Chapter 1

Shouters and Whisperers : Random Wireless Sensor Networks with Asymmetric Two-Tier Connectivity

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1.1 Introduction

There has recently been great interest in networks with small-world [13] and power-law [3, 2] connectivity. Such networks have been shown to possess several characteristics that are very desirable for communication networks, e.g., short path lengths, high robustness, etc. However, most of the work has focused on networks in abstract spaces where the geographical constraints of two or three dimensional space are not a concern. This is an appropriate model for networks such as the internet and the World Wide Web, but not for wireless networks such as networks of randomly deployed sensors. A wireless communication device sends out its signal in all directions and, in the absence of physical obstacles, all receivers within range pick up the message. Thus, the connectivity of wireless networks in open environments shows strong clustering, and arbitrary random connectivity (or rewiring) is not a good model. Recently, many researchers have applied methods from percolation theory and critical systems to wireless networks, and have demonstrated the presence of typical phase-transition phenomena [11, ?, 6, 9]. In particular, some studies have looked at the effect of communication range (hop length) on connectivity, path length, and other network properties. However, this work has focused mainly on homogeneous networks where all nodes have identical transmission range — augmented in some cases [6] by base station nodes with wired infrastructure. In this paper, we look at random wireless networks comprising two classes of nodes with distinct transmission ranges, producing a two-tiered connectivity pattern represented by a directed graph. The primary motivation is to determine if the presence of a few nodes with a longer transmission range can improve the performance of otherwise homogeneous networks, much as the presence of a few long-range links do in small-world networks. Ultimately, this can lead to more efficient networks with greater robustness and longer lifetimes.

1.2 System Model

We consider a $L \times L$ square region with wireless sensor nodes deployed randomly over the area with density λ. These sensor nodes are capable of transmitting in a wireless fashion over a finite range. The baseline network has a homogeneous population of nodes, each with transmission radius R_w. We use a very simple connectivity model where node A is *connected* to a node B if the latter is within A's transmission radius. Thus, all edges in the baseline network are undirected. It is well-known that such networks exhibit percolation, i.e., a phase transition in connectivity with increasing node density and transmission radius [11, ?, 6, 9]. For a fixed node density, above a certain threshold radius, R — termed the *percolation threshold* — almost all nodes become part of a single connected graph called the *percolation cluster*, while just below R, nodes only form small, localized graphs. The radius R_w is chosen such that the network is just above the percolation threshold, ensuring a connected network with high probability.

Due to the inherently localized connections, the baseline network is analogous to the "regular" network model used by Watts and Strogatz [13] as the point-of-departure for small world networks. Both show a high degree of clustering and long path lengths between nodes. However, due to the undirected nature of wireless transmission, a small world version of our baseline network cannot be obtained by random rewiring of edges. Instead, we consider networks where a fraction η of the nodes increase their transmission radius to $R_s > R_w$, adding some longer range edges to those of the original network. The nodes with the larger radius are termed *shouters*, and the remaining nodes are called *whisperers*. The network is called a *whisperer-shouter (WS) network*. Unlike the basline network, WS networks are directed graphs.

We define: *whisperer radius*, $\beta = R_w/L$, *shouter amplification factor*, $\gamma = R_s/R_w$, *shouter fraction*, η, *shouter density*, $\lambda_s = \eta\lambda$, and *whisperer density*, $\lambda_w = (1-\eta)\lambda$, as the basic network parameters, and study the dependence of network properties on γ and η for a fixed L and β. Note that both $\eta = 0$ and

$\gamma = 1$ correspond to homogeneous networks.

Unlike networks that can be rewired randomly, WS networks are always highly clustered due to their localized connectivity pattern. However, two quantities of interest do vary in opposite directions as η and/or γ is changed. These are the *mean shortest path length* between nodes and *congestion*. The path from node i to node j in a wireless network comprises a series of "hops" along the edges of the network's graph. A message travelling from i to j therefore entails a retransmission by an intermediate node for every hop, and the *hop count*, H_{ij}, of the shortest path from i to j consitutes a reasonable measure of the distance travelled by the message. More realistic measures may also account for the length and/or energy cost of each hop. Another factor affecting the message is the congestion it must endure at each node between i and j. A good first-cut measure of congestion at a node, k, is its in-degree, I_k: the number of nodes it can "hear".

