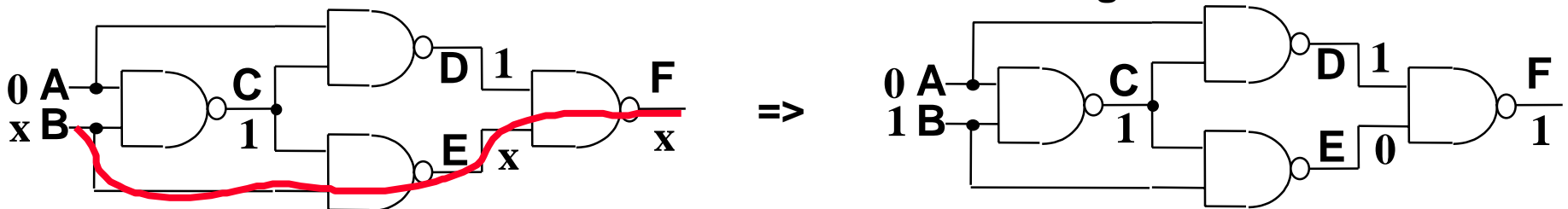
Step 3. If the inversion parity is even  $\Rightarrow$  return  $(A, V_k)$ . Otherwise  $\Rightarrow$  return  $(A, V_k')$ .

\* A path is a  $x$ -path if all of its lines have value  $x$ .

Ex: Objective =  $(F, 1)$ .

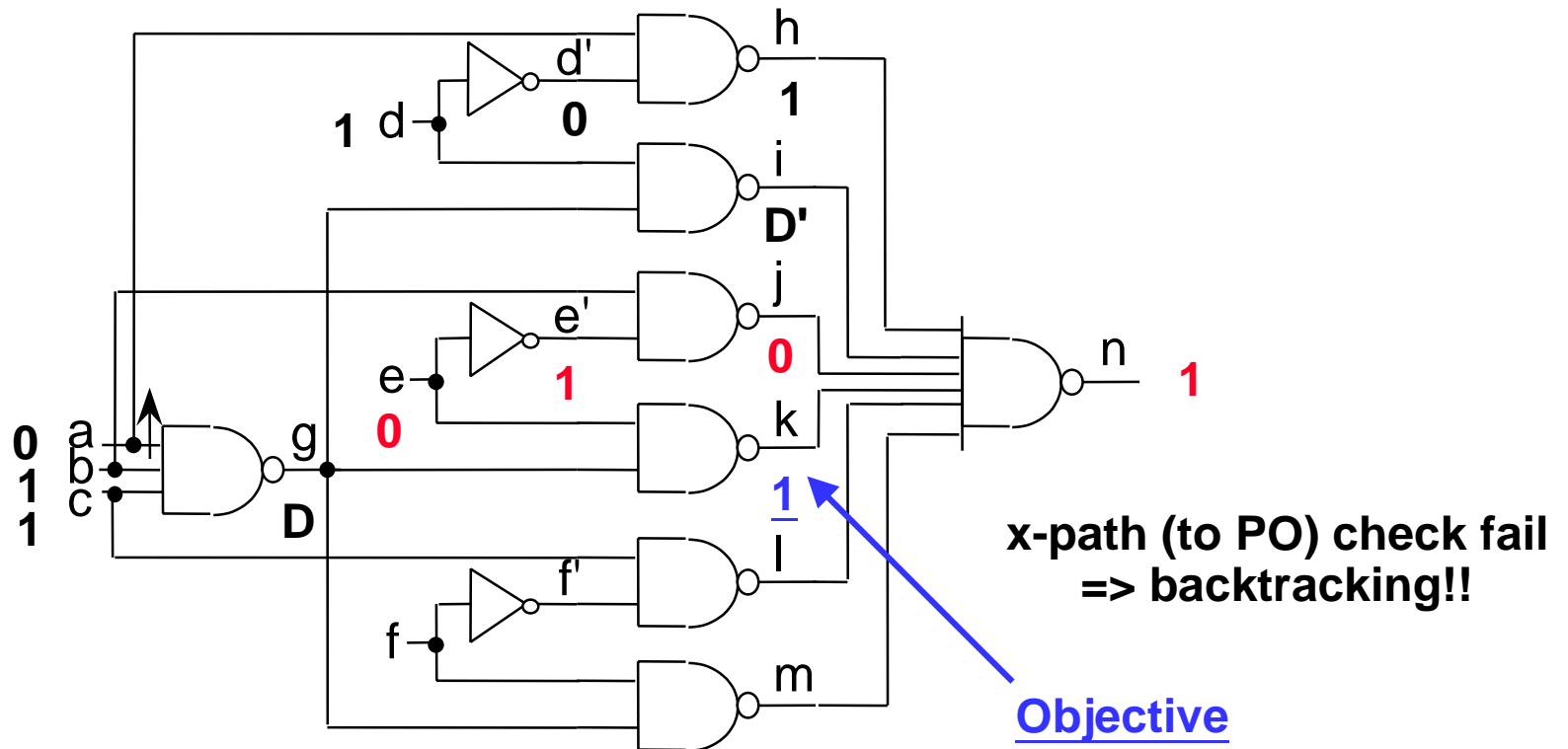


The first time of backtracing



The second time of backtracing

# PODEM: Example 1

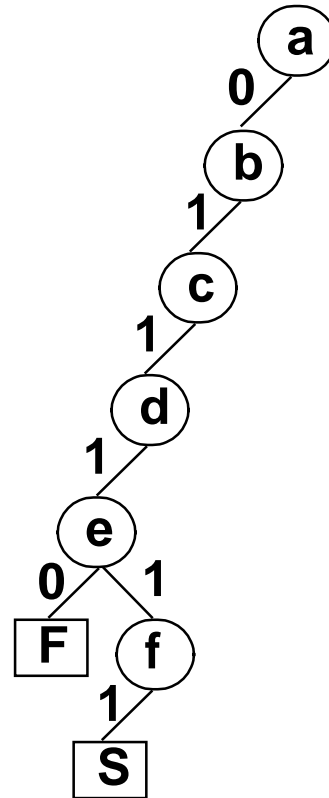




# PODEM: Value Computation

Objective	PI assignment	Implications	D-frontier	Comments
<b>a=0</b>	<b>a=0</b>	<b>h=1</b>	<b>g</b>	
<b>b=1</b>	<b>b=1</b>		<b>g</b>	
<b>c=1</b>	<b>c=1</b>	<b>g=D</b>	<b>i,k,m</b>	
<b>d=1</b>	<b>d=1</b>	<b>d?0</b> <b>i=D</b>	<b>k,m,n</b>	
<b>k=1</b>	<b>e=0</b>	<b>e?1</b> <b>j=0</b> <b>k=1</b> <b>n=1</b>	<b>m</b>	<b>x-path check fail !!</b>
	<b>e=1</b>	<b>e?0</b> <b>j=1</b> <b>k=D</b>	<b>m,n</b>	<b>reversal</b>
<b>l=1</b>	<b>f=1</b>	<b>f?0</b> <b>l=1</b> <b>m=D</b> <b>n=D</b>		

# PODEM: Decision Tree

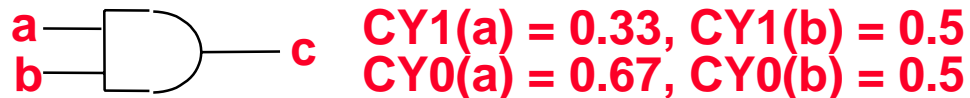


\* **decision node:** the PI selected to assign value.  
**branch:** the value assigned to the PI.

