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Efficient associative memory using small-world architecture

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Abstract

Most models of neural associative memory have used networks with broad connectivity. However, from both a neurobiological viewpoint and an implementation perspective, it is logical to minimize the length of inter-neural connections and consider networks whose connectivity is predominantly local. The "small-world networks" model described recently by Watts and Strogatz provides an interesting approach to this issue. In this paper, we show that associative memory networks with small-world architectures can provide the same retrieval performance as randomly connected networks while using a fraction of the total connection length. © 2001 Published by Elsevier Science B.V.

Keywords: Associative memory; Small-world networks

1. Introduction

Associative neural memories have been investigated extensively [5], and many results on their dynamics, capacity, etc., have been established [4,1]. Unlike fully connected artificial neural networks, biologically plausible associative memories must have sparse connectivity, reflecting the situation in the cortex and hippocampus. Networks with randomly diluted connectivity have been studied in detail [3,12,6], but cortical connectivity is captured better by modular networks comprised of several modules with dense internal connections and sparse inter-modular connections [8,11].

In this paper, we pursue the idea that it is not only the *degree* of sparseness but also its *structure* that determines the performance of a neural associative memory. In

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particular, we look at "small world networks" [13], which have highly clustered connectivity but short inter-node path lengths due to a small number of long-range connections. Our results show that such networks are more efficient in terms of resource utilization than randomly connected sparse networks.

2. Background and motivation

The notion of small world networks was first introduced in a social experiment by Stanley Milgram [9]. Milgram demonstrated that, despite the high amount of clustering in social networks (meaning that two acquaintances are likely to have other common acquaintances), any two individuals could be "linked" through a surprisingly small number of others. This idea is commonly referred to as "6 degrees of separation", implying that the average "distance" between any two people in the world is 6.

Recently, Watts and Strogatz have described a formulation for an analytical model of the small world. The network is a regular n-node ring lattice where each node is connected to its $k/2$ nearest neighbors on either side. A fraction p of these connections are then re-wired to other randomly selected nodes. For low p, this creates a network with primarily local connectivity and a few randomly placed long-range connections termed "short cuts" (see Fig. 1). The network is characterized by two quantities: (1) The *mean path length* between nodes, $L(p)$, and (2) The *clustering coefficient*, $C(p)$, which denotes the fraction of possible edges in a typical neighborhood which actually exist. The model is interesting because, for a surprisingly small value of p, the network remains highly clustered (or cliquish) like a regular lattice, but has small characteristic path lengths like a random graph. This is the *small world regime*.

Standard models of neural associative memory usually assume broad connectivity (full or diluted random) [1], without which the recall process often gets stuck on

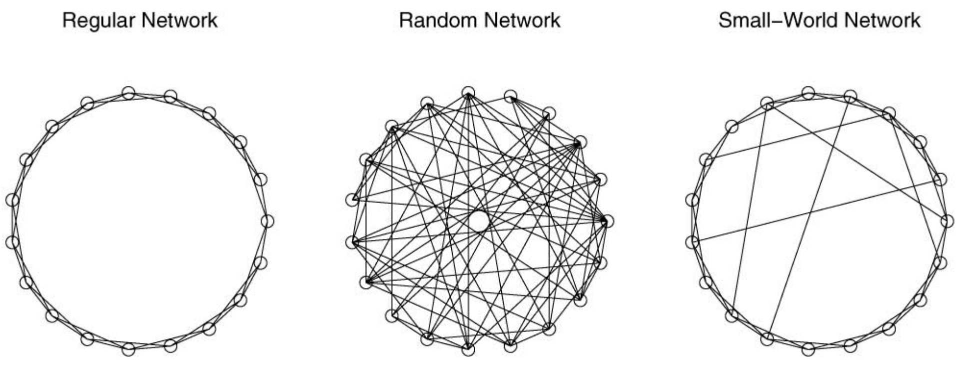


Fig. 1. Network connection topologies. Note that these graphs are undirected, and that in the associative memory networks, each edge is directed.

patterns with large sub-domains of error [10]. The hypothesis behind the research reported here is this: *The smallest value of p for which a neural associative memory with a small-world architecture performs almost as well as a randomly connected memory of the same connectivity is much less than* 1. This is a significant issue since the total connection length in a small-world network is much smaller than a random network of the same nominal connectivity (synapses/neuron). Thus, if the hypothesis is correct, a small-world network can achieve high performance with far less connection length than a randomly connected network, saving in terms of energy and space. Indeed, it has been shown that the nervous system of *C. Elegans* shows small world properties [13], and there is evidence that networks of Hodgkin–Huxley neurons with small-world connectivity are optimal for producing fast synchronized oscillations [7].

