A Spiking Neural Model for the Spatial Coding of Cognitive Response Sequences

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***Abstract*— The generation of sequential responses is a fundamental aspect of cognitive function, encompassing processes such as motor control, linguistic expression, memory recall and thought itself. There is considerable evidence that complex cognitive responses (such as voluntary actions) are constructed as chunked sequences of more elementary response primitives or synergies, which can themselves be seen often as sequences of simpler primitives. Almost all neural models of sequence representation are based on the principle of recurrence, where each successive item is generated by preceding items. However, it is also interesting to consider the possibility of purely spatial neural representations that result in sequential readout of pre-existing response elements. Such representations offer several potential benefits, including parsimony, efficiency, flexibility and generalization. In particular, they can allow response sequences to be stored in memory as chunks encoded by fixed point attractors. In this paper, we present a simple spiking neuron model for the flexible encoding and replay of response sequences through the impulsive triggering of coding patterns represented as fixed point attractors. While not intended as a detailed description of a specific brain region, the model seeks to capture fundamental control mechanisms that may apply in many parts of the nervous system.**

***Index Terms*—Cognitive control, sequence learning, spiking neural networks, attractor networks.**

I. Introduction

A Developmental, dynamical and embodied view of cognition [1]-[15] inevitably requires mechanisms by which cognitive systems can "bootstrap" from simple behaviors to more complex ones over the course of development. These behaviors include not only external ones, i.e., actions, but also internal ones, such as recognition of objects, memory recall, decision-making, ideation, etc. All of these ultimately correspond to temporally evolving, synergistic patterns of activity across interacting complex networks in the brain-body system comprising neural and musculoskeletal elements [1]-[15]. A useful way to think of cognitive systems is thus as generators of response sequences at multiple spatial and temporal scales, with more complex responses – both internal and external – constructed from the combination of simpler ones through development and learning. These combinations can be superpositional or sequential. For example, complex voluntary actions over extended periods can be seen as sequences of simpler actions, each of which may, in turn, be generated by a superposition of scaled and time-shifted stereotypical patterns of coordinated muscle activities called *synergies* [16]-[24]. Though these synergies combine through superposition, sequential combination remains extremely important since the basic synergy patterns are of short durations (at most a few hundred milliseconds), whereas actions (or other cognitive responses) can unfold over much longer periods. In some situations, the sequentiality becomes explicit, as in playing a musical instrument, learning a list of words, or composing sentences. The characteristics of such sequential learning and recall have been studied extensively through experiments in both monkeys and humans [25]-[28]. Several neurocomputational models of sequential processing have also been developed [29]-[31], but the focus in most f these models has been on learning and recalling specific sequences through explicitly temporal associations. In this paper, we describe a simple but flexible model for encapsulating sequences efficiently as spatial activity patterns, which is potentially extremely useful for building complex sequential responses from simpler ones through developmental learning.

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II. Background and Motivation

Almost all neural models of sequence representation have used a "chaining" approach, where successive items in the sequence are generated by association with previous items. However, it is interesting to consider whether response sequences can be represented by purely spatial patterns, and read out as sequences when needed. This is the essence of the competitive queuing (CQ) model proposed by Grossberg [32] to explain the phenomena of primacy and recency in the free recall of lists from working memory, and more recently by Grossberg and Pearson [33] as the basis of the LISTPARSE model. Experiments by Averbeck et al [34] have provided intriguing evidence that extended action sequences such as those involved in drawing or writing may be encoded initially as specific spatial patterns of activity, and then read out through an execution mechanism. However, the significance of this mechanism extends beyond its possible use in motor control. Since all cognitive function involves sequences of preconfigured response primitives – words, phrases, concepts, thoughts, memories, ideas, movements, muscle synergies, etc., the flexible and efficient representation of such sequences must be a fundamental function of the nervous system – in particular, the cerebral cortex. The explicit encoding of sequences in memory through associative chaining is very expensive and rather cumbersome, since each element of the sequence and every association between elements must be represented explicitly. In contrast, spatial patterns of activity can easily be learned as fixed point attractors in recurrent networks [35], and are extremely convenient as distributed neural representations. When the responses needed are *stereotypical* sequences of *preconfigured* response elements such as muscle synergies [16]-[24] or chunks [36]-[38], spatial representations are especially useful. Such responses occur, for example, in drawing and writing, playing familiar pieces of music, speaking familiar phrases, and in a vast array of other responses. They may also be important for the representation of temporal structure in semantic constructs, which is a poorly understood issue in cognitive neuroscience.