Clearly, as the fraction and/or radius of shouters increases in a WS network, hop counts between nodes decline and the in-degrees of individual nodes increase. Thus, there is a trade-off between the reduction in path lengths (hop-count) and increase in congestion (node in-degree). Also, since the presence of shouters increases redundancy of paths between nodes, WS networks are likely to be more robust to loss of nodes. Based on this, we consider the behavior of WS networks in η-γ space to address two questions:

1. **Q1:** Is there a region where a large reduction in hop-count can be achieved at the cost of only a small increase in congestion?

2. **Q2:** To what degree is the connectivity of WS networks more robust to loss of nodes, and how does this depend on network parameters?

The first question looks for an effect analogous to the small-world phenomenon where great reduction in path length can be achieved with minimal loss of clustering, while the latter considers the sort of robustness to node failure found in scale-free networks [2].

1.3 Analysis and Simulation

Consider a directed graph, $G_N = \{E, V\}$, corresponding to a WS network, with node set V and edge set E. Let V_s and V_w denote the sets of shouter and whisperer nodes, respectively. The area covered by a transmission from a whisperer is $a_w = \pi R_w^2$, while that covered by a shouter is $a + s = \pi R_s^2$. For a given node i let, w_i denote the number of whisperers within radius R_w and s_i the number of shouters within the radius R_s of i. The in-degree of this node, $I_i = w_i + s_i$. If the shouters are chosen randomly from the initial node population, both whisperers and shouters can be seen as having Poisson distributions with parameter λ_w and λ_s, respectively. It follows that
$prob(w_i = k_w) = \frac{(\lambda_w a_w)^{k_w}}{k_w!} e^{-\lambda_w a_w}$

$prob(s_i = k_s) = \frac{(\lambda_s a_s)^{k_s}}{k_s!} e^{-\lambda_s a_s}$
Thus, $I_i = w_i + s_i$ is also Poisson
$prob(I_i = k) = \frac{(\lambda_w a_w + \lambda_s a_s)^k}{k!} e^{-\lambda_w a_w + \lambda_s a_s}$
and the expected value of in-degree of connectivity $\langle I \rangle = \lambda_w a_w + \lambda_s a_s$.

Using the parameter definitions above, $\lambda_w = N_w/L^2 = (N - N_s)/L^2 = (1-\eta)N/L^2 = (1-\eta)\lambda$, $\lambda_s = \eta\lambda$, $a_w = \pi L^2(\beta)^2$, and $a_s = \pi L^2(\gamma\beta)^2$, so that

$$\langle I \rangle = [(1-\eta)\lambda + \eta\lambda\gamma^2]\beta^2\pi L^2 = (1 + \eta(\gamma^2 - 1))\lambda\pi\beta^2 L^2$$

Thus, the mean node in-degree has a linear dependence on η (the fraction of shouters) and a quadratic dependence on β (the normalized radius) and γ (the shouter amplification factor). The empirical data presented below verifies this, though the results are only approximate due to correlations between nodes.

The variation in the mean shortest path-length between nodes as a function of network parameters is more complicated to evaluate analytically, and here we use only empirical data obtained via simulation.