3. Network model

To test the hypothesis stated above, we use a network of N neurons which assume ± 1 binary states. Neural interconnectivity is specified by a graph, G on the vertices $[N] \times [N]$ where a connection, or synapse, from neuron i to j exists if $\{i,j\} \in G$. The network is trained with M randomly generated patterns of length N and the interconnection weights are given by

$$w_{ij} = \frac{1}{N} \sum_{\mu=1}^{M} \xi_i^{\mu} \xi_j^{\mu} c_{ij}, \tag{1}$$

where

$$c_{ij} = \begin{cases} 0 & \text{if } \{i,j\} \notin G, \\ 1 & \text{if } \{i,j\} \in G. \end{cases} \tag{2}$$

Initially, the network is a one-dimensional lattice with periodic boundary conditions (a ring), with each neuron feeding its output to its $k = \alpha N$ nearest neighbors, where α is the overall connectivity of the network. Each vertex (neuron) is then visited, and with probability p, each edge is rewired to a randomly chosen vertex in the network, as described in [13]. This introduces long-range or 'short-cut' connections. The network changes from regular connectivity at $p = 0$ to random connectivity at $p = 1$. The constraint of exactly k efferent connections per neuron reduces the probability of isolated vertices, and, if $k \gg \ln N$, the graph will remain connected [2].

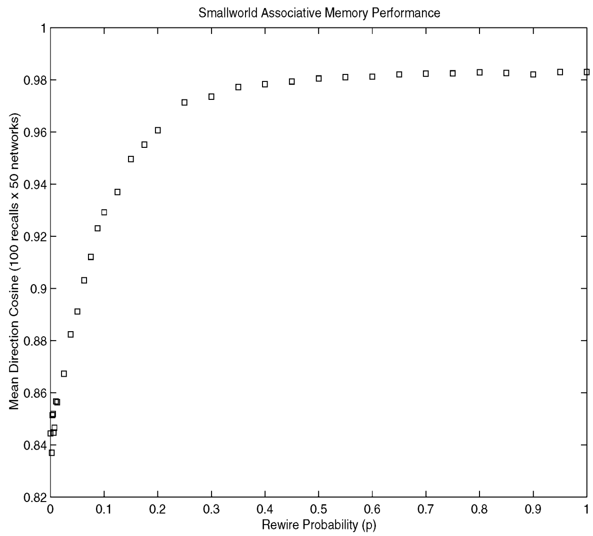
The state of each neuron in the system is updated in discrete time by the following rule:

$$S_i(t+1) = sgn\left(\sum_{j \neq i}^{N} w_{ij} S_j(t) \right), \tag{3}$$

where $sgn(\ )$ is the signum function and $S_j(t)$ is the output of neuron j at time t. Updating is done synchronously, and the network dynamics continue until convergence.

4. Simulations

The network was simulated for $N = 1000$ neurons, $M = 25$ memories, and $\alpha = 0.15$. The performance and dynamics of the network are investigated for $p = 0$ (only short-range connections), $p = 1$ (random connections), and intermediate values of p (the small-world regime). In all cases, test patterns are presented with 25% initial error.