In this paper, we present a CQ-type model that allows the coding of responses sequences with specific temporal profiles, and show how this model allows representations that are both general and flexible. This approach offers several advantages:

- Representing sequences as spatial patterns makes them much simpler to store, recall and use.

- Spatial representations are more efficient than explicitly sequential ones because several items are represented in a single pattern.

- Multiple sequences involving the same items can be obtained from the same basic pattern simply by by varying the modulation pattern. This *content invariance* can be the basis of both generalization and innovation. For example, once a certain pattern of movements has been learned, it can be reproduced easily with variable timing and novel permutations of it can be explored easily to discover other useful sequences.

- The model dissociates the timing of the sequence from its contents, allowing each to be learned separately. Thus, a relatively small number of canonical item patterns and modulation patterns can combinatorially and *systematically* produce a rich array of responses.

- The duration and rhythm of sequences can be varied very easily and flexibly by simply varying the modulation pattern *spatially*. In contrast, learning the same sequence with different timings in a chaining system is much more difficult and is more limited by the capacity of the system.

- The chunked patterns allow sequential structures to be loaded and used as single entities in working memory, and possibly in other systems.

- This scheme can be applied hierarchically to produce sequences at various spatiotemporal levels, greatly amplifying the richness of the system.

III. Model Description

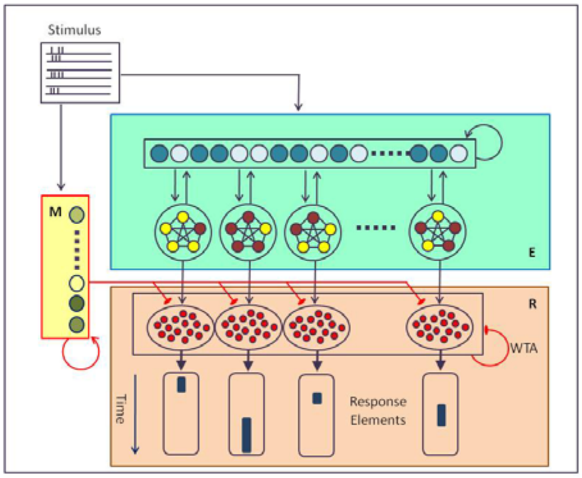

Figure 1: Architecture of the model showing the E, M and R systems. See text for details.

A. *Overall Architecture*

The architecture of our model is shown in Figure 1. The system comprises the following three subsystems:

The Encoding System (E): This system encodes response sequences as patterns of activity across a two-layer recurrent network. Each sequence is represented as an attractor pattern embedded in the network through Hebbian learning as described in the next section. Layer 1 of E receives a brief impulse stimulus, which triggers a specific attractor pattern across both layers. Neurons in Layer 2 project to the Response System, determining the composition and order of the response sequence generated.

The Modulation System (M): This system provides signals that determine the inactivation rate of neurons in the Response System, R. These signals are represented by a spatial activity pattern in M, which can also be encoded as an attractor.

The Response System (R): Conceptually, the Response System comprises two levels: 1) An *activation layer* that receives input from E, and has neurons specialized for each response element; and 2) A layer of actual response elements that perform the primitive functions (e.g., simple reaching movements). In this paper, we show simulations only for the activation layer to focus on essential functionality. Typically, response elements would be fairly complex networks in their own right.

