A Self-Organizing Heuristic for Building Optimal Heterogeneous Ad-Hoc Sensor Networks

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Abstract— **Much of the research in the area of sensor networks is focused on homogeneous networks. Of late, there has been a steadily increasing amount of work on heterogeneous networks, mainly due to their flexibility and better fit into potential applications. In this paper, we present a heuristic, developed using a reverse engineering approach, that can be used to build efficient heterogeneous ad-hoc sensor networks based on a generic network model. A genetic algorithm is used to generate a set of heterogeneous sensor networks optimized for short paths and congestion. A thorough analysis of the optimal network set is done to extract rules and a heuristic is developed to embody these rules. The heuristic is then used to produce high-performance networks without genetic algorithms. We present simulation results and analysis of the heuristic networks and compare their performance with optimal heterogeneous networks as well as homogeneous networks.**

I. Introduction

As mobile units, including phones, computers, PDA's and bluetooth-enabled devices have proliferated, interest in ad-hoc networks, i.e., networks without a fixed infrastructure, has increased dramatically. A challenging case of ad-hoc wireless networks are large-scale random sensor networks [1] comprising hundreds, thousands or even more wireless sensor nodes with limited communication range deployed randomly over an extended area. These networks have been the focus of extremely active research over the past decade and hold great promise in several military, environmental and commercial application areas [2]. Almost all research on sensor networks is based on a homogeneous network model — networks in which all the sensor nodes are identical and transmit signals with the same power, creating symmetric connectivity. This is a generally useful simplifying assumption but it puts a strong constraint on networks and, given the random distribution of nodes, can often lead to systems that are sub-optimal in performance. Heterogeneous networks where each node can customize its transmission power are more general and potentially more efficient, though they do create considerable complexity in terms of configuration. The utility of heterogeneity in different aspects of sensor networks has been corroborated by several recent studies [3], [4].

A. Motivation and Related Work

In typical models of large-scale sensor networks, all nodes are assumed to transmit with equal power, which means that they all have the same *transmission radius*. Such homogeneous networks are a natural but rather limited model for large-scale random sensor networks. They lead to symmetric connectivity which can be analyzed mathematically, and require no complex protocols for node-level decision-making about transmission power. However, in sufficiently dense networks (or in networks with significant variation of node density), identical transmission radii can be very inefficient, leading to unnecessary congestion in some locations and/or disconnection in others. Heterogeneous networks, where each node has its own transmission radius can be much more efficient, but require an efficient protocol for setting these radii. As with all applications in large-scale networks, the primary requirement for such a protocol is scalability which, in turn, means that all decision-making must be based on local information. In this paper, we address the question: *How can each node in a large-scale random sensor network select a transmission radius so that some global properties are optimized?*

The primary purpose of a sensor network is usually event monitoring or detection, with energy efficiency an important secondary consideration. Control of transmission radius has typically been used as a means of minimizing energy consumption while maintaining sufficient connectivity. However, other properties of the network are often equally important. For example, sensor networks perform significant data aggregation, and the path length between nodes is critical to achieving reliable real-time functionality. Similarly, congestion at specific nodes can have significant effect on network performance. Thus, in the present paper, we focus on optimizing the *mean shortest path length* between nodes without undue increase in *congestion*. The approach we use is generic and can also be used to optimize other application-specific network characteristics.

The earliest work on topology control of wireless sensor networks was done by Ramanathan et al. [5], who developed distributed algorithms LINT and LILT based on maintaining a specified node degree for the network nodes by means of power adjustment. Srivastava et al. have proposed extensions for these algorithms [6]. Li and Hou present the DRNG and DLMST algorithms based on the relative neighborhood graph and the minimum spanning tree respectively in [7]. Li et al. present a different strategy in [8]. A detailed comparison and critique of most of the topology control algorithms for heterogeneous ad-hoc networks is found in [9]. All these algorithms are oriented mainly toward reducing the overall power requirements of the network to increase system lifetime.

MobileGrid [10] is an algorithm based on a parameter

called the contention index which places emphasis on network capacity and optimal performance. The same authors present a scalable distributed topology control algorithm for heterogeneous sensor networks in another paper [11]. This algorithm starts with a strongly connected maximum topology and generates a minimum power topology with the same reachability and minimum nodal transmission range. The common factor in all these algorithms is their graph-theoretic nature. Most of the algorithms generate connected network topologies with lower average node transmission range and lower power requirement. However, not all the algorithms are scalable; most of them work best in sparse networks and are not suited for high density network structures.