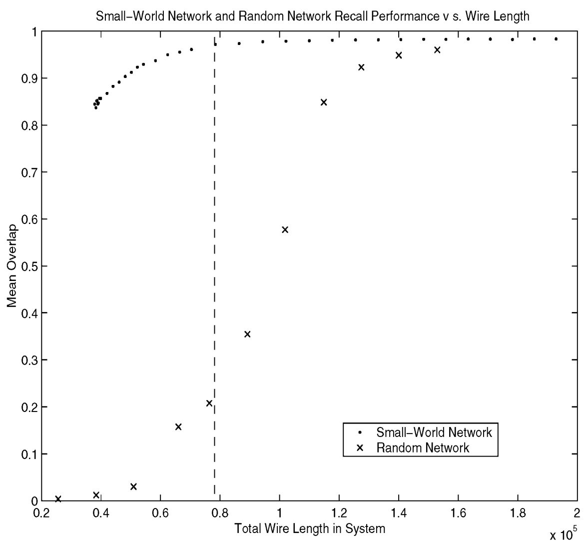


Fig. 2. (left) Network performance as a function of rewire probability; (right) Performance vs. wire-length for small-world (●) and random (×) networks. Wire length is calculated assuming a network radius of 1.

Fig. 2 shows the performance of the network as a function of rewiring probability and wire length. The comparison with the randomly connected network of equivalent wire length (and, therefore, lower connectivity) clearly shows the benefit of the small-world architecture.

As mentioned above, networks with purely local connectivity fail primarily because of the emergence of stable high-error domains in the network. A small-world architecture helps break up these domains by injecting signals from distant low-error do-

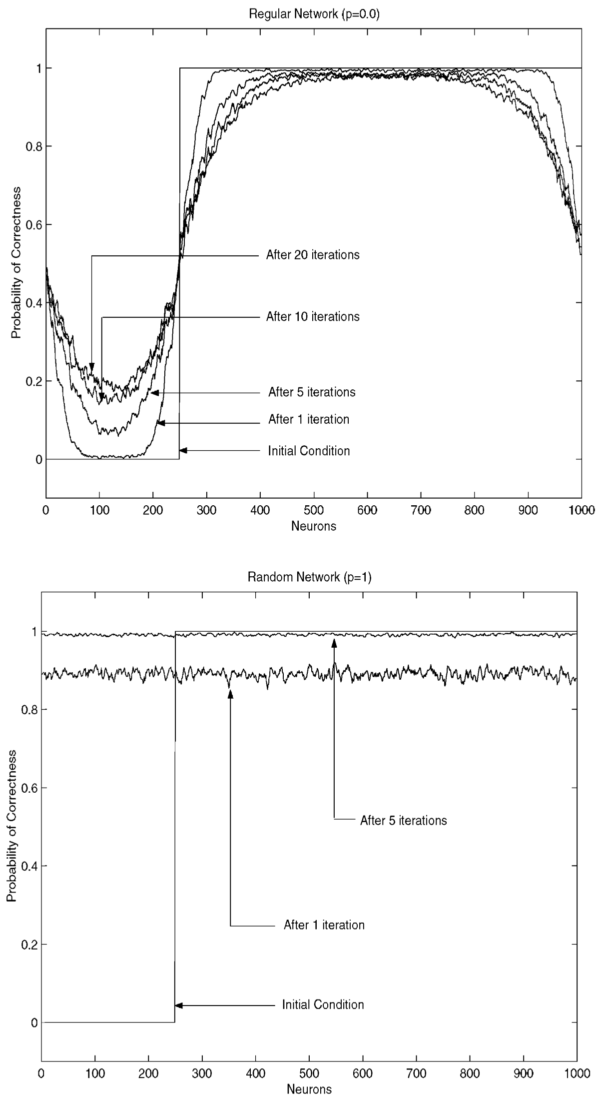


Fig. 3. (left) Regular network recall dynamics: Continuous curves for this and other corresponding graphs are obtained by using a nearest neighbors averaging filter; (right) Random network recall dynamics.

mains. To verify this, we simulate networks where the initial pattern has a contiguous 25% region corrupted (neurons 0 to 249). For each iteration, we monitor which neurons are firing correctly and which are firing incorrectly.