B. Encoding System Model

The E system is implemented as two layers of spiking neurons. The neurons use the Izhikevich model [39], which is computationally efficient and biologically plausible. The neuron equations are:

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I \quad (1)$$

$$\frac{du}{dt} = a(bv - u) \quad (2)$$

with resetting after each spike:

$$\text{if } v \geq +30mV, \quad \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \quad (3)$$

Here, v represents the membrane potential of the neuron, u represents a membrane recovery variable, which provides negative feedback to v, and a, b, c and d are parameter values that determine the neuron's intrinsic behavior. In particular, we use Izhikevich's class 2 excitability spiking model with $a=0.2$, $b=0.26$, $c=-65$, and $d=0$. The range of output spiking period is from 25ms to 2ms if the input level I changes from *0.4* to *24*. Figure 2 shows the relationship between the stimulus input and the period of spiking neuron.

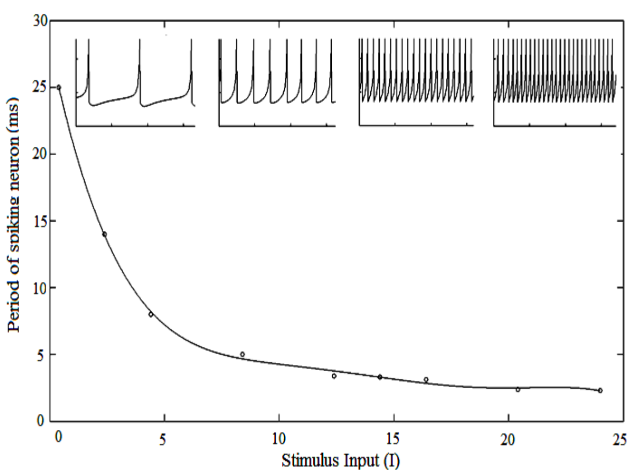

Figure 2: The relationship between the stimulus input and the period of spiking neuron. The inset graphs show neuron spiking at different input levels.

As shown in Figure 2, the system has two layers conceptually. Layer 1, termed the *input layer*, receives stimulus from upstream systems (e.g., prefrontal cortex), and has bidirectional connectivity with the output layer. The neurons in Layer 2, called the *output layer*, are divided into several modules with dense internal connectivity. There are no inter-module connections in the current implementation.

All neurons in each group are identical, but have different weights to the downstream system. As a result, by turning on and off different members of the group for a given response element, the generator gives different input values by combining all the outputs in each group. Attractors are used to set the weights within the network. Value of weight can be measured by Hebbian learning rule, as depict by equation (4),

$$w_{ij} = \begin{cases} \frac{1}{n}\sum_{k=1}^{P} y_i^k y_j^k, & i \neq j; \\ 0, & otherwise \end{cases} \quad (4)$$

where w_{ij} is the weight of the connection from neuron j to neuron i, n is the dimension of the input vector, P is the number of training patterns, and y^k_i is the k^{th} input for neuron i.

After properly setting the weights, given a brief stimulus with a specific spatial pattern, the recurrent network retrieves the most similar stored pattern and latches it until next stimulus. The output layer has significantly fewer neurons than the input layer, ensuring that correct retrieval in the latter is sufficient to trigger the correct output component of the attractor as well. In the simulations shown, the input and output layers have 400 and 100 spiking neurons, respectively. The output layer is divided into 10 modules, each with 10 neurons.

The excitatory input I^e and stimulus input I^s of neuron i at time t is given by:

$$I_i^e(t) = \sum_{j=1}^{N}\sum_{n} w_{i,j}\alpha(t - t_j^n - \Delta) \quad (5)$$

$$I_i^s(t) = \sum_{j=1}^{N}\sum_{n} w_{i,j}\alpha(t - t_{j(Stim)}^n) \quad (6)$$

where,

$$\alpha(t) = \frac{1}{\tau}\exp(\frac{-t}{\tau}) \quad (7)$$

N is the number of total neuron, w_{ij} is the synaptic conductance from j to i, Δ is the synaptic delay, and τ is a time constant. The spike times of the j^{th} presynaptic neuron are denoted by t^n_j. The input for neuron i at time t is:

$$I_i(t) = I_i^e(t) + I_i^s(t) \quad (8)$$

The attractors used for the current simulations are approximately orthogonal, each with about 9% neurons active. The stimulus duration is 20*ms*, after which the attractor is kept active purely through recurrent connections. Figure 3 shows the actual spiking patterns obtained in the output layer when the attractors 1 through 10 are triggered successively at intervals of 50*ms*.