The rest of the paper is organized as follows: Section II describes the network model used and provides an introduction to the optimization strategy; the implementation and working of the optimization strategy are elucidated in section III; the basis for the proposed heuristic and its development are covered in section IV; simulation results are presented and discussed in section V with section VI providing the conclusion to the paper along with possible future extensions.

II. Problem Description

The main issue in ad-hoc heterogeneous sensor networks is the selection of radius for the nodes since the ad-hoc nature of the networks precludes using node location as a parameter for design. The problem is to choose or assign individual node radii in such a way that some desired network properties are optimized. Since sensor networks typically have a large number of nodes, centralized global optimization is precluded by scalability concerns. Each node must select its own radius based on local information, making this an example of the classic self-organization problem of obtaining global optimality from local decisions.

A. Network Model

The network occupies a square region of unit area with n nodes randomly distributed uniformly in this area. The nodes are assumed to be *uniformly heterogeneous*, i.e., all the nodes can vary their transmission radius over the same range, $[r_{min},\ r_{max}]$. In order to make this assumption more practical, we modify it such that each node can choose a radius from the set of discrete values, $\{r_{min} = r_1, r_2, r_3, \ldots, r_n = r_{max}\}$. This corresponds to having sensor nodes that can operate in pre-defined modes with different communication radii and choose their operating modes so as to optimize the network performance.

As mentioned earlier, we focus on obtaining connected heterogeneous sensor networks with low mean shortest path length (MSPL) and low congestion. These properties were chosen because they are important for most network applications. The mean shortest path length, μ_k, of a network, N_k, is defined as the average of the lengths of the shortest paths (in hops) between all pairs of nodes in the network. Since it is not always possible to guarantee full connectivity , a penalty term is used in calculating MSPL when disconnections are encountered. The indegree of a node in the network is representative of its congestion and the mean indegree for all nodes is taken as the congestion value, η_k, for the network.

Connectivity can be virtually ensured for homogeneous networks by choosing the transmission radius of nodes at or above the *percolation radius*, r_{perc} — the radius at which a random network suddenly shifts from a disconnected and fragmented state to a connected state with probability one [12]–[14]. Since sensor network applications typically use high node density, the percolation radius is small and feasible. In our heterogeneous case, we choose r_{min} to be slightly lower than percolation radius and r_{perc} as one of the possible choices for node radius in the set $\{r_{min} = r_1, r_2, r_3, \ldots, r_n = r_{max}\}$.

B. Optimization Strategy

The goal of the work presented here is to find local decision rules that optimize the global characteristics of network congestion and MSPL. All previous topology control schemes have used heuristic decision rules based on logical analysis. However, in all but the simplest cases, it is very difficult to come up with such rules from scratch or to know the degree to which they actually work to produce optimal results. We, therefore, propose an alternative strategy that works as follows:

1) Define a *configuration space*, C, and cost (or fitness) function, J, for networks of the class considered.
2) Generate a large number of random node distributions, D_k, with specified densities, λ_k, and for each D_k, obtain M optimized node radius assignment configurations, C^*_{ki}, $i = 1, 2, ..., M$, using a genetic algorithm (or some other stochastic optimization approach capable of finding global optima). Obtaining optimal configurations from several runs on the same distribution allows rejection of non-global optima.
3) Verify that the optimality of the C^*_{ki} is a property of the particular configurations and not of the radius distribution parameters by comparing with *surrogate configurations* drawn from the same radius distributions. If this verification fails, it indicates that simply using a particular radius distribution is enough to produce global optimality. However this is unlikely in most cases.
4) Analyze the optimal configurations from Step 2 to determine what properties of these particular configurations — destroyed in surrogate ones — are responsible for their optimality. This is the most difficult step, but working with a comprehensive list of network features for analysis makes it feasible.
5) Once the differences between optimal and surrogate non-optimal configurations have been clarified, develop local — possibly approximate — rules that allow nodes in any network to make choices leading to global configurations with properties characteristic of optimal ones.
6) Implement and verify the performance of the heuristic by comparing networks obtained through it with networks obtained via the genetic algorithm.

