Using Noise to Synchronize Chaotic Neural Oscillators

Ali A. Minai, ECECS Department, University of Cincinnati, Cincinnati, OH
Tirunelveli Anand, ECECS Department, University of Cincinnati, Cincinnati, OH

Abstract

*The utility of noise in nonlinear systems is an issue of broad interest, with profound implications about the organization of behavior in complex systems. In this paper, we present an instance where 'noise-like' aperiodic input to a group of chaotic neural oscillators can spontaneously produce a highly organized and potentially useful behavior — synchronization — which is much more **difficult** to produce with a 'signal-like input. In particular, oscillators subject to a fixed signal input can be synchronized quite rapidly by the addition of noise, and, in most cases, increasing noise amplitude enhances the reliability and speed of synchronization.*

1. Introduction

Emergent synchronization among nonlinear oscillators is a phenomenon of great potential significance in biological and physical systems [4, 6, 5, 19, 20, 25, 27, 28, 3]. The conditions under which continuous-time oscillators synchronize have been studied extensively [26, 19, 20, 11, 7, 2, 27, 28, 3]. In this paper, we present an elegant and relatively generic mechanism for synchronizing populations of discrete-time oscillators, such as those used to model neural networks [8,1] and other spatially extended systems [10].

Synchronization has been studied phenomenologically in discrete-time coupled map lattices and globally coupled map networks by Kaneko and others [9, 10, 18]. Networks of chaotic elements have also been studied as associative memories [8,1]. However, the focus has been on the characterization and application of the clustered periodic attractors which arise naturally in these spatially extended systems. Our work is related most closely to previous research on random maps [32, 22], which showed that nonlinear maps with randomly varying parameters can produce synchronized or clustered dynamics.

2. Neural Oscillator Model

The neural oscillator model we have studied was proposed and studied by Wang [29, 30], and is essentially a discrete-time version of Wilson-Cowan oscillators [31]. Oscillations arise from the interaction of an excitatory element with state x^t and an inhibitory element with state y^t at time t. The equations for state transition are:

$$x^{t+1} = f(w_{xx}x^t - w_{xy}y^t + u^t - \theta_x) \qquad (1)$$

$$y^{t+1} = f(w_{yx}x^t - w_{yy}y^t + v^t - \theta_y) \qquad (2)$$

where the w's are weight parameters, u^t and v^t are external inputs, the θ's are fixed thresholds, and $f()$ is a sigmoid function. We take $f(p;\mu) = tanh(\mu p)$. We also assume $w_{xx} = w_{xy}$ = a and $w_{yx} = w_{yy}$ = b. Wang [30] has shown that, with a $\geq$ 2b and $u^t = v^t = 0$, changing μ produces a series of period-doubling bifurcations to chaos.

With the specification of $f()$ and restriction of the weights, and setting v^t, θ_x and θ_y to zero, the oscillator can be described by a l-dimensional equation:

$$z^{t+1} \equiv F(z^t) = tanh[\mu(az^t + u^t)] - tanh[\mu bz^t] \qquad (3)$$

where $z^t = x^t - y^t$ [30,16]. An oscillator with this equation will be described as a *$\mu/a/b$ oscillator.* The dynamics of z^t has two basins of attraction separated by z^t = 0, and a period-doubling cascade occurs in both basins as μ is increased. It is simple to show that if the initial condition, z^0, and u^t have the same sign for all t, the dynamics remains confined to the corresponding basin. Unless otherwise stated, we will assume that $z^0 > 0$ and $u^t \geq 0$, so that $z^t > 0$ for all t.

For a fixed input, u, the oscillator can exhibit a wide variety of behaviors. If μ is set such that the oscillator is chaotic for u = 0, then increasing u leads to a period-halving cascade, leading eventually to a 2-cycle for a very broad range of u (see Figure 1). It is this bifurcation behavior in response to external stimulus which makes synchronization possible.