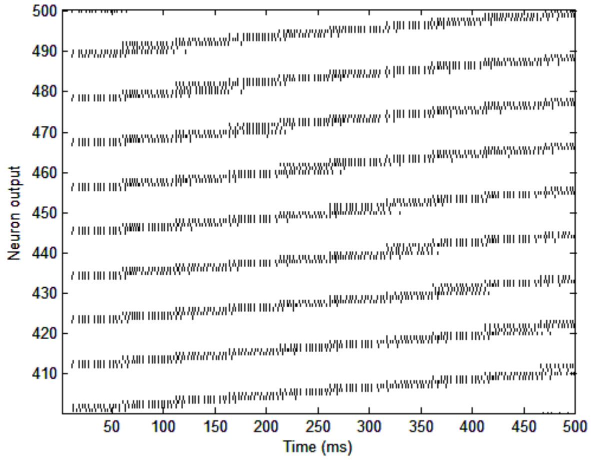

Figure 3: Spiking patterns of the output layer as the attractors are activated successively. The system has 10 near-orthogonal attractors, each of which remains active for 50 ms. Note that the number of active output layer neurons in each attractor varies.

C. Response System Model

The system has 10 groups of 50 neurons each corresponding to 10 different response elements. The neurons are modeled as modified RS (regular spiking) Izhikevich neurons [39]. The modification involves the inclusion of a resource depletion variable that ensures that a neuron remains active only for a certain duration.

The neurons within each group are connected to each other through weak random inhibitory connections. Also all the neurons in the group are inhibited by neurons from other groups with strong inhibitory connections. When the groups are stimulated with inputs of different magnitudes, the group with the largest input is the first to reach firing threshold, and its activity inhibits all the other groups as long as it is active. However, the resource depletion process ensures that the group eventually turns off and the next most excited group can become active, thus producing the sequence.

The input to each group comes from neurons of the corresponding group in the output layer of the encoding system. The input to neuron j in group i of the response system is denoted by x_{ij} and is given by:

$$x_{ij}(t) = \sum_{k \in M_i} w_{ik} s_k(t) \qquad (9)$$

where w_{ik} is the synaptic weight from the k^{th} neuron in attractor module (10 modules) M_i which projects to group i in the Response System, and s_k is the spiking output of neuron k. Each neuron in M_i connects to all neurons in group i with the same weight, but the weights for each neuron in M_i is different. Thus, the total magnitude of stimulus from M_i to groups i neurons depends on which neurons in M_i are active in the current attractor in the E system. The input current to each neuron j in each group i is calculated as:

$$I_{ij}(t) = G(t) * x_{ij}(t) \qquad (11)$$

where * indicates the convolution operator and $G(t)$ is the synaptic double exponential kernel function given by

$$G(t) = \frac{\tau_1 \tau_2}{(\tau_1 - \tau_2)}\left(e^{-\frac{t}{\tau_1}} - e^{-\frac{t}{\tau_2}}\right) \qquad (12)$$

where time to peak is given by

$$t_{peak} = \frac{\tau_1 \tau_2}{(\tau_1 - \tau_2)} \ln \frac{\tau_1}{\tau_2} \qquad (13)$$