It should be noted that the genetic algorithm is used *only* to find optimal networks so that the node-level properties responsible for this optimality can be extracted from them. Once these properties are discovered, they are embodied in *local rules* for setting node radii *without using a genetic algorithm*. The main difference from previous work is that the local rules are discovered through a systematic reverse engineering process rather than by clever ad-hoc intuition, and are supported by at least some evidence that their application produces near-optimal networks.

III. Genetic Algorithm - Finding the Optimal Network

For the genetic algorithm, a network is coded as an ordered vector, $V = [r^1\ r^2\ ...\ r^n]$ of the transmission radii, r^j, for all nodes, $j = 1, 2, ..., n$, in the network, where n is the total number of nodes in the network. This is termed the network's *chromosome*. For a particular layout, D, of the nodes, the initial population of *candidate networks*, $\{N_i\}$, $i = 1, 2, ..., m$, is formed by randomly assigning each node in each network a radius uniformly from the range $[r_{min},\ r_{max}]$. Note that for the genetic algorithm we relax the constraint of discrete radius values, allowing the genetic algorithm to choose any radius in the range $[r_{min},\ r_{max}]$. This helps in finding solutions closer to the true optimum. The *cost*, J_i, of network N_i is defined as the product of the network's MSPL and congestion:

$$J_i = \mu_i \eta_i$$

Note that this is the converse of a fitness measure, and the genetic algorithm seeks to minimize it. In this paper we use "fitter networks" to refer to networks with lower cost. The goal of the genetic algorithm is to produce the best assignment of radii for the specified node layout, and not to find an optimal layout.

In each generation beginning with the first, all candidate networks are evaluated. Then $m_r \leq m/2$ pairs of the fittest ones are chosen for reproduction, and offspring are formed by crossing over the chromosomes of each parent pair. The next generation is comprised of the m_r newly formed offspring and $m_s = m - m_r$ randomly chosen candidate networks from the previous population that also undergo mutation with a small probability. This procedure is repeated for nearly 500 generations or until the cost of the fittest network has flattened out over time.

The genetic algorithm is repeated for a particular node layout with different initial populations to produce a set of near-optimal solutions for that layout. The algorithm is used to create similar near-optimal solution sets for several node layouts, which are then analyzed for patterns or motifs to develop heuristic rules. Note that, while the solutions found by the GA are near-optimal rather than truly optimal, we will refer to them as "optimal" in this paper.

IV. Analysis of the Optimal Network Set

The analysis of the optimal networks found by the GA led to two interesting initial observations:

1) Most of the nodes have small transmission radii, and many have radii between the minimum and the percolation radius.
2) Only a few key nodes have large transmission radii.

The observations make intuitive sense. A larger node radius results in a greater number of neighbors and increased reach which reduces the MSPL, but also leads to increase in network congestion. If many nodes have large radii, the increase in congestion is likely to more than offset the decrease in the MSPL. Since the genetic algorithm optimizes for both congestion and short path length, the optimal solution is a network in which the gain in increasing node radius, and the reduction of short path length as a result, outweighs the cost of increased congestion. One way to do this is to allow most nodes to *shrink* their radii as much as possible and only letting a few well-placed nodes to have large radii, which is what is observed. Indeed, the second observation indicates the presence of hub-like nodes in the network, analogous to those seen in scale-free networks [15], [16]. The optimality of an extremal bimodal radius distribution also recalls the optimality of bimodal connectivity distributions in random networks with high robustness [17], [18].

Having identified the type of radius distribution likely to produce good networks, the main issue is the determination of a criterion for deciding which nodes get large or small radii. We analyzed the evolved optimal network to find an appropriate rule as described below.

A. Development of the Heuristic

Intuitively, the optimal choice of node radius for the heterogeneous networks is likelier to be based on a simple network/node property than a complicated quantity. We performed detailed data analysis using simple node properties and checked for correlation between the properties and radius selection. The results showed that for a given node j, there were two relevant parameters:

1) *Density of neighboring nodes:* The number, N^j, of neighbors within close proximity of j.
2) *Separation between target nodes:* The maximum separation, d^j, among nodes within j's transmission radius.

Analysis of the optimal solution sets obtained from the GA shows that most large radius nodes in the optimal networks either have a large separation among their target nodes or high density of nearby nodes. However, sometimes there are nodes that possess *both* these desirable characteristics, and these are the best candidates for large transmission radii. Thus, we use both parameters to determine a good radius for each node.