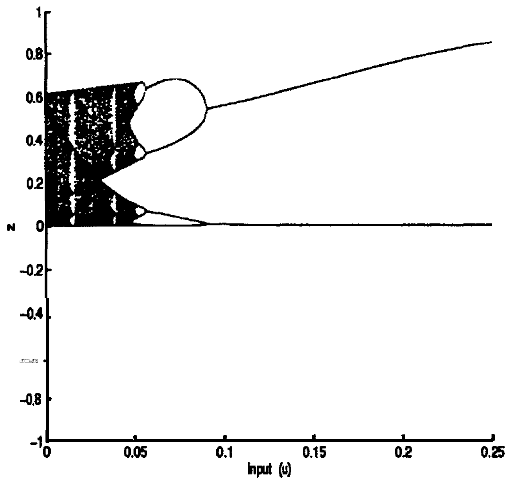

Figure 1. Bifurcation diagram for a 5/5/1 oscillator with respect to changing input, u. As the input Is increased, the oscillator goes from chaos to a stable P-cycle via a series of period-halving bifurcations. Negative inputs with a negative initial condition lead to a similar bifurcation pattern with $z<0$ (not shown).

3. Synchronization Dynamics

To show how a noise-like input can synchronize a population of initially unsynchronized identical oscillators, we begin with a simple type of input, u^t, which takes one of two values, u_l or u_h, for random lengths of time, k_r. The values are chosen such that u_l puts the system in the chaotic regime, and $u_h > u_l$ puts it in the 2-cycle regime. Consider a population of N desynchronized oscillators with different initial conditions and subject to u = u_l for k_0 time steps. Since the system is chaotic, the oscillators follow separate paths, corresponding to N degrees of freedom. Suppose that at step $t_1 = k_0 + 1$, u changes to u_h for k_1 time steps. The dynamics enters the 2-cycle regime, where there are 2 domains of attraction — one for each phase of the cycle. Let these be denoted by D_1 and D_2. If k_1 is long enough, the oscillators will divide into 2 phase-locked clusters, C_1 and C_2. If $z_i^{k_0} \in D_1$, oscillator i ends up in C_1, and if $z_i^{k_0} \in D_2$, it goes to C_2. Note that, while the input remains in the period-2 regime, the clusters will remain strictly out of phase, never occupying the same domain. At time step $t_2 = k_0 + k_1 + 1$, input u reverts to u_l for k_2 steps, putting the system into the chaotic regime. Due to determinism, the two clusters remain phase locked internally, but the phase constraint *between* the clusters is lost. At time step $t_3 = k_0 + k_1 + k_2 + 1$, the input changes again to u_h. Each cluster now falls into one of the two domains, D_1 and D_2, but *both* clusters may fall into the same domain too, leading to synchronization in the period-2 regime. Subsequent return to the chaotic phase will not destroy this synchronization.

Essentially, the synchronization mechanism described above works by alternating periods of clustering with rigid inter-cluster phase constraints (periodic phase) and relaxation of these constraints (chaotic phase). The deterministic nature of the system ensures that the number of clusters never grows, leading to ultimate synchronization. It is clear that the mechanism would work just as well with cycles of any period, though the time needed to achieve synchronization might increase with the period. There are also other constraints imposed by the specific parameters of $f(\,)$ and the distribution of the k_r values [17], but the generic scenario is valid.