Over several networks, and 25 corrupted patterns per network, we obtain a probability for each particular neuron firing correctly during a given time cycle. Figs. 3 and 4 show these results for selected iterations in the regular, random, and small world settings. From the graphs, it is apparent that the random network (Fig. 3—right) is

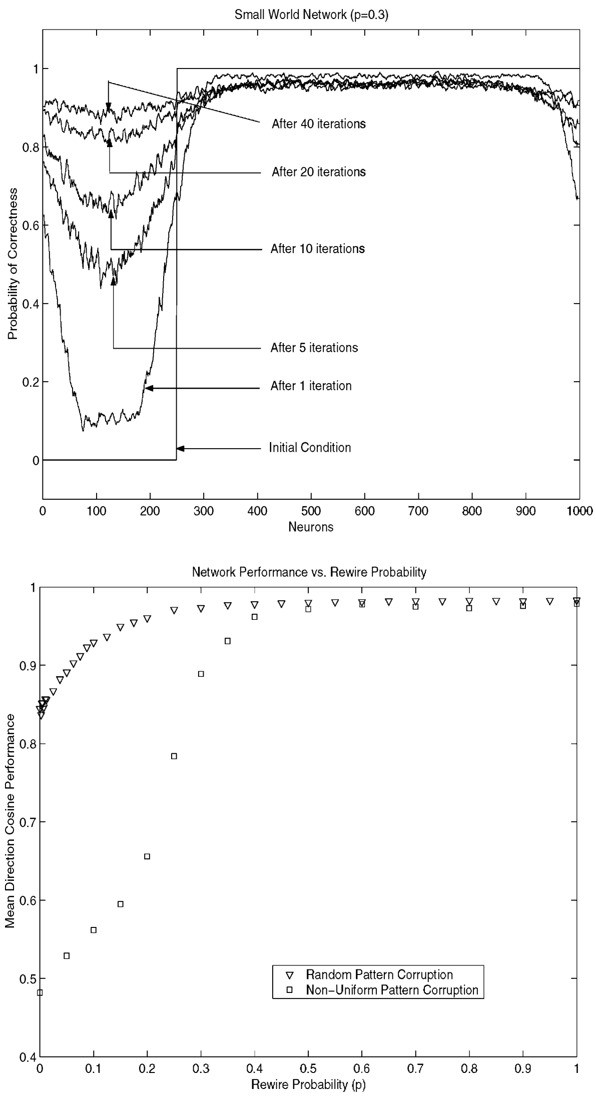


Fig. 4. (left) Small world network ($p = 0.3$) Recall Dynamics; (right) Network performance vs. Rewire probability. Points averaged over 10 networks, 25 patterns per network.

able to correct the vast majority of the error bits in one iteration, despite the separation of the ring into an initial 'error region' and a 'correct region'. The regular network (Fig. 3—left), however, is not able to propagate a correcting signal to the center of the error region, even after many iterations. In the small world regime (Fig. 4—left), we see how the addition of a small percentage of long range connections greatly improves the network's signal propagation ability, and, hence, it's performance. Fig. 4 (right) shows the performance of the network as a function of p. It is clear that near optimal performance is obtained for p as low as 0.4.

5. Discussion

The simulations presented in this paper show that an associative memory network's performance does not simply depend on the overall connectivity, but also on the *structure* of the connectivity. By beginning with a regular lattice, and rewiring a fraction of the neighborhood connections randomly, we can quickly approach the performance of a uniformly distributed random network with only a fraction of the total connection length. We observe a phase transition at relatively low p, much like the transition observed for mean path length in the Watts/Strogatz model. We also are able to observe the way in which signal propagation struggles in the regular lattice, and how the addition of shortcut links greatly improves this propagation. In terms of modular networks, we can hypothesize that densely interconnected modules with sparse inter-connections lead to network characteristics similar to those of small-world networks.

The results presented here are of significant importance for both biological and artificial neural networks. It is well-known that network connectivity in the cortex and other brain regions is mainly local, with relatively sparse long-distance projections. This is because these longer connections are more expensive, and take up more space. A critical issue, then, is to determine how much long-distance connectivity is enough to give the network all (or almost all) the computational power of a randomly connected network. Our results on small-world networks suggest that a relatively small degree of long-distance connectivity may suffice.

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