# A More Intelligent Backtracing

- To guide the backtracing process of PODEM, controllability for each line is measured.
  - **CY1(a)**: the probability that line a has value 1.
  - **CY0(a)**: the probability that line a has value 0.
- Ex:  $f = ab$ . Assume  $CY1(a)=CY0(a)=CY1(b)=CY0(b)=0.5$ .  
 $\Rightarrow CY1(f)=CY1(a) \times CY1(b)=0.25$ ,  $CY0(f)=CY0(a)+CY0(b)-CY0(a) \times CY0(b)=0.75$ .
- How to guide the backtracing using controllability?
  - **Principle 1**: Among several unsolved problems, first attack the hardest one.
  - **Principle 2**: Among several solutions of a problem, first try the easiest one.

ex:



**Objective=(c,1)  $\Rightarrow$  Choose *path c-a* to backtracing.**

**Objective=(c,0)  $\Rightarrow$  Choose *path c-a* to backtracing.**

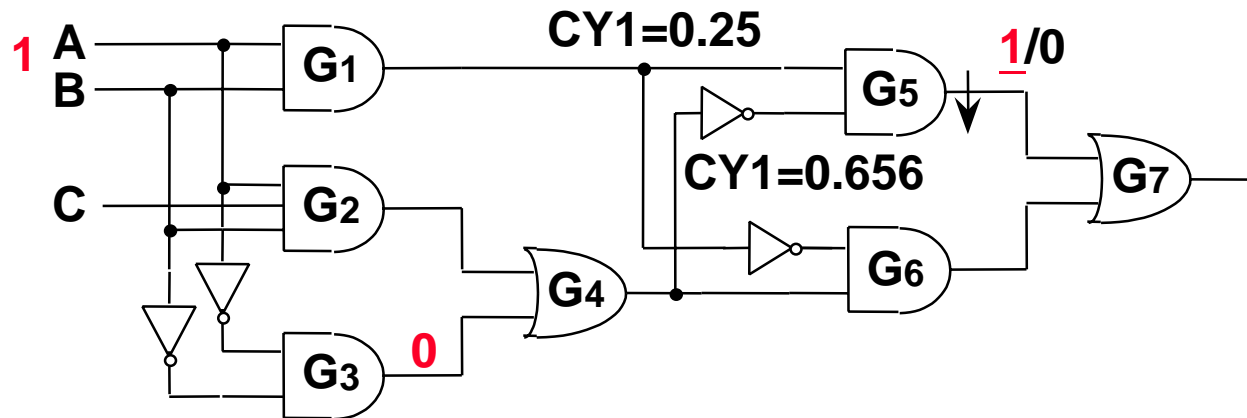
# PODEM: Example 2 (1/3)

Initial objective=(G5,1).

G5 is an AND gate => Choose the hardest-1 => Current objective=(G1,1).

G1 is an AND gate => Choose the hardest-1 => Arbitrarily, Current objective=(A,1).

A is a PI => Implication => G3=0.



The controllabilities are calculated by a testability measure program TEA. Initially, CY1 and CY0 for all PIs are set to 0.5.

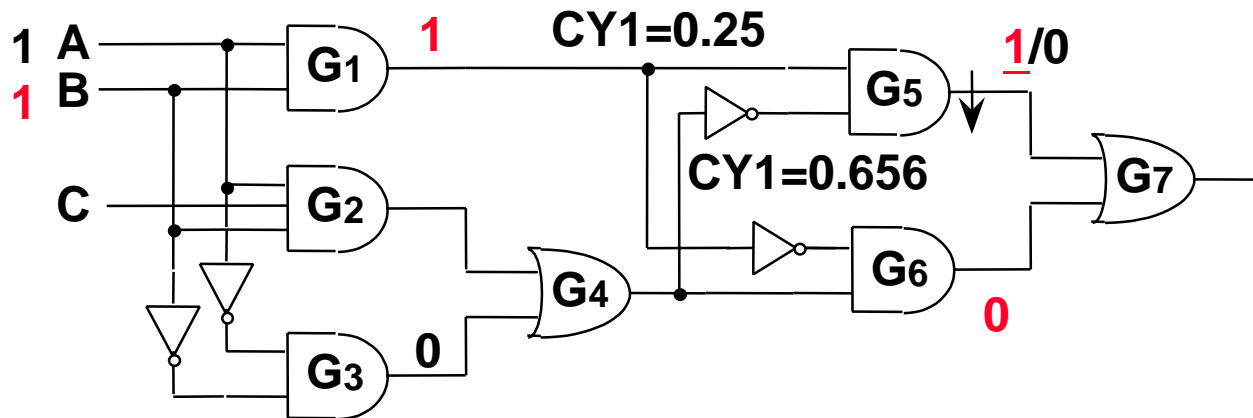
# PODEM: Example 2 (2/3)

The initial objective satisfied? No!  $\Rightarrow$  Current objective= $(G5,1)$ .

$G5$  is an AND gate  $\Rightarrow$  Choose the hardest-1  $\Rightarrow$  Current objective= $(G1,1)$ .

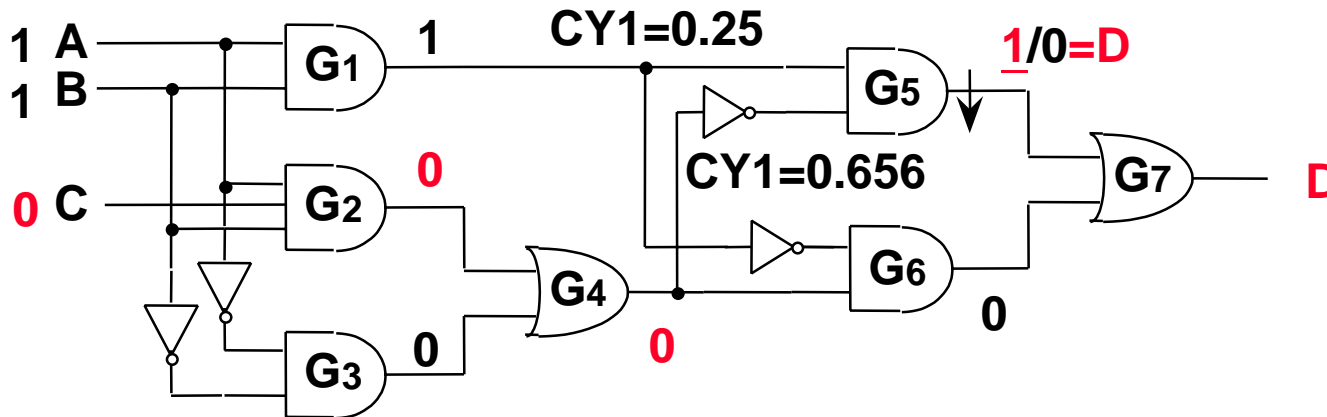
$G1$  is an AND gate  $\Rightarrow$  Choose the hardest-1  $\Rightarrow$  Arbitrarily, Current objective= $(B,1)$ .