The above analysis requires that, after each switch, the input remain stable long enough for transients to disappear. In fact, however, this is not necessary. A very broad range of rapidly varying inputs can also synchronize the system. As discussed above, synchronization occurs due to the interplay of two competitive effects: Convergence of trajectories in the periodic regimes, and mixing of trajectories in the chaotic regime. The former is needed to cluster oscillators into phase-locked groups, while the latter ensures that there is only one such group instead of many. Since the system constantly jumps between the periodic and chaotic regimes, the convergence (or otherwise) of its global behavior is a statistical effect. If the system spends all its time in the convergent regime it will organize into several clusters, but not necessarily into one synchronized cluster. In contrast, if it spends too much time in the mixing (chaotic) regime, the clustering never takes hold. Thus a judicious balance of convergence and divergence is needed to obtain global synchronization. This can be given a more rigorous description using the symbolic dynamics of the oscillators, as discussed elsewhere [15,17]. Both experiment and theory indicate that stochastic inputs with simple distributions (e.g., Gaussian or uniform) produce very broad, well-differentiated regimes of synchronized behavior (see Figure 3 and 4). Periodic inputs, on the other hand, do not produce such simple behavior (Figure 5) because, in some cases, oscillator groups can become phase-locked to the input, leading to attractor clustering as seen in globally coupled maps [10,8]. Synchronization can, thus, be produced much more easily and reliably with an aperiodic 'noise-like' input than with a periodic 'signal-like' one! This is somewhat reminiscent of Pecora and Carroll's method for synchronizing phase-shifted continuous-time periodic oscillators by a chaotic driving input [21].

The scenario described above is clearly quite generic. Virtually any population of discrete-time maps capable of period-doubling bifurcations can be synchronized simply by repeatedly 'scrubbing' the system across the several bifurcations. Of course, maps which have fixed-point regimes can be synchronized trivially by moving the system into and out of the fixed point regime. The results presented here show that transition to a fixed point is not necessary; switching between chaos and periodic behavior is enough. Indeed, the system does not even need to actually get into a cycle — only into the periodic regime with its drastically reduced choice of phases (See Figure 2). This is of great significance since many extended systems do not have the option of converging to a fixed point or low-period cycle in order to synchronize.

One interesting aspect of the synchronization scenario described here is that a fixed input will not lead to synchronization while a time-varying one will. Thus, if a set of oscillators is getting a fixed input, the addition of some noise to it will produce synchronization. Furthermore, due to the mechanism which causes synchronization, there are two distinct regimes of behavior as discussed below.

Let the oscillator population input consist of two parts: a fixed component, s and a time-varying 0-mean component, q^t, so that $u^t = s + q^t$. Also, let the smallest fixed input needed to put the oscillators into a cycle of sufficiently small period be s^*. The first regime of behavior then corresponds to $s < s^*$. Here, the amplitude of q^t which must be added to cause synchronization decreases as $s \rightarrow s^*$, since the closer s is to s^*, the less extra input is needed to move it in and out of low-period cyclical behavior.

The second regime occurs when $s > s^*$, so that the oscillators are already in a low-period cycle. Now the function of q^t is to move them in and out of the chaotic zone, and the larger s is, the larger the amplitude of q^t required for synchronization. The fixed input s^* represents an "optimal" value where the addition of even very small amount of noise can synchronize the oscillators.

4. Results and Discussion

Figure 3 shows the actual behavior of 50 5/5/1 oscillators with q^t represented by 0-mean Gaussian white noise with variance c^2. The two regimes described

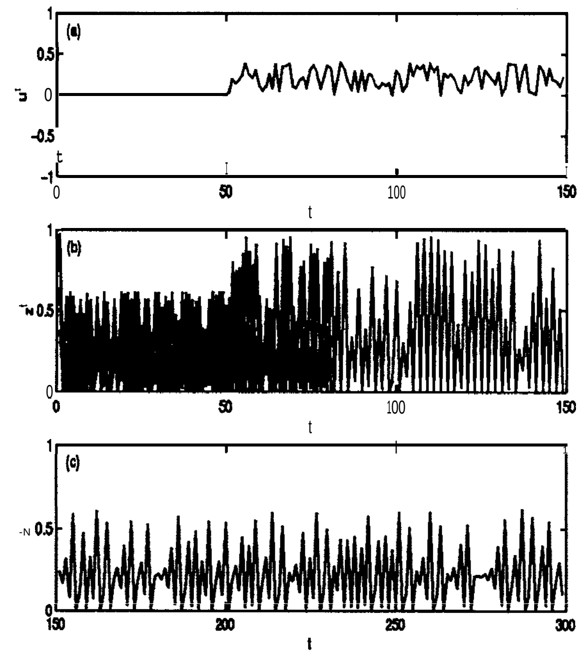

Figure 2. Time series for seven 5/5/1 oscillators subject to a fixed input of 0.2 and uniform random noise between ±0.2. Graph (a) shows the Input, which was non-zero only between steps 50 and 150. Graph (b) shows the time series of the oscillators' state for the time before and during the application of the input. Graph (c) shows the time series of the oscillators' state after the input was switched off at step 150. The result is synchronized chaotic dynamics. Note also that the dynamics synchronized without ever becoming periodic.