B. The Radius Heuristic

After deployment, each node independently evaluates the distribution of nodes in its neighborhood and chooses a radius according to the heuristic described below. The neighborhood information can be made available to the nodes either by incorporating GPS capabilities in them or by using the signal strength of "hello" messages received from nearby nodes to calculate distances. The nodes require only static information

in order to use the heuristic and the complexity involved is thus minimal. The heuristic is fully scalable and can be used for static self-organization of the network. The case when node mobility is possible and the use of dynamic information in the heuristic have not been considered in this paper.

Let $R = \{r_{min} = r_1, r_2, \ldots, r_n = r_{max}\}$ denote the set of n different radius values that can be chosen by a node. For a given node, j, a fitness measure, f_i^j, is calculated for each value $r_i^j \in R, i = 1, 2, ..., n$. The node then assumes radius value $r^j = r_{i^*}^j$ such that $f_{i^*}^j \geq f_i^j, i = 1, 2, ..., n$.

The fitness measure f_i^j, for a given radius value r_i^j, is the weighted sum of two individual components: 1) the *base fitness measure*, b_i^j, and 2) the *gain fitness measure*, g_i^j. For the given radius r_i^j, let N_i^j and d_i^j denote the number of neighbors of the node and the maximum separation among the node's neighbors, respectively. Then the base fitness b_i^j and the gain fitness g_i^j are computed as follows.

Base Fitness: The base fitness b_i^j of node j for radius r_i^j has two components, $b_i^{N^j}$ and $b_i^{d^j}$. The first component, $b_i^{N^j}$ is the number of neighbors of the node when its radius is r_i^j, normalized by the expected number of neighbors. The second component $b_i^{d^j}$ denotes the maximum neighbor separation at that radius, i.e, the maximum distance between any two neighbors of the node when its radius is r_i^j, normalized by the diameter. Let λ denote the node density. Then,

$$b_i^j = b_i^{N^j} + b_i^{d^j}$$

$$b_i^{N^j} = \frac{N_i^j}{(2\pi r_i^j \lambda) - 1}, \quad b_i^{d^j} = \frac{d_i^j}{2r_i^j}$$

Gain Fitness: Similar to the base fitness measure, the gain fitness g_i^j of node j for a given radius r_i^j has two components. The first component,$g_i^{N^j}$, is the *increase* in number of neighbors for the radius r_i^j compared to the previous radius r_{i-1}^j scaled by the expected number of neighbors at minimum radius. The second component, $g_i^{d^j}$, is the *increase* in the maximum neighbor separation at this radius r_i^j compared to the previous radius r_{i-1}^j normalized by the maximum diameter of the node. Thus,

$$g_i^j = g_i^{N^j} + g_i^{d^j}$$

$$g_i^{N^j} = \frac{N_i^j - N_{i-1}^j}{(2\pi r_1^j \lambda) - 1}, \quad g_i^{d^j} = \frac{d_i^j - d_{i-1}^j}{2r_n^j}$$

The gain fitness at minimum radius, $g_1^j = 0$. The overall fitness f_i^j for radius r_i^j is the weighted sum of its base and gain fitnesses,

$$f_i^j = \alpha_i b_i^j + \beta_i g_i^j$$

The base fitness is weighted using α_i so that, under comparable conditions, the base fitness of smaller radii will be more than that of larger ones. This acts as a penalty for choosing a higher radius if there is no disproportionate benefit in doing so. For every increment of node radius from r_{i-1}^j to r_i^j, $1 < i \leq n$, there is an expected increase in the values of N_i^j and d_i^j that contributes to g_i^j. The weighting β_i is used so that a normal gain fitness value does not add enough to the overall fitness to overcome the penalty imposed for the higher radius. Only if the radius increase provides *greater than expected* benefit does the gain fitness increase overall fitness. The maximum value of f_i^j, $1 \leq i \leq n$, is the best fitness value $f_{i^*}^j$ and the corresponding radius is chosen by the node as its transmission radius.