$B$  is a PI  $\Rightarrow$  Implication  $\Rightarrow G1=1, G6=0$ .



# PODEM: Example 2 (3/3)

The initial objective satisfied? No!  $\Rightarrow$  Current objective=(G5,1).  
The value of G1 is known  $\Rightarrow$  Current objective=(G4,0).  
The value of G3 is known  $\Rightarrow$  Current objective=(G2,0).  
A, B is known  $\Rightarrow$  Current objective=(C,0).  
C is a PI  $\Rightarrow$  Implication  $\Rightarrow$  G2=0, G4=0, G5=D, G7=D.



**No backtracking!!**

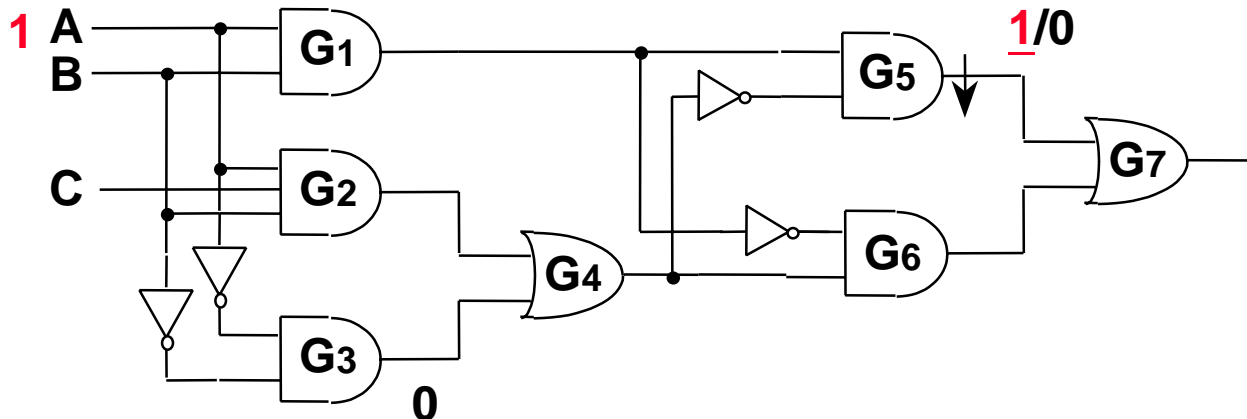
# If The Backtracing Is Not Guided (1/3)

Initial objective=(G5,1).

Choose path G5-G4-G2-A => A=0.

Implication for A=0 => G1=0, G5=0 => Backtracking to A=1.

Implication for A=1 => G3=0.



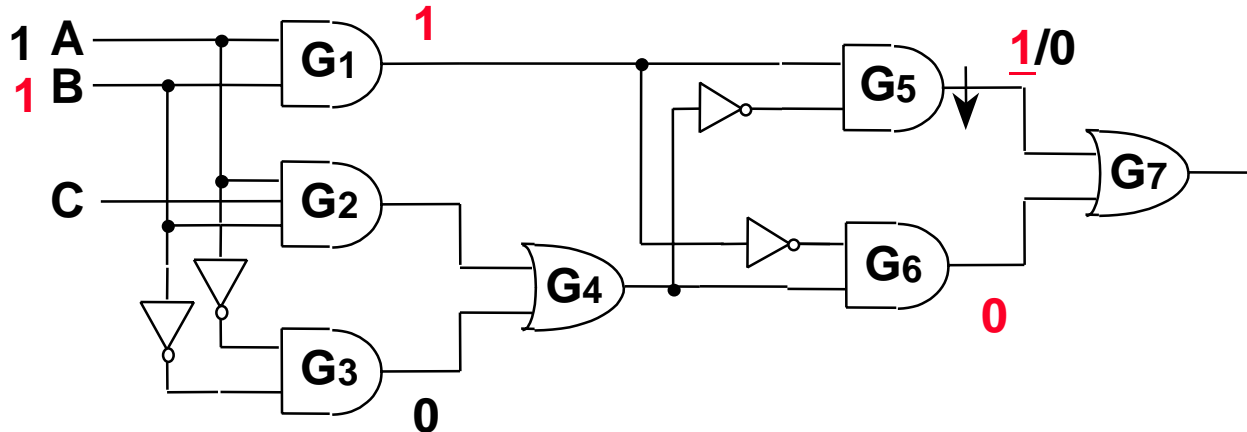
# If The Backtracing Is Not Guided (2/3)

The initial objective satisfied? No!  $\Rightarrow$  Current objective= $(G5,1)$ .

Choose path  $G5-G4-G2-B \Rightarrow B=0$ .

Implication for  $B=0 \Rightarrow G1=0, G5=0 \Rightarrow$  Backtracking to  $B=1$ .

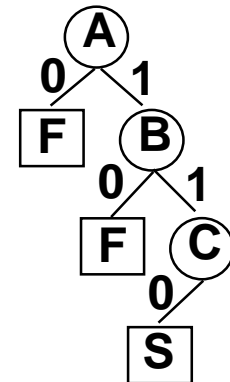
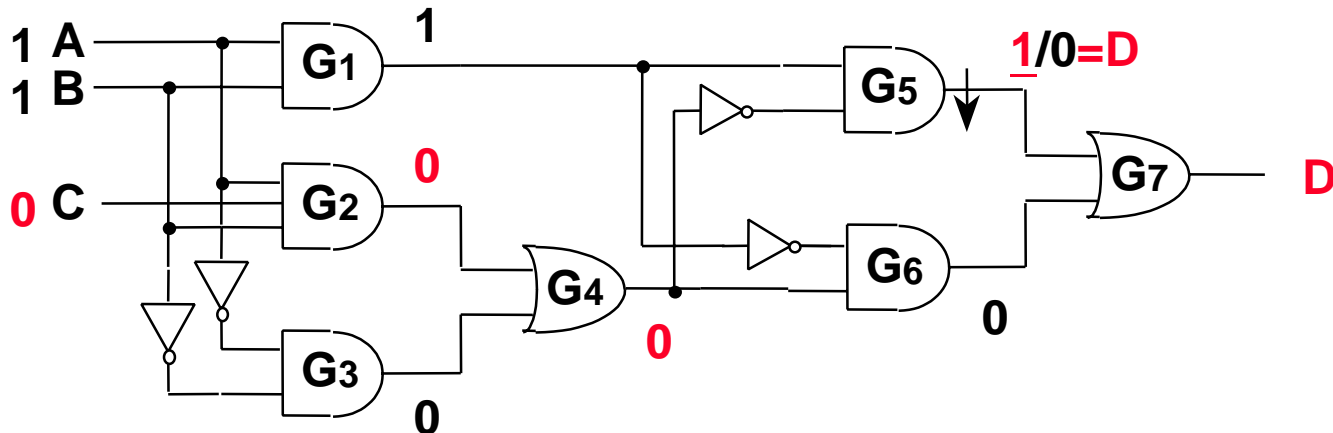
Implication for  $B=1 \Rightarrow G1=1, G6=0$ .





