Impact of Heterogeneity on Coverage and Broadcast Reachability in Wireless Sensor Networks

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***Abstract*— While most existing research efforts in the area of wireless sensor networks have focused on networks with identical nodes, deploying sensors with different capabilities has become a feasible choice. In this paper, we focus on sensor networks with two types of nodes that differ in their capabilities, and discuss the effects of heterogeneity of sensing and transmission ranges on the network coverage and broadcast reachability. Our work characterizes how the introduction of a few sensor nodes with better capabilities can reduce the number of total required sensors without sacrificing the coverage and the broadcast reachability. Analytical results are validated via simulations. Our work can be extended to more complicated heterogenous wireless sensor networks with more than two types of sensors, while homogenous networks can also be seen as a special case of heterogeneous wireless sensor networks where the two types of sensors are the same. This work can serve as a guideline for designing large-scale sensor networks cost-effectively.**

***Index Terms*— Coverage, Heterogeneity, Homogeneity, Reachability, Wireless Sensor Networks.**

I. Introduction

A wireless sensor network (WSN) is a collection of small, cheap and low powered sensor nodes which can dynamically form a network without any underlying infrastructure support [1], [2]. As sensor nodes are emerging as a key tool to gather information from diverse physical phenomena, a number of applications such as habitat monitoring, health caring, battlefield surveillance and enemy tracking have been proposed and discussed [2]. Most existing research focus on the homogeneous WSN where all the sensors are identical in terms of sensing, communication, computation, and power capabilities [3] [4], and homogeneous architecture is easy to model and resilient to individual sensor failures. However, the presence of a few more powerful sensors can improve the network reliability and stability with marginal or no increase in the cost of network deployment [5]. This makes the heterogeneous WSN increasingly important, and the research in this area is highly desirable.

Recently, researchers [4], [6], [5] in sensor networks have proposed to deploy sensor nodes with different capabilities as part of the same network. Intuitively, the introduction of some sensor nodes with greater capabilities can help enhance the overall network performance. However, questions of where, how many, and what types of heterogeneous resources to deploy remain largely unexplored [5]. This raises the issue of quantifying the effects of deploying heterogeneous sensor nodes on quality of service of the whole network.

One of the fundamental problems in sensor networks is sensing *coverage*. In general, it answers the questions about the quality of service (surveillance) that can be provided by a particular sensor network [7] [8]. To be specific, it reflects how well a sensor network is monitored or tracked by the sensors [9]. The *coverage* is defined as *the probability that any target point in the sensed area is within the sensing range of any nearby sensors*. Here, we focus our work on the case that a number of sensor nodes, deployed in a field to detect certain intrusion activities. An example of such a scenario may be seismic or acoustic sensors deployed in a battle field to detect enemy intrusion [9].

It should be noted that the introduction of sensor nodes with longer transmission range might complicate the operation of the network with the introduction of asymmetric links, since a node with longer transmission range might reach another node with shorter transmission range, but the node with shorter transmission range might not be able to reach the node with longer transmission range. Previous works on the connectivity have only considered bi-directional links for both homogenous range assignment [10], and heterogenous range assignment [11]. In this paper, we focus on the network *broadcast reachability*, which is defined as the *probability that a packet broadcast from a sensor with longer transmission range can reach all the other sensors in the network*, if we assume perfect scheduling and no collision at the MAC layer. Broadcast reachability has practical implications in broadcast, where a symmetric link is not required. Intuitively, heterogeneity leads to a more significant improvement in broadcast reachability than in connectivity, where *connectivity* is defined as *the probability that a packet broadcast from any sensor can reach all the other sensors in the network*.

By abstracting large scale sensor networks into graphs, important properties of the WSNs can be investigated. In this paper, we focus on sensor networks comprising two types of nodes that differ in their sensing and transmission ranges. It is no doubt that sensors with more complex detector or signal processing units can have better sensing capabilities. Thus, in our work, we model a better sensing capability as longer sensing range. Then, we quantify the impact of node characteristics on the network *coverage* and *broadcast reachability*. This provides analytical framework on the actual deployment of nodes in heterogenous WSNs.

The primary motivation for our work could be described as follow. Suppose we need to deploy a sensor network in a given area and achieve the requirement that 80% of the targeted area should be 3-covered or 1-covered. Two types of sensor nodes are given, one type of the sensors is simple with limited sensing and transmission ranges. The other type of sensors is more powerful in term of sensing and transmission ranges. Then, how to choose the number of each type of sensors to provide the required guarantee while keeping the cost minimum? The goal of this paper is to provides some insights to the design of WSN.