In order to better understand the working of the heuristic, consider the following scenario: Let a and b be nodes having disjoint neighbor set at minimum radius, that is, a and b have no common neighbors at $r^a = r^b = r_{min}$. Let δ_{ab} be the Euclidean distance between nodes a and b. Now, for both the nodes a and b, for a value of radius $r_k \geq \delta_{ab}, r_k \in R$, there is an increase in both the number of neighbors N_k and the maximum separation among neighbors d_k. This contributes to the gain fitness g_k and an increase in the overall fitness f_k for both a and b. Thus, the selection of the higher radius r_k is more likely for the nodes a and b. The final selection of radius for nodes a and b is determined by the set of weights α_i and $\beta_i, i = 1, 2, ..., n$ which incorporate a penalty for higher radius selection and also account for the expected increase in fitness values due to the increased radius value. It is important to select the weights properly since a low penalty will lead to more number of nodes selecting higher radius thus increasing congestion while a high value will result in a partially disconnected and/or clustered network.

In a nutshell, the base fitness contributes mainly to the selection of a smaller radius value that is best for the node. The gain fitness helps select a higher radius value for nodes in cases where it would be more beneficial. Thus, the principle of the heuristic can be summarized as follows: Choose a good smaller radius unless choosing a larger one produces extraordinary increase in fitness.

V. Simulation and Results

Extensive simulation was done to evaluate the performance of the heuristic on four network quality measures: fitness (as defined above), MSPL, mean node radius, and robustness. While fitness is the primary criterion for evaluating the network performance, mean node radius is important since it corresponds to the overall energy consumption in the network and also provides a simpler comparison with homogeneous networks. A fitter network with higher mean node radius may, in some sense, be considered less desirable than one with lower fitness but lower mean node radius. Also, sensor nodes are subject to failure during operation, and the ability to keep operating in the presence of random node failure is an important property for any practical network. We quantify this as the robustness of the network. The main focus is on comparing the performance of the networks produced by the heuristic with that of "optimal" networks obtained via the GA and with homogeneous networks.

The simulations were done for networks with 200 nodes. We found that the percolation radius for such a network is nearly 0.11 and we have used $r_{min} = 0.09$ and $r_{max} = 0.15$ for the heuristic and the GA networks. The value $r = 0.11$ was found to be the best overall node radius for the homogeneous case, i.e., a barely connected network was

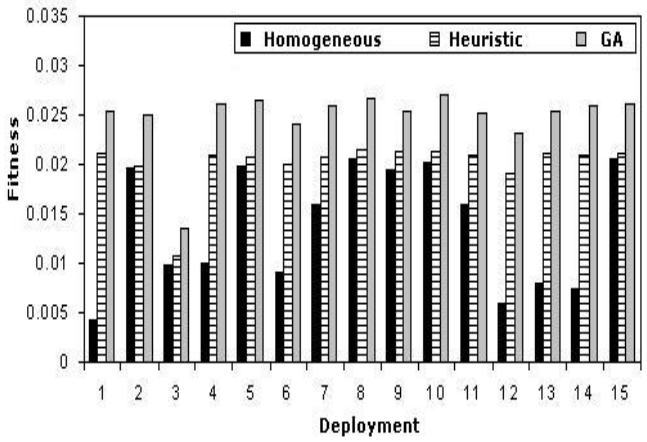

Fig. 1. Comparison of Network Fitness

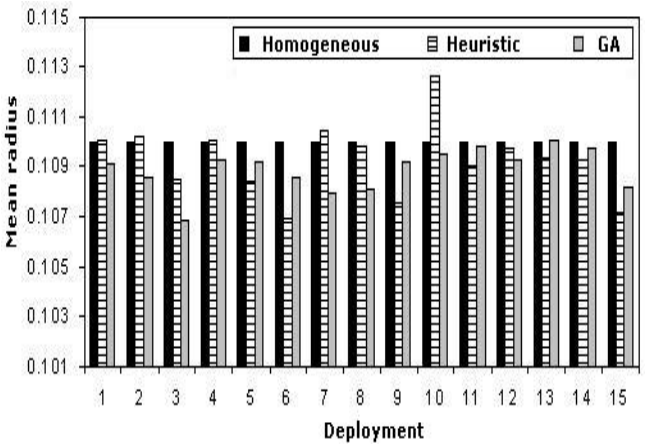

Fig. 3. Comparison of Mean Node Radius

best. Figure 1 shows the fitness of 15 different networks for the homogeneous ($r = 0.11$), GA, and the heuristic cases, while Figure 2 shows the plot of the network MSPL for the same networks. Each group of bars corresponds to a different deployment, that is, different uniform random placements of the 200 nodes. It is seen that the heuristic produces networks with better fitness and better network MSPL than homogeneous networks. Homogeneous networks at $r = 0.11$ are not always connected and hence their MSPL values are high due to the penalty incurred for disconnected nodes as seen in Figure 2. The important point to be noted here is that while the MSPL of heuristic networks compares well with that of GA networks, the fitness of heuristic networks, though far better than that of homogeneous networks, lags behind the GA networks considerably. This is caused by the heuristic networks having higher congestion than the GA networks, partly reflecting the fact that the GA performs global optimization while the heuristic only does so locally.