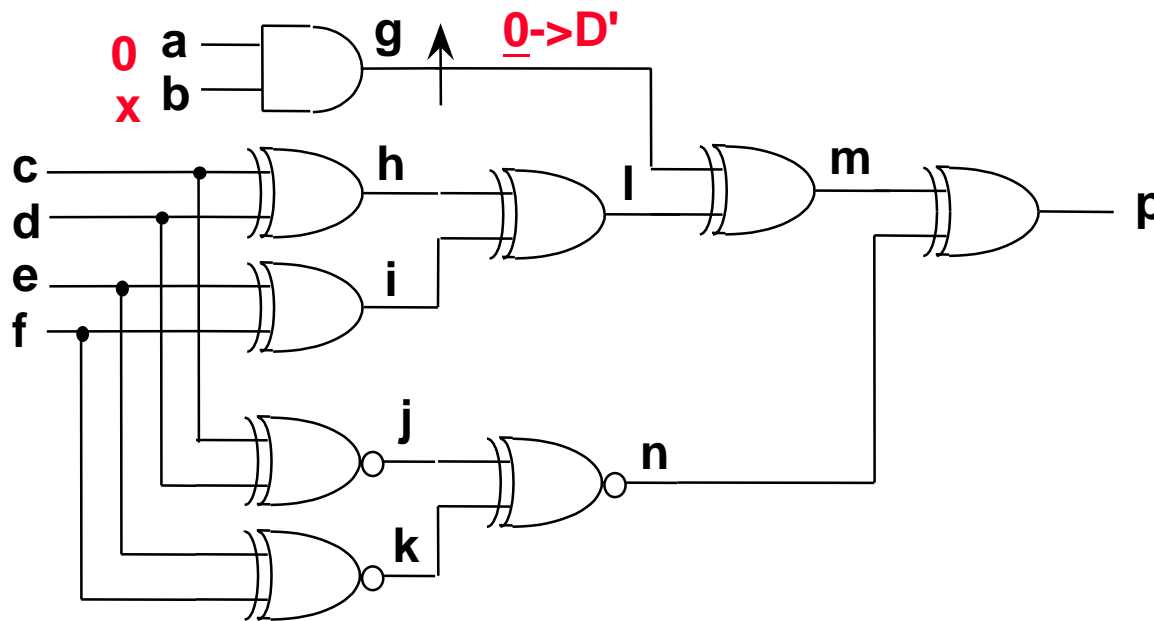
# If The Backtracing Is Not Guided (3/3)

The initial objective satisfied? No!  $\Rightarrow$  Current objective= $(G5,1)$ .  
Choose path  $G5-G4-G2-C \Rightarrow C=0$ .  
Implication for  $C=0 \Rightarrow G2=0, G4=0, G5=D, G7=D$ .



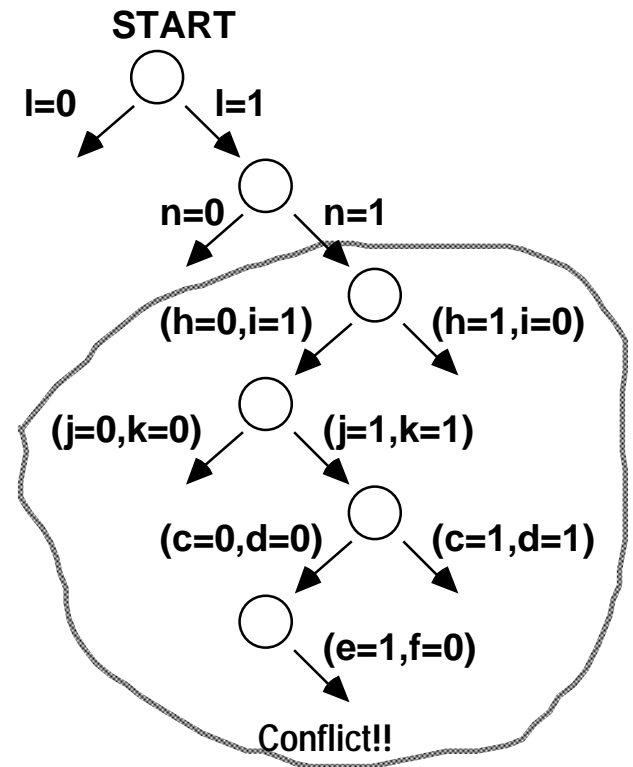
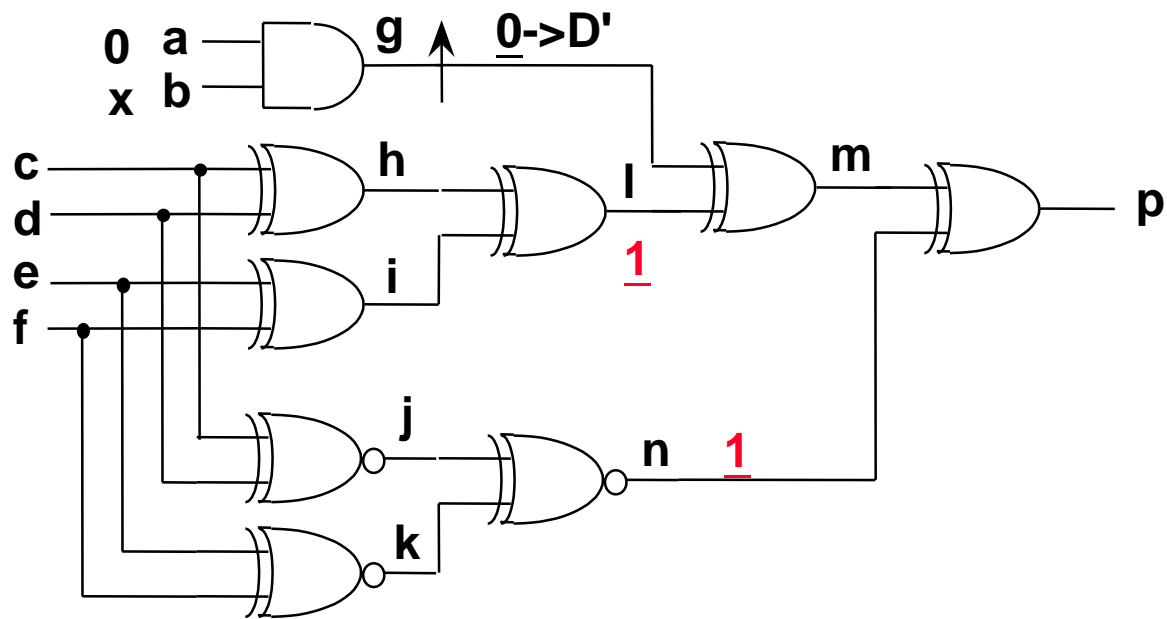
Two times of backtracking!!