above are clearly visible, and s^* corresponds roughly to the fixed input needed to put the oscillators into a P-cycle. Note that negative input values do occur in this example, so the dynamics is not confined to the positive basin only. However, the bifurcation scenario for negative inputs is similar to that for positive ones, and leads to synchronization in the same way. The pronounced nonlinear character of the boundary between synchronization and non-synchronization for $s < s^*$ is caused by the presence of negative inputs.

Along the c-axis for any fixed s, there is a relatively sharp transition from non-synchronization to almost certain synchronization as c crosses a threshold noise amplitude. Thus, too little noise will not cause synchronization but an increase in noise will — an in-

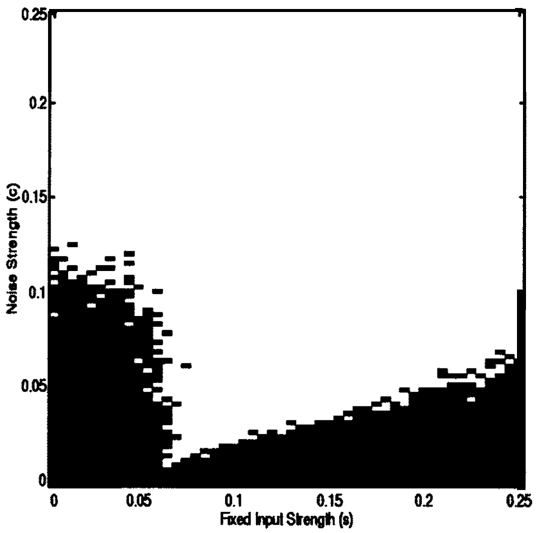

Figure 3. The synchronization regimes of fifty 5/5/1 oscillators subject to a fixed input, s, and O-mean Gaussian noise with variance c^2. Black indicates no synchronization while white indicates synchronization. Each data point represents a single run with the input applied for 500 steps following 100 initial relaxation steps.

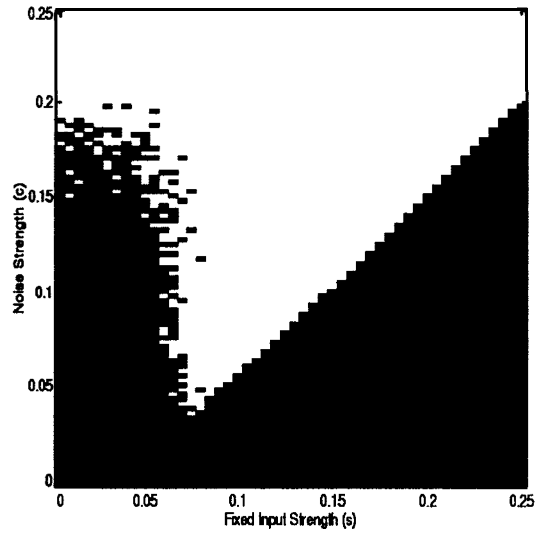

Figure 4. The synchronization regimes of fifty 5/5/1 oscillators subject to a fixed input, s, and 0-mean uniform noise between $\pm c$. Other parameters as in Fig. 3.

stance where more noise is better than less. Figure 6(a) shows the probability of synchronization for different noise amplitudes and fixed s levels. Figure 6(b) shows how the time needed to obtain synchronization depends on noise amplitude. Clearly, increase in noise amplitude enhances the speed as well as the reliability of synchronization.