The dynamics of the membrane potential for neuron j in group i is given by:

$$\frac{dv_{ij}}{dt} = 0.04 v_{ij}^2 + 5 v_{ij} + 140 - u_{ij} + I_{ij}^{exc} - I_{ij}^{inh} \qquad (14)$$

where I_{ij}^{exc} and I_{ij}^{inh} are the excitatory and inhibitory inputs, respectively, and u_{ij}, given by

$$u_{ij} = u_{ij}^q + f(u_{ij}^r) \qquad (15)$$

represents the inactivation variable for the neuron. The first component, u_{ij}^q, is the standard inactivation variable in the RS model, while u_{ij}^r represents a spike-dependent variable representing the depletion of a resource needed for activity. This variable increases only when the neuron spikes, and is modulated by a sigmoid function:

$$f(u) = \frac{\alpha}{1 + e^{-\beta u + \theta}}$$

Where α, β and θ are parameters determining the effect of resource depletion on firing rate. The equation for u_{ij}^q is:

$$\frac{du_{ij}^q}{dt} = a(b v_{ij} - u_{ij}^q) \qquad (16)$$

$$a = 0.01, \; b = 0.26$$

$$v_{ij}(t) = c \quad \text{if } (v_{ij}(t) \geq 30) \qquad (17)$$

$$c = -65$$

$$s_{ij}(t) = \begin{cases} 1 & v_{ij}(t) \geq 30 \\ 0 & \text{otherwise} \end{cases} \qquad (18)$$

$$u_{ij}^q = u_{ij}^q + d \quad \text{if } (v_{ij}(t) \geq 30) \qquad (19)$$

$$d = 2$$

$$u_{ij}^r = u_{ij}^r + r_i \quad \text{if } (v_{ij}(t) \geq 30) \qquad (20)$$

where r_i indicates the resource depletion rate for each group, which is identical for all the neurons in a group. s_{ij} indicates a spike of the j^{th} neuron in i^{th} group.

$$I_{ij}^{inh} = \sum_{\substack{k != i \\ k = j}}^{N_g} w_{ij}^{inh} z_k \qquad (21)$$

is the inhibitory stimulus from all the other groups, which is the sum of currents, z_k, from inhibitory interneurons targeting group k, weighted by w_{ij}^{inh} t and N_g is the total number of groups. The current from the inhibitory interneuron for each group is calculated as:

$$z_i(t) = G'(t) * s_{ij}(t) \qquad (22)$$

$$G'(t) = \frac{\tau_1^{'}\tau_2^{'}}{(\tau_1^{'} - \tau_2^{'})}(e^{-\frac{t}{\tau_1^{'}}} - e^{-\frac{t}{\tau_2^{'}}}) \qquad (23)$$

where $G^{'}(t)$ is the transfer function of synaptic transmission from the inhibitory neuron to the neurons in the other groups, whose time to peak is given by:

$$t_{peak} = \frac{\tau_1^{'}\tau_2^{'}}{(\tau_1^{'} - \tau_2^{'})}\ln\frac{\tau_1^{'}}{\tau_2^{'}} \qquad (24)$$

IV. Results

The system described above was implemented with several embedded response sequences. Figure 4 shows the recovery of one (especially simple) sequence when the attractor activated by the stimulus triggers response elements in the order of their indices. Figure 5 shows the result when the attractor pattern in E is the same as in Figure 4, but the activity pattern in M is different, causing response elements to have widely varying activity durations. Figure 6 shows two more response sequences generated by other attractor patterns in E. In each figure, the spikes generated are plotted as a raster pattern, with each spike indicated by a dot.

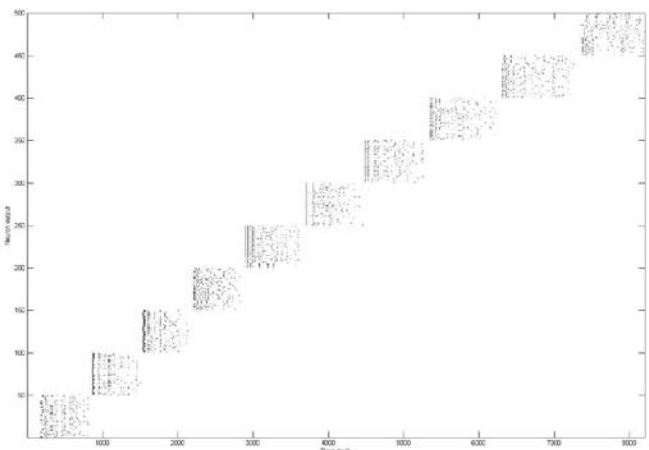

Figure 4: Generating a sequence where successive response groups are activated.