# ECAT Circuit: PODEM



**No backtracking !!**

# ECAT Circuit: D-Algorithm

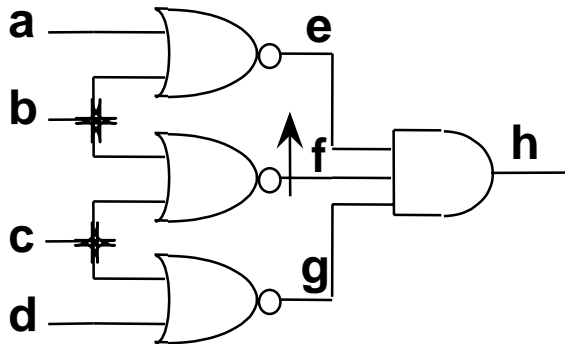


# Features of PODEM

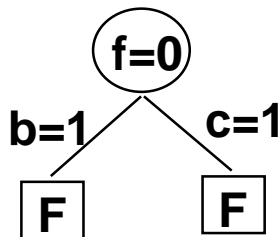
- PODEM examines all possible input patterns implicitly but exhaustively (branch-and-bound) as tests for a given fault.  
=> It is a complete TG.
- PODEM does not need
  - consistency check, as conflicts can never occur;
  - the J-frontier, since there are no values that require justification;
  - backward implication, because values are propagated only forward.
- Backtracking is implicitly done by simulation rather than by an explicitly save/restore process. (State saving and restoring is a time-consuming process.)
- Experimental results show that PODEM is generally faster than the D-algorithm. [4]

# Redundant Faults

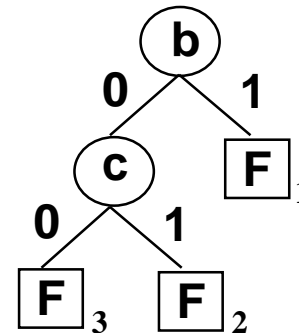
- Presence of the fault does not change the functionality of the circuit under test.
- Ex:



- Good function:  $h = (a+b)' (b+c)' (c+d)' = a'b'c'd'$
- Faulty function =  $(a+b)' (c+d)' = a'b'c'd'$
- Perform test generation



decision tree for D-algorithm



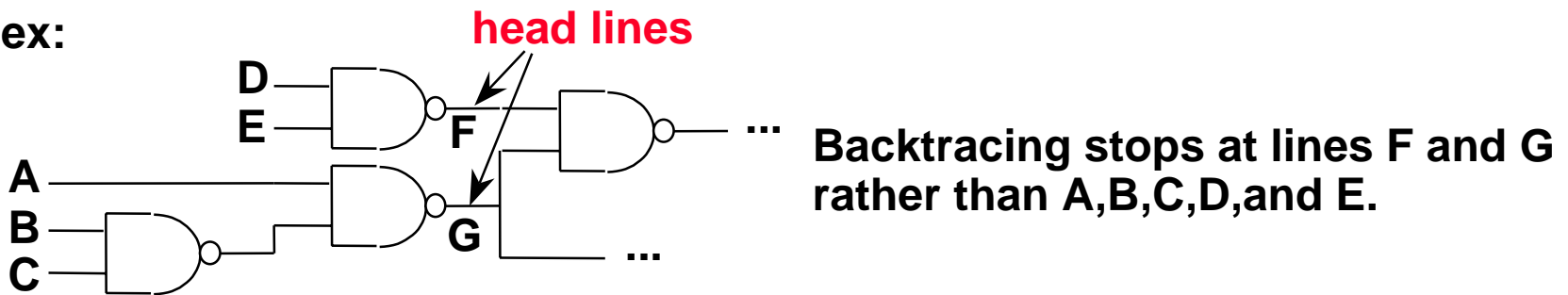
decision tree for PODEM

# FAN (Fanout-Oriented TG)

- FAN introduces two major extensions to the backtracing concept of PODEM:

1. Rather than stopping at PIs, backtracing in FAN may stop at internal lines.  
=> will reduce the number of backtracking.

ex:



2. Rather than trying to satisfy one objective, FAN uses a multiple-backtrace procedure that attempts to simultaneously satisfy a set of objectives. (In PODEM, a PI assignment satisfying one objective may preclude achieving another one, and this leads to backtracking.)

# ATPG

- **ATPG (Automatic Test Pattern Generation):**

Generate a set of test patterns for a set of target faults.

- **Basic scheme:**

Initialize vector set to NULL

Repeat

    Generate a new test vector

    Evaluate fault coverage for the test vector

    If the test vector is acceptable Then add it to vector set

Until required fault coverage is obtained

- **To accelerate the ATPG:**

Random patterns are often generated first to detect easy- to-detect faults, then a deterministic TG is performed to generate tests for the remaining faults.

# Sequential Test Generation

- **For Circuits with Unknown Initial States**

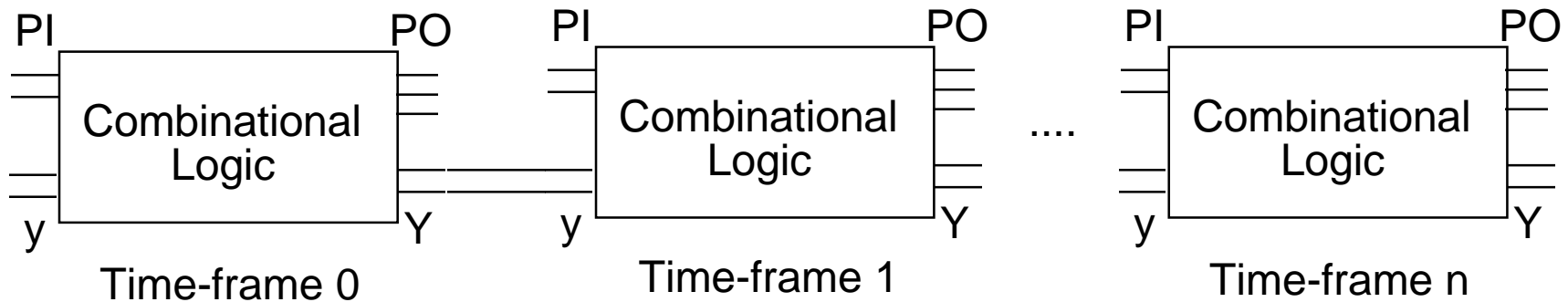
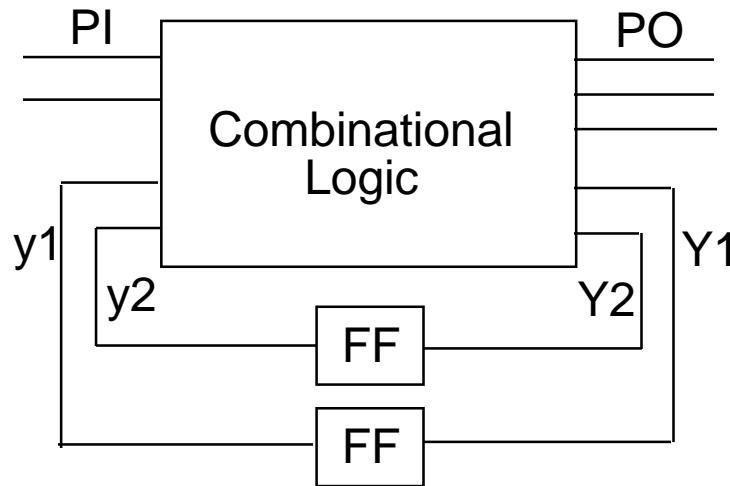
- Time-frame Expansion Based: Extended D-algorithm (IEEE TC, 1971), 9-V Algorithm (IEEE TC, 1976), EBT (DAC, 1978 & 1986), BACK (ICCD, 1988), ...
- Simulation-Based: CONTEST (IEEE TCAD, 1989), TVSET (FTCS, 1988), ...

- **For Circuits with Known Initial States**

- STALLION (IEEE TCAD, 1988), STEED (IEEE TCAD, May 1991), ...