The plot of mean node radius of the homogeneous, GA and heuristic networks for different deployments is shown in Figure 3. Here we see that the heuristic networks have a mean node radius comparable to homogeneous and even GA networks. This means that the heuristic networks do well in terms of minimizing overall energy consumption.

Higher congestion in the heuristic networks translates into an increased number of edges or connections among the nodes. Networks with comparable mean node radius are expected to have comparable congestion but the heuristic networks have unexpectedly higher congestion. Intuitively, a node or a network managing to make more connections at the same radius should be more robust. This is verified through the calculation of robustness for the different networks. We measure robustness to node failure by randomly deleting 5% of the nodes and then calculating the network fitness. Figure 4 shows the plot of network fitness (averaged over multiple trials) versus percentage of nodes deleted for a particular node deployment. It is seen that the heuristic networks are very robust and outperform the GA networks considerably, while both are much better than homogeneous networks. Heuristic networks not only do better in terms of the percentage change in fitness but emerge as the fitter ones after node deletion for nearly all the deployments.

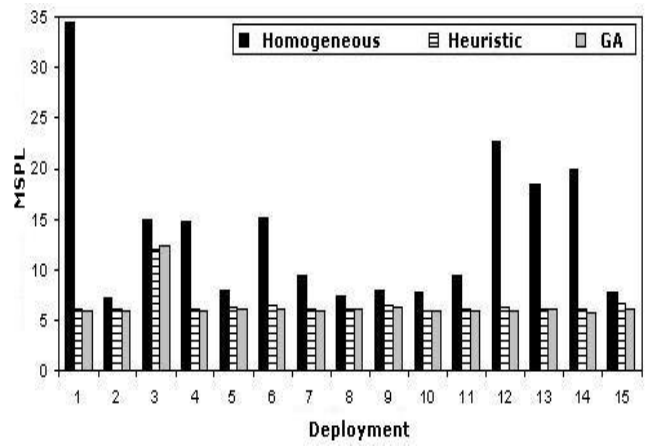

Fig. 2. MSPL Comparison

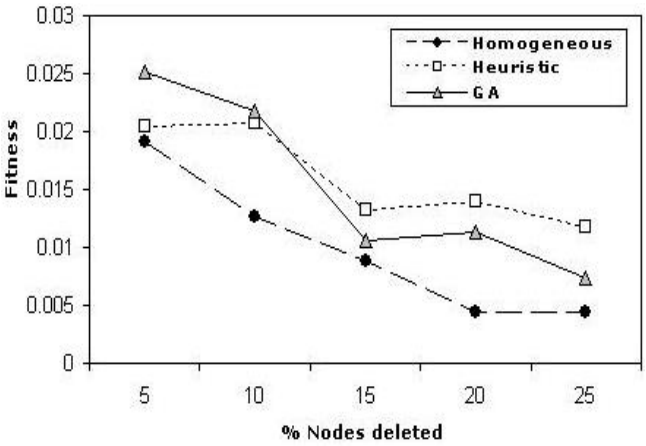

Fig. 4. Comparison of Robustness

VI. Conclusion

We have shown through simulation results that a heuristic rule proposed for setting transmission radii of nodes in a random sensor network produces heterogeneous networks that are fit, robust and have energy requirements comparable to homogeneous networks. While the GA networks score over the heuristic networks in terms of fitness alone, heuristic networks, being more robust, prove to be the better overall solution in terms of the three important network quality measures — fitness, energy consumption and robustness. Our heuristic is simple and scalable and can be adapted to other applications.

One extension to this research work would be the introduction of dynamics in the network while choosing the operating radius. Each node, after an initial radius selection, can use the information about the radius choices of nearby nodes to further improve the network properties in a single or a series of steps. Another question to be answered would be the applicability of existing routing schemes to the heuristic networks and the need to adapt existing schemes or develop new routing schemes for the heuristic networks. We plan to extend our work further along the above mentioned lines and also hope that the ideas and work presented in this paper encourage others to pursue a similar line of research.

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