To obtain meanigful empirical data, we first determined the smallest transmission radius such that a homogeneous network of the given size would be almost certainly connected. As mentioned above, such networks undergo a phase transition from small clusters to a single connected component (percolation cluster) at a threshold radius. We chose our baseline homogeneous network such that each node had a transmission radius slightly higher than this percolation threshold, ensuring that the network would be connected even without shouters. We then systematically varied the shouter fraction, η, and the shouter gain, γ, to obtain data for mean shortest path hop-count, $\langle H(\eta, \gamma)$, and mean node in-degree, $\langle I(\eta, \gamma)\rangle$. Figure 1 shows the empirical results obtained for 250-node networks. As expected, mean in-degree shows a linear dependence on η and a quadratic dependence on γ. It is also apparent that the answer to **Q1** (above) is negative: There is no region with greatly reduced hop-count but low in-degree. This is verified in Figure 2, which plots

$$C = \langle I(\eta, \gamma)\rangle\langle H(\eta, \gamma)\rangle - \langle I(0, 1)\rangle\langle H(0, 1)\rangle$$

C can be seen as a rough measure of total congestion seen by messages traversing the shortest paths between nodes. However, there is a remarkably large region where C is close to zero, indicating a total congestion no worse than that seen for homogeneous networks.

To address **Q2**, we randomly deleted a fraction, δ, of nodes in networks and measured the effect on the connectivity of the remaining nodes by calculating two values: 1) The fraction of surviving nodes still connected to the percolation cluster; and 2) The fraction of node pairs that are connected (multi-hop) before and after the deletion. Figures 3 and 4 show the results for four situations: 1) Homogeneous (baseline) networks with $\eta = 0$, $\gamma = 1$; 2) High γ (=3), low η (=0.03) networks; 3) Low γ (=1.1), high η (=0.5) networks; and 4) Networks with moderate γ (=2) and η (=0.2). As can be seen from the figures, networks with moderate values of η and γ show significantly greater robustness by both

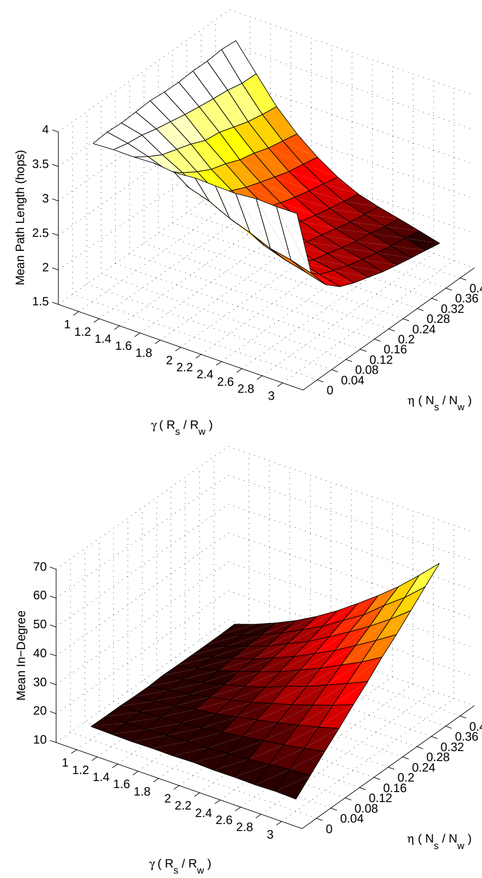

Figure 1.1: Hop-Count and In-Degree vs η and γ

measures. The other networks show no greater robustness than the homogeneous network. These results can be explained as follows. In the high γ-low η networks, the few shouters account for most of the connectivity beyond that of the baseline network, and these are lost quickly as nodes fail randomly. The low γ-high η networks are not significantly different from homogeneous networks with slightly larger transmission radius, and behave similarly. The moderate γ-moderate η networks, however, provide significantly greater redundancy in a well-distributed way, so the loss of a fraction of shouters still leaves most of the benefit intact.