Figure 4 shows the synchronization region for uniform, $U[-c, +c]$, noise instead of Gaussian. Once again, the domain of synchronization is broad and well characterized with $s^* \approx 0.065$. Also, the linear boundary of the synchronization region for $s > 0.065$ or so confirms the theory presented above, showing that $c \sim s$ in the $s > s^*$ regime, as expected. The boundary is clearer than in the Gaussian case because the Gaussian distribution does not have finite support. The nonlinear boundary for $s < 0.065$ is due to negative input values.

Figure 5 shows the results when the noise is replaced by a period-8 sinusoid. There are still several regions of synchronization, but they are much more complex and difficult to characterize because the periodic input phase-locks with the bifurcation dynamics in some region of s-c space.

Intuitively, the main result presented here may appear paradoxical. One would expect that adding an aperiodic drive to a system of incoherent (albeit identical) chaotic oscillators would accentuate their incoherence rather than suppress it. However, the deterministic nature of the system and the bifurcation process conspire to achieve coherent behavior by repeatedly merging smaller clusters of oscillators. The role of chaos in the process relates to the fundamental (and often controversial) issue of whether chaos provides any benefit to a dynamical system. It has been argued that chaotic dynamics represents a state of infinite possibilities which allows an information processing system (such as the brain) rapid access to a large number of different semantically significant ordered behaviors [24]. A similar argument can be made in the case of synchronization. When a system in a 2^k-periodic regime with 2^k phase-differentiated clusters moves into the chaotic regime, the clusters remain, but the phase relationship between them is destroyed. When the system goes back to the 2^k-periodic regime, the clusters can potentially condense onto fewer than 2^k phases. In this sense, chaos helps to produce synchronization by repeatedly providing increased clustering choices to the system, while the determinism of the system precludes any increase in the number of clusters. Indeed, the

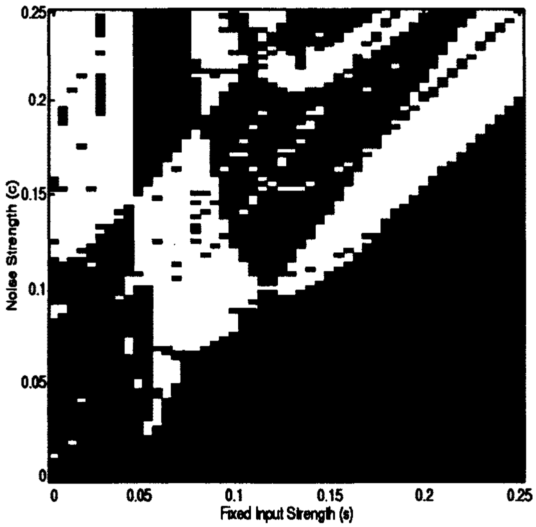

Figure 5. The synchronization regimes of fifty 5/5/1 oscillators subject to a fixed input, s, and a discretized sinusoidal input with period 8 and peakamplitude c. Other parameters as in Fig. 3

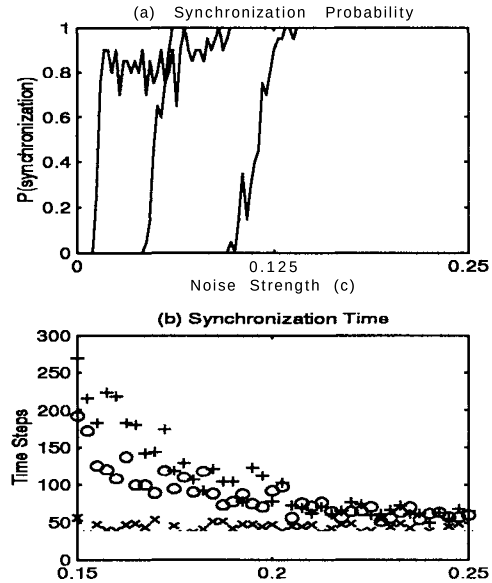

Figure 6. Graph (a) shows the probability of synchronization for fifty 5/5/1 oscillators oscillators subject to a fixed input, s, and 0-mean Gaussian noise with variance c^2. The curves are for s = 0.0 (rightmost curve), s = 0.065 (leftmost curve), and s = 0.2 (middle curve). Graph (b) shows the dependence of the time to synchronization on the noise strength, c. The three cases are s = 0.0 (o), s = 0.065 (+), and s = 0.2 (x). Each data point was averaged over 20 independent runs.