These figures clearly illustrate how static input patterns from the E and M systems can generate specific sequential responses. Thus, stereotypical response sequences and their timing can be represented in the E and M systems as spatial attractor patterns, and the system can generalize to variations of these responses (e.g., at different tempos) by systematically changing parts of the activity patterns in the E and M systems. This provides an extremely convenient, efficient and productive way to represent a large repertoire of response sequences involving the same response elements as a set of fixed-point attractors in E and M. For example, if the neurons in E and R represented population codes for direction of hand movement [40], each sequence would represent a specific shape (e.g., a square) which could be produced with variations (e.g., larger square, rectangles, etc.) simply by varying the activity pattern in M, while more complex variations (e.g., drawing the square backwards) could be encoded by varying the activity pattern in E.

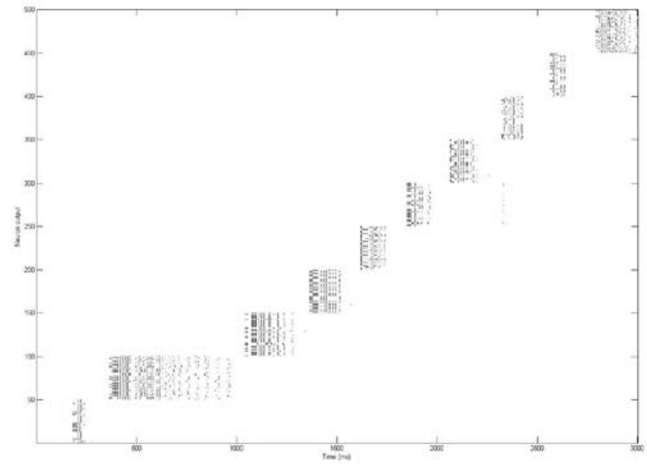

Figure 5: The same sequence as that in Figure 4, but with a different tempo. The E attractor is identical for the two cases, only the M pattern varies.

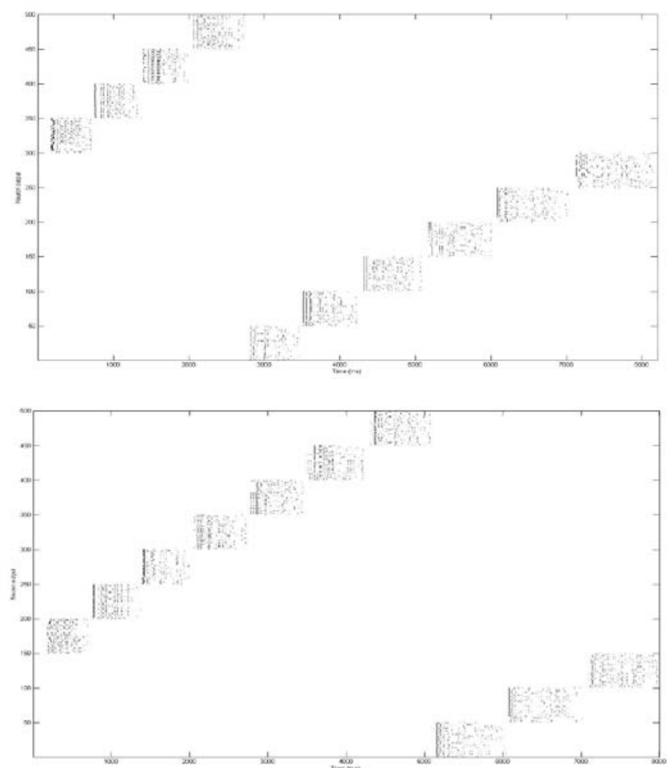

Figure 6: Sequences generated for two other attractors in the E system.

V. Conclusion

This paper has described a very simple spiking neuron model for representing stereotypical response sequences with specific timing using purely spatial codes. This allows for easy generalization of learned codes over different variations in order and tempo, and can provide the basis for constructing complex responses from simpler ones through development and learning.

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