The reminder of the paper is organized as follows. In section II, we analyze the network coverage problem in both homogeneous and heterogenous cases. Then, we discuss the network broadcast reachability problem in a heterogenous network with two different types of sensors in section III. We present some related work in section IV and, finally, the paper is concluded in section V.

II. Coverage Analysis

In this section, we first study the coverage problem in homogeneous networks for completeness, and then we analyze the coverage problem in the heterogeneous case with two types of sensor nodes. This work can be extended to wireless sensor networks with more than two types of sensor nodes.

A. Analysis for Homogenous Networks

In homogenous WSNs, all the nodes have the same communication range and sensing range, denoted by r_c and r_s respectively. We present a coverage analysis which applies to random node deployment in both two dimensional and three dimensional space. Suppose that sensor nodes are deployed randomly in a square sensed area of size $A = L \times L$. The node density is $\lambda = \frac{N}{A}$, where N is the total number of deployed nodes [10]. To simplify the analysis, we assume a simple deterministic radio and sensing model [12], where a neighbor node can be reached when it is within the range of r_c, and a target can be sensed if the target is located within the range of r_s. Considering fluctuations in the sensing and transmission range will be addressed in future work.

We first consider the case of single-sensor detection, where a target point in the sensed area is considered as being covered when it is located within the sensing range of at least one nearby sensor. This situation is defined as 1-coverage [12]. For the two dimensional case, the 1-coverage probability is the probability that there is at least one sensor node located within the circle of area $\pi {r_s}^2$ centered at a target point in the sensed area [12]. For the three dimensional case, the 1-coverage probability is the probability that there is at least one sensor node within the sphere of volume $\frac{4\pi {r_s}^3}{3}$ centered at a target point in the sensed space. For a uniformly distributed sensor

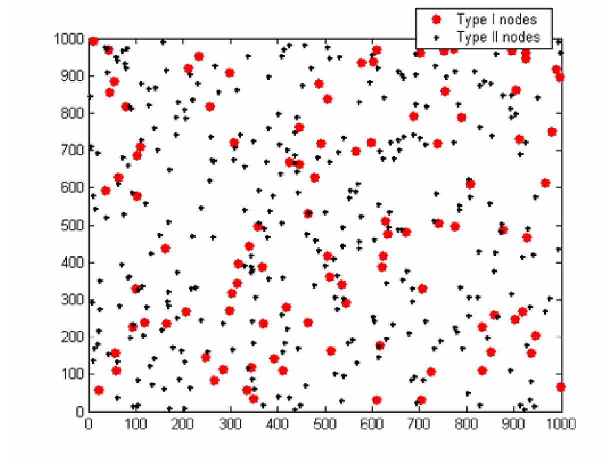

Fig. 1. Heterogenous network deployment

network with node density λ, the probability that there are m nodes within the area or space of S is Poisson distributed [10]:

$$P(m) = \frac{(S\lambda)^m}{m!} e^{-S\lambda}, \tag{1}$$

where $S = \pi {r_s}^2$ for two dimensional node deployment and $S = \frac{4\pi {r_s}^3}{3}$ for three dimensional deployment.

The probability that the monitored area or space is 1-covered, denoted by $p_{cover-1}$, can be expressed as:

$$p_{cover-1} = 1 - P(0) = 1 - e^{-\lambda S}. \tag{2}$$

Next, we consider the case where an event detection involves joint sensing by multiple sensors around the target. For example, trilateration-based localization can be seen as 3-joint sensing detection, where if a sensor is in the coverage of 3 anchor sensors that are equipped with GPS, the location of the sensor can be determined. Let k denote the number of sensors required for joint detection. To detect an event with k-joint sensing, the number of nodes within the circle area $\pi {r_s}^2$ or the sphere of $\frac{4\pi {r_s}^3}{3}$ must be at least k. Thus, the probability that the network deployment area is k-covered, denoted by $p_{cover-k}$, is:

$$p_{cover-k} = 1 - \sum_{m=0}^{k-1} P(m) = 1 - \sum_{m=0}^{k-1} \frac{(\lambda S)^m}{m!} e^{-\lambda S}. \tag{3}$$

where $S = \pi {r_s}^2$ in two-dimensional case, and $S = \frac{4\pi {r_s}^3}{3}$ in three-dimensional case.