# Iterative Logic Array (ILA) Model for Sequential Circuits

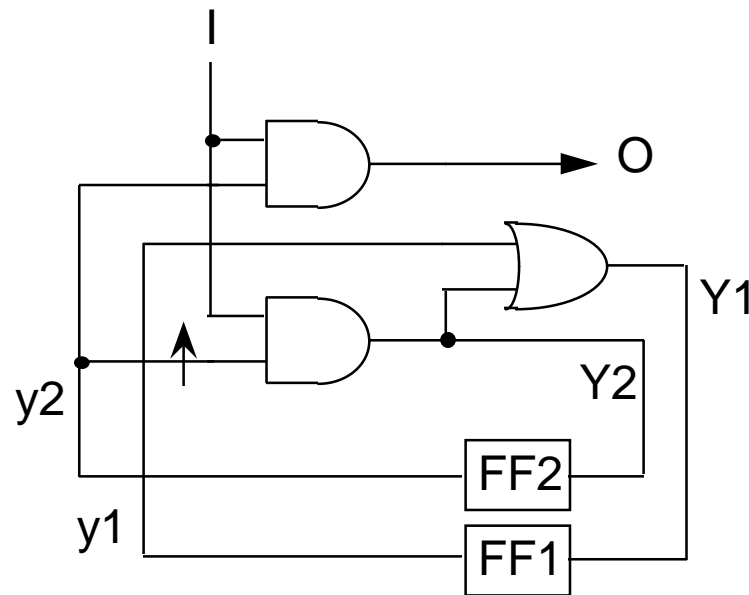


# **Extended D-Algorithm [1]**

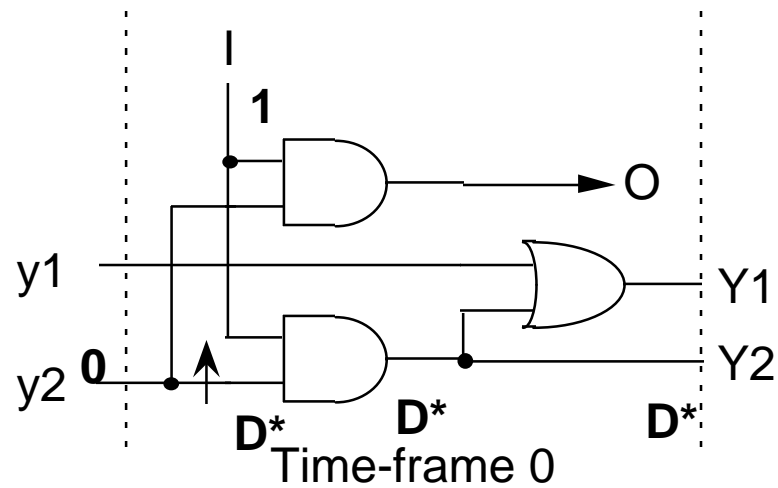
**(Kubo, NEC Research & Development, Oct. 1968)  
(Putzolu and Roth, IEEE TC, June 1971)**

- 1. Pick up a target fault  $f$ .**
- 2. Create a copy of a combinational logic, set it time-frame 0.**
- 3. Generate a test for  $f$  using D-algorithm for time-frame 0.**
- 4. When the fault effect is propagate to the DFFs, continue fault propagation in the next time-frame.**
- 5. When there are values required in the DFFs, continue the justification in the previous time-frame.**