1.4 Conclusions and Future work

In this preliminary study of random wireless networks with two-tier connectivity, we have shown that they hold some promise with regard to robustness, but do

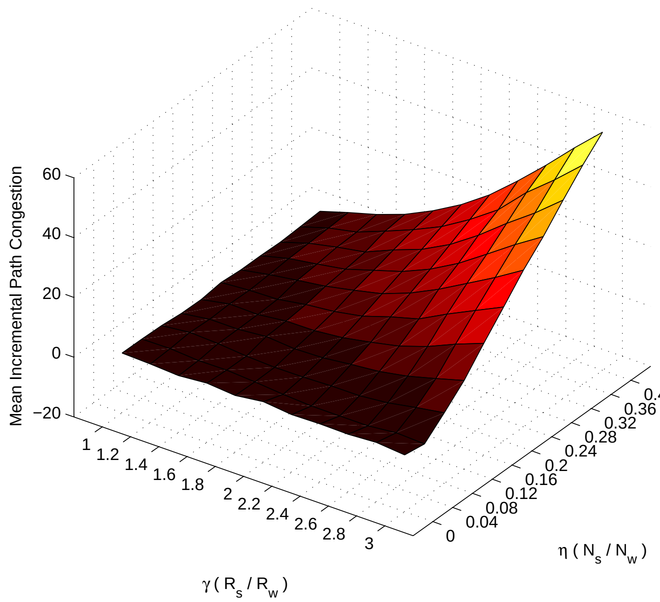

Figure 1.2: Congestion

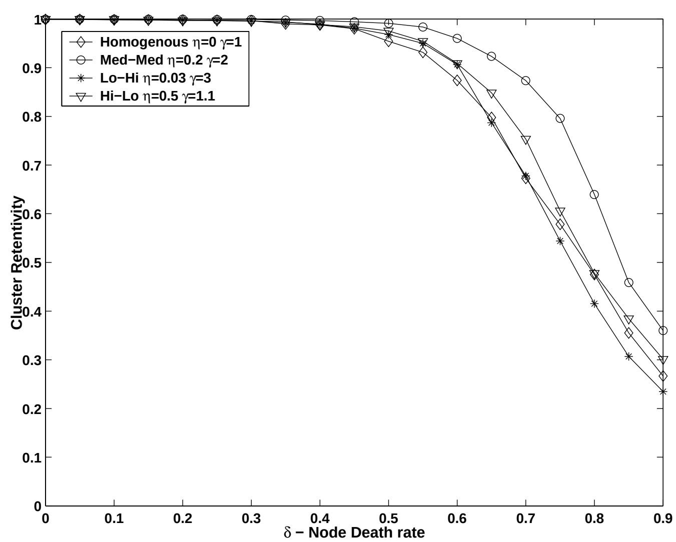

Figure 1.3: Size of the residual percolation cluster vs. fraction of failed nodes. Cluster size is shown as fraction of surviving nodes

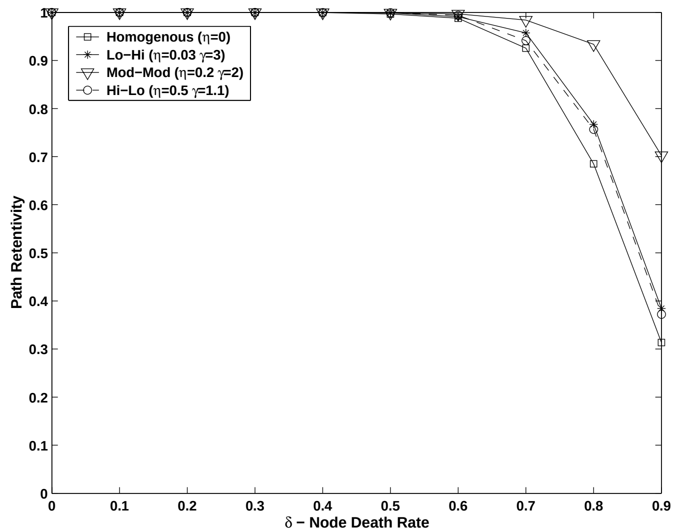

Figure 1.4: Fraction of surviving node-pair connections vs. fraction of failed nodes

not provide any significant gains in terms of path length and congestion. However, it must be emphasized that the connectivity in our networks was obtained randomly, and it is possible that optimized two-tier networks would show more significant benefits. Results on such networks will be reported in the future.

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