whole process can be seen as a type of annealing, where clustered periodic states (representing suboptimal energy minima) are repeatedly 'softened up' (by making them chaotic) and followed by relaxation to periodic states with fewer clusters. However, the use of chaos — which is deterministic — rather than thermal energy — which produces stochastic motion — ensures that the number of clusters never goes up. Previous work [32, 22] has analyzed the synchronization or clustering of randomly varying nonlinear maps in terms of averaged Lyapunov exponents. While that analysis does not directly address the role of chaos in ensuring global synchronization rather than clustered attractors, the method can be used to analyze global synchronization with some restrictions [15,13].

Some previous reports of noise-induced synchronization in chaotic maps [14] have focused on systems where noise acts essentially as a perturbative agent. Synchronization in such cases occurs primarily by the chance convergence of oscillator trajectories due to the effect of finite precision [23,12]. This is reflected in the exponential dependence of the synchronization time on the precision of state representation [12] and, presumably, on the number of oscillators being synchronized. By contrast, in the system described in this report, noise acts as an agent of bifurcation, and synchronization stems from the real convergence of trajectories in the ordered (periodic) regimes rather than the chance convergence produced by finite precision. This is illustrated by the fact that synchronization occurs within a few hundred (or fewer) steps even for large populations of oscillators (Figure 6). We have also successfully added independent O-mean Gaussian noise with variance as high as 0.00001 to each oscillator without disruption of the synchronization effect [15]. This provides strong evidence against the possibility of merely numerical synchronization [32].

In summary, we have shown that a population of identical discrete-time neural oscillators can be synchronized rapidly by a common aperiodic input, provided that the oscillators possess chaotic and periodic regimes in response to changes in stimulus. This suggests that noise can play an important role in the organization of dynamical behavior in complex systems.

Acknowledgement: The authors would like to thank Xin Wang for providing reprints of his work.

References

[1] M. Adachi and K. Aihara. Associative dynamics in a chaotic neural network. *Neural Networks*, **10:83–98**, 1997.

[2] D. Cairns, R. Baddeley, and L. Smith. Constraints on synchronizing oscillator networks. *Neural Computation*, **5:260–266**, 1993.

[3] S. Campbell and D. Wang. Synchronization and desynchronization in a network of locally couple **wilson-cowan** oscillators. *IEEE* Trans. on *Neural Networks*, **7:541–554**, 1996.

[4] R. **Eckhorn**, R. Bauer, W. Jordan, M. **Brosch**, M. Munk, and R. Reitboeck. Coherent oscillations: A mechanism of feature linking in the visual cortex? multiple electrode and correlation analysis in the cat. *Biol. Cybern.*, **60:121–130**, 1988.

[5] A. **Engel**, P. **König**, A. Kreiter, and W. Singer. Synchronization of oscillatory **neuronal** responses between striate and exstriate visual cortical areas of the cat. *Proc. Nut. Acad. Sci. USA*, **88:6048–6052**, 1991.

[6] C. Gray, P. **König**, A. **Engel**, and W. Singer. Oscillatory responses in cat visual cortex exhibit **inter**-columnar synchronization which reflects global stimulus properties. *Nuture*, **338:334–337**, 1989.

[7] D. Hansel and H. Sompolinsky. Synchronization and computation in a chaotic neural network. *Phys.* Rev. *Lett.*, **68:718–721**, 1992.