According to the characteristics of Poisson stream, if a Poisson stream is split into k sub-streams such that the probability of a job going to be the ith sub-stream is p_i, each sub-stream is also Poisson with a mean rate of $p_i\lambda$ [13]. So, it is also possible to incorporate a node failure rate in our analysis. If we assume that all the sensor nodes have the same failure probability, denoted by p_f, it is simple to replace the previous node density λ with $\lambda(1 - p_f)$ in equations (2) and (3).

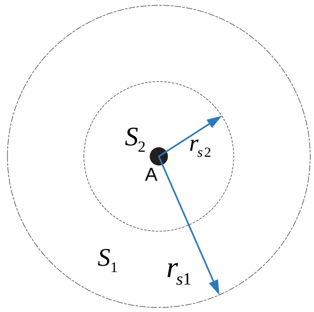

Fig. 2. Coverage in heterogeneous case

B. Analysis for Heterogenous Networks

In this part, we focus on heterogenous WSNs with two types of sensors. We define the sensors with greater sensing capability as Type I nodes and the sensors with lower sensing capability as Type II nodes. We assume that the Type I nodes' sensing range is r_{s1}, and that of Type II nodes' is r_{s2} ($r_{s1} \geq r_{s2}$). The densities of Type I and Type II nodes are λ_1, and λ_2, respectively. Fig. 1 shows a possible network deployment where both the type I nodes and type II nodes' distribution conforms to a two dimensional Poisson distribution. To achieve 1-coverage in two-dimensional case in the heterogenous network with two types of sensors, there must be at least one Type I node located within the area $S_1 = \pi {r_{s1}}^2$ or at least one Type II node located within the area $S_2 = \pi {r_{s2}}^2$ around each target point. As shown in Fig. 2, for the arbitrary point A to be 1-covered, there must be at least one type II node around A within S_2, or one type I node around A within S_1. The probability that there is no type I nodes around A within type I nodes' sensing coverage area S_1 is $P_1(0) = e^{-\lambda_1 S_1}$. Similarly, the probability that there is no type II nodes around A within type II nodes' sensing coverage area is: $P_2(0) = e^{-\lambda_1 S_2}$, Then, the probability that no nodes (either type I or type II nodes) is monitoring point A is $P_1(0)P_2(0)$. Thus, the probability that there is at least one node (either type I or type II node) could monitor point A is $1 - P_1(0)P_2(0)$. If $p_{cover-1}$ denotes the 1-coverage probability for a randomly chosen point, the probability that the point could be 1-covered can be expressed by the following equation:

$$p_{cover-1} = 1 - P_1(0)P_2(0). \tag{4}$$

For the k-joint sensing case, the probability of k-coverage, denoted by $p_{cover-k}$, is defined as the probability that for any target point in the monitored area, there are at least k sensor nodes that can sense it, and these can be any combination of Type I and Type II sensors. For example, in Fig. 3 (a), point P is in the sensing range of two type II sensors and one type I senor represented by black nodes and white nodes respectively. Thus, point P is 3-covered. Similarly in Fig. 3 (b), (c), and (d), point P is also 3-covered. Thus,

$$p_{cover-3} = 1 - \sum_{m=0}^{3-1} \left[\sum_{j=0}^{m} P_1(j)P_2(m-j) \right] \tag{5}$$

For the general case, the probability of k-coverage, can be

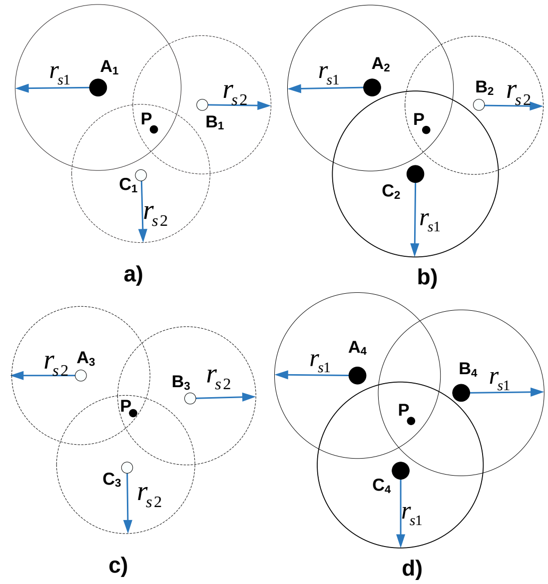

Fig. 3. Three-joint sensing case

expressed as:

$$p_{cover-k} = 1 - \sum_{m=0}^{k-1} \left[\sum_{j=0}^{m} P_1(j)P_2(m-j) \right]. \tag{6}$$