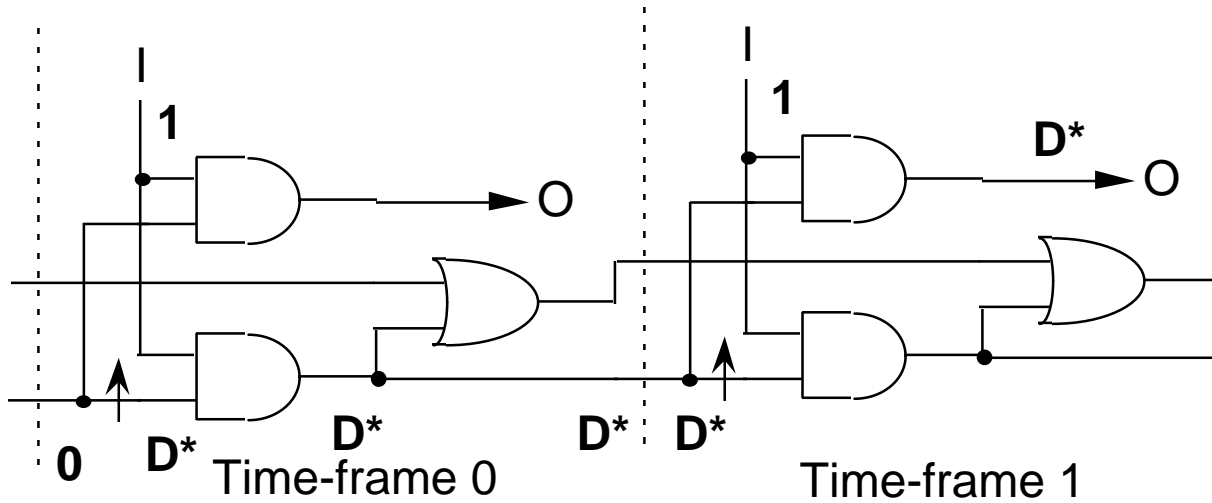
# Example for Extended D-Algorithm



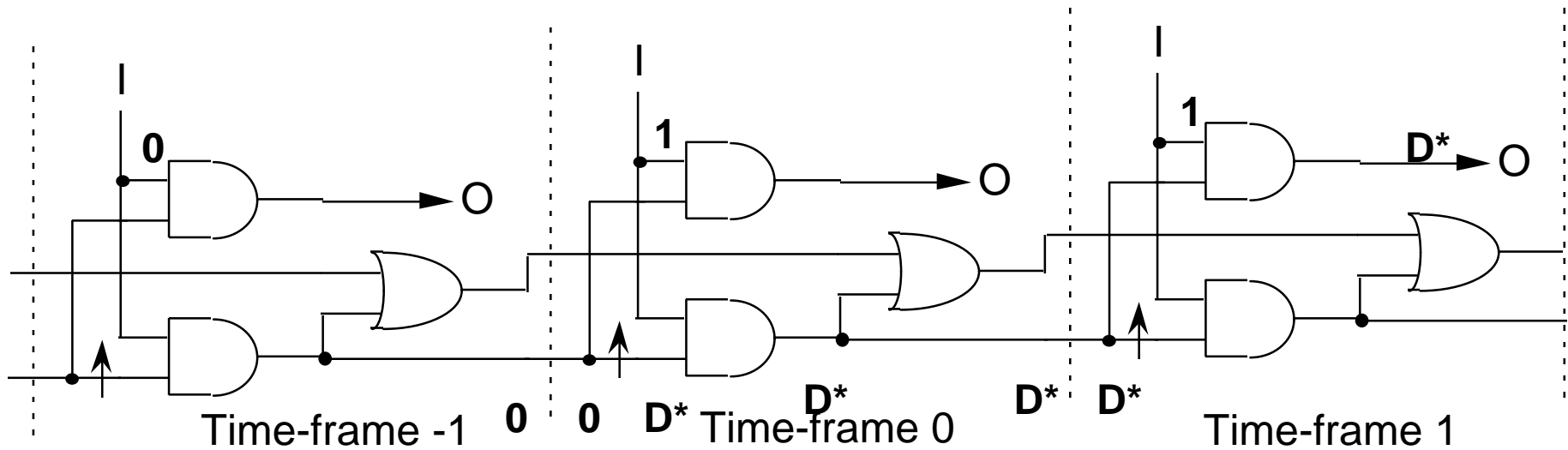
# Example: Step 1



# Example: Step 2



# Example: Step 3

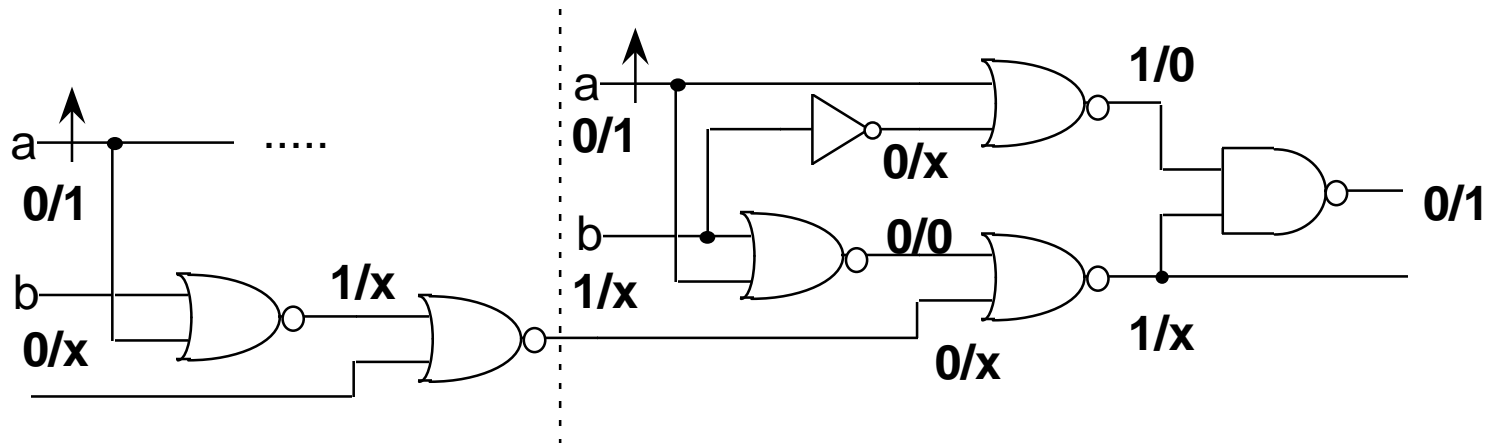
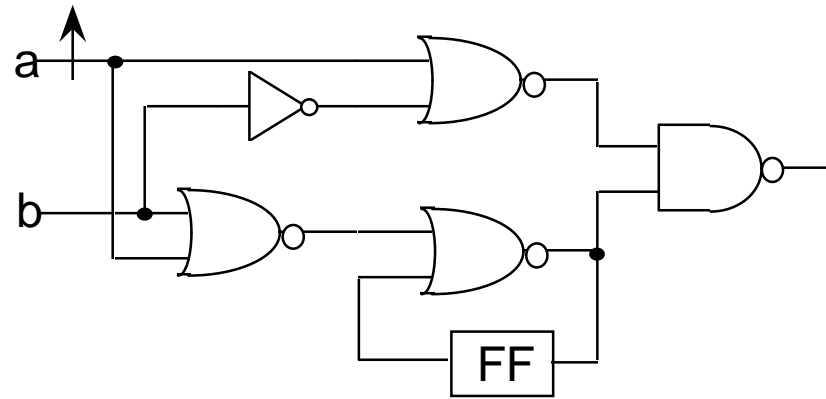


# 9-V Sequential TG <sup>[2]</sup>

(Muth, IEEE TC, June 1976)

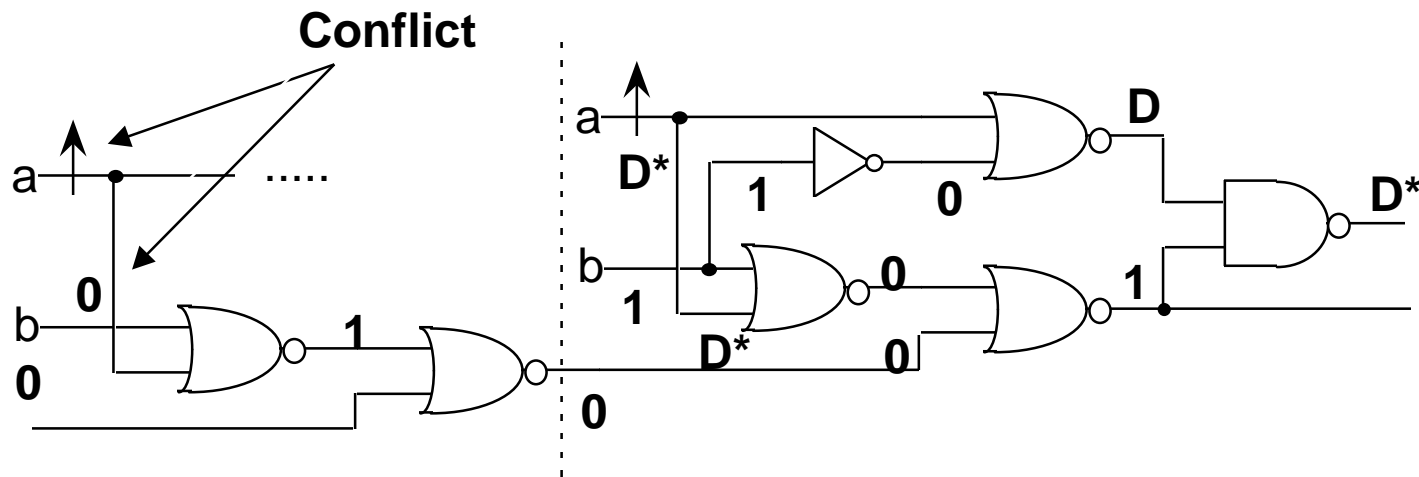
- **Extended D-algorithm is not complete.**
- **If nine-value, instead of five-value, is used, it will be a complete algorithm.** (Since it takes into account the possible repeated effects of the fault in the iterative array model.)