[8] S. **Ishii**, K. Fukumizu, and S. Watanabe. A network of chaotic elements for information processing. *Neural Networks*, **9:25–40**, 1996.

[9] K. Kaneko. Spatiotemporal chaos in one- and two-dimensional coupled map lattices. *Physica D*, 37:60-82, 1989.

[10] K. Kaneko. Clustering, coding, switching, hierarchical ordering, and control in a network of chaotic elements. *Physica D*, **41:137–172**, 1990.

[11] P. **König** and T. Schillen. Stimulus-dependent assembly formation of oscillatory responses: I. synchronization. *Neural Comp.*, **3:155–166**, 1991.

[12] L. **Longa**, E. **Curado**, and F. Oliviera. **Roundoff**-induced coalescence of chaotic trajectories. *Phys.* Rev. *E*, **54:R2201–R2204**, 1996.

[13] L. **Longa**, S. **Dias**, and E. **Curado**. Lyapunov exponents and coalescence of chaotic trajectories. *Ph ys. Rev. E*, **56:259–263**, 1997.

[14] A. **Maritan** and J. Banavar. Chaos, noise, and synchronization. *Phys. Rev. Lett.*, **72:1451–1454**, 1994.

[15] A. Minai and T. **Anand.** Chaos-induced synchronization in discrete-time oscillators driven by a random input. To appear in *Phys. Rev. E, 1997.*

[16] A. Minai and T. **Anand.** Stimulus-induced bifurcations in discrete-time neural oscillators. In review, 1997.

[17] A. Minai and T. **Anand.** Symbolic dynamics of synchronization in discrete-time chaotic oscillators. In preparation, 1997.

[18] V. Nekorkin, V. Makarov, V. Kazantsev, and M. **Veralde**. Spatial disorder and pattern formation in lattices of coupled bistable elements. *Physicu D*, **100:330–342**, 1997.

[19] L. Pecora and T. Carroll. Synchronization in chaotic systems. *Phys. Rev. Lett.*, **64:821–824**, 1990.

[20] L. Pecora and T. Carroll. Driving systems with chaotic signals. *Phys. Reu. A*, **44:2374–2383**, 1991.

[21] L. Pecora and T. Carroll. Pseudoperiodic driving: Eliminating multiple domains of attraction using chaos. *Phys. Rev. Lett.*, **67:945–948**, 1991.

[22] A. Pikovsky. Statistics of trajectory separation in noisy dynamical systems. *Phys. Lett. A*, **165:33–36**, 1992.

[23] A. Pikovsky. Comment on "chaos, noise, and synchronization". *Phys. Rev. Lett.*, **73:2931**, 1994.

[24] C. Skarda and W. Freeman. How brains make chaos in order to make sense of the world. *Behuv. Bruin. Sci.*, **10:161–195**, 1987.

[25] C. von der Malsburg and J. Buhmann. Sensory segmentation with coupled neural oscillators. *Biol. Cybern.*, **67:233–246**, 1992.

[26] C. von der Malsburg and W. Schneider. A neural cocktail-party processor. *Biol. Cybern.*, **54:29–40**, 1986.

[27] D. Wang. Emergent synchrony in locally coupled neural oscillators. *IEEE Trans. Neural Networks*, 6:941-948, 1995.

[28] D. Wang and D. **Terman**. Locally excitatory globally inhibitory oscillator networks. *IEEE Trans. Neural Networks*, **6:283–286**, 1995.

[29] X. Wang. Period-doublings to chaos in a simple neural network: An analytic proof. *Complex Systems*, 5:425-441, 1991.

[30] X. Wang. *Discrete-time neural networks us dynamical systems.* **PhD** thesis, University of Southern California, 1992.

[31] H. Wilson and J. Cowan. Excitatory and inhibitory interactions in localized populations. *Biophys. J.*, 12:1–24, 1972.

[32] L. Yu, E. Ott, and **Q.** Chen. Fractal distribution of floaters on a fluid surface and the transition to chaos for random maps. *Physicu D*, **53:102–124**, 1991.