# Example: Nine-Valued TG





# If Five-Valued TG Is Used



**The test can not be generated by five-value TG.**

# Problems of Mixed Forward and Reverse Time Processing Approaches

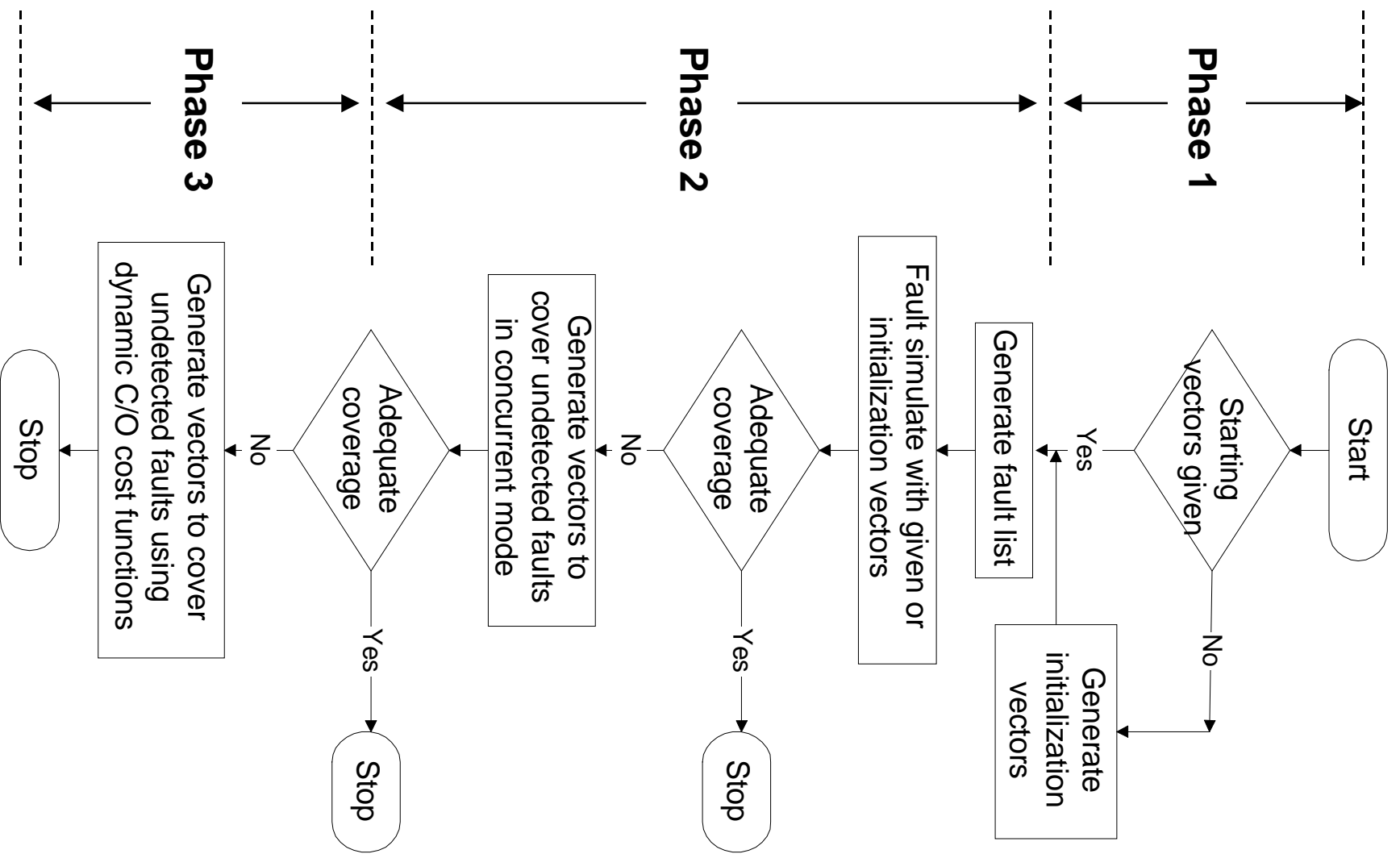
- **The requirements created during the forward process (fault propagation) have to be justified by the backward process later.**
  - **Need going both forward and backward time frames.**
  - **May need to maintain a large number of time-frames during test generation.**
  - **Hard to identify "cycles" .**
  - **Implementation is complicated.**

# **CONTEST: A Concurrent Test Generator for Sequential Circuits [3,4]**

(Agrawal and Cheng, IEEE TCAD, Feb. 1989)

- **Simulated-based test generation.**
- **It is subdivided into three phases :**
  - Initialization
  - Concurrent fault detection
  - Single Fault detection
- **For different phases, different cost functions are defined to guide the searching for vectors.**

# Flow Chart of CONTEST



# Simulation-Based Approaches

## Advantages:

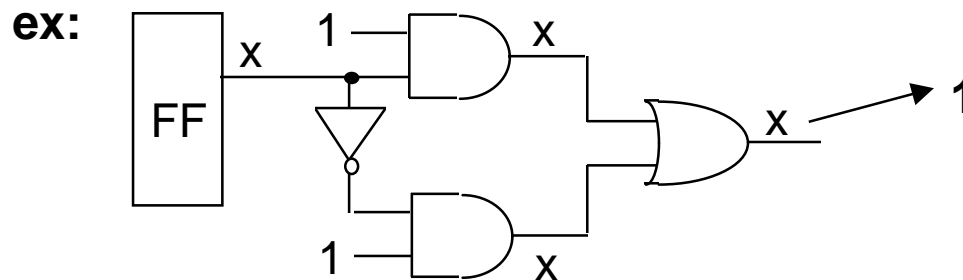
- **Timing is considered.**
- **Asynchronous circuits can be handled.**
- **Can be easily implemented by modifying a fault simulator.**

## Disadvantages:

- **Can not identify undetectable faults.**
- **Hard-to-activate faults may not be detected.**

# Difficulties of Sequential Test Generation

- **Initialization is difficult.**
  - Justifying invalid states
  - Long initialization sequence
  - Simulator limitations



- **Timing can not be considered by time-frame expansion.**
  - Generated tests may cause races and hazards.
  - Asynchronous circuits can not be handled.

# **Test Generation Assuming A known Initial State**

- **Initialization is avoided.**
- **Assumption is valid for pure controllers that usually have a reset mechanism (reset PI, resettable flip-flops, ...).**

# **STALLION** [5]

(Ma et al, IEEE TCAD, Oct. 1988)

- 1. Generate state transition graph (STG) for fault-free circuit.**
- 2. Create a copy of the combinational logic, set it time-frame 0. Generate a test for the fault using PODEM for the time-frame.**
- 3. When the fault is propagated to PPOs (but not POs), find a fault propagation sequence T to propagate the fault effect to a PO using the STG.**
- 4. When there are values required in PPIs, say state S, find a transfer sequence T0 from initial state S0 to S using the STG.**
- 5. Fault simulate sequence T0+T. If it is not a valid test, go to 3 to find another sequence.**



# STALLION

## Advantages:

- **Transfer sequences are easily derived from STG.**
- **Good performance for controllers whose STG can be extracted easily.**

## Disadvantages:

- **Fault-free transfer sequence may not be valid.**
- **Extraction of STG may not be feasible for large circuits.**

## Heuristics:

- **Construct partial STG only.**
- **If the required transfer sequence can not be derived from partial STG, augment the partial STG.**

# References

- [1] G. R. Putzolu and T. P. Roth, "A Heuristic Algorithm for the Testing of Asynchronous Circuits", IEEE Trans. Computers, pp. 639-647, June 1971.
- [2] P. Muth, "A Nine-Valued Circuit Model for Test Generation", IEEE Trans. Computers, pp. 630-636, June 1976.
- [3] V. D. Agrawal, K. T. Cheng, and P. Agrawal, "A Directed Search Method for Test Generation Using a Concurrent Simulator", IEEE Trans. CAD, pp. 131-138, Feb. 1989.
- [4] K. T. Cheng and V. D. Agrawal, "Unified Methods for VLSI Simulation and Test Generation", Chapter 7, Kluwer Academic Publishers, 1989.
- [5] H-K. T. Ma, et al, "Test Generation for Sequential Circuits", IEEE Trans. CAD, pp. 1081-1093, Oct. 1988.