Given the node density and the sensing range, the coverage probability of the total sensed area can be estimated. We have performed Monte-carlo simulation using a VC++ simulator to get the empirical coverage probability. For each parameter choice, the corresponding number of type I and type II sensors are random randomly deployed in a 1000×1000 square area, and all the simulation results shown here are averaged over 100 runs. It should be noted that to avoid border effects, we adopt a periodic boundary conditions and a toroidal distance metric [10], instead of Euclidean distance in our simulations. Fig. 4 gives the analytical and simulation results on the coverage probability in both homogenous and heterogenous node configurations. For heterogeneous configurations, the number of Type II nodes is fixed at 300 with the sensing range equal to 40 and the number of Type I nodes with the sensing range equal to 120 varies from 0 to 150, while in homogenous case, all the sensors are Type II nodes. From the figure, it is clear that the coverage probability in the heterogenous case increases much more dramatically with an increase in Type I nodes than the homogeneous case. Based on our results, let us reconsider the problem we presented earlier. In order to get the 1-coverage probability of 0.8 in homogenous case we need to deploy about 120 type II sensors in addition to the original fixed 300 ones; while in heterogeneous case, we only need to deploy about 10 type I nodes to provide the same coverage probability. Furthermore, In order to get the 3-coverage probability of 0.8 in homogenous case we need to deploy over 2000 additional type II sensors; while in heterogeneous case, we only need to deploy about 70 type I nodes to get the same effect. Analytical and simulation results shows that our analysis is fairly accurate and substantiates our

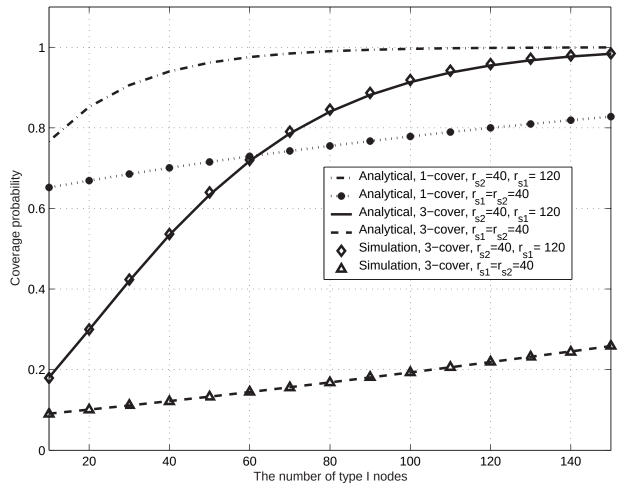

Fig. 4. Coverage probability under homogenous and heterogenous case

intuition.

III. Network Broadcast Reachability Analysis

In WSNs, packets such as queries are broadcasted throughout the whole network. For heterogenous WSNs, it is feasible for more powerful nodes to undertake more responsibilities. It is important to know if or how possible a packet broadcasted from a more powerful node can reach the entire network. The standard definition of connectivity based on symmetric links is not especially useful in quantifying the broadcast reachability because heterogeneous WSNs are fundamentally asymmetric. Instead, we look at another characteristic called broadcast reachability, which considers the reachability of nodes in terms of broadcasting.

We consider 2-dimensional networks with two types of nodes: Type I sensor nodes have transmission range r_{c1}, while Type II nodes' transmission range is denoted by r_{c2}. Without loss of generality, we assume that $r_{c1} \geq r_{c2}$. The node density and the number of Type I and Type II nodes are denoted by λ_1, λ_2, n_1 and n_2 respectively, where $\lambda_1 = \frac{n_1}{L \times L}$ and $\lambda_2 = \frac{n_2}{L \times L}$. The total number of nodes in the network is denoted by N, and $N = n_1 + n_2$.

We assume that Type I nodes are given more responsibility, and they sometimes broadcast packets throughout the network. Broadcast reachability is therefore the probability that a packet broadcasts from a Type I node can reach all nodes in the WSN. Here, we derive the broadcast reachability of the network as the probability that all the nodes in the network are reachable by the broadcast originated from a Type I node. For any node in the WSN, denoted by node A, the probability that it is reachable by the broadcast from any Type I node other than node A is defined as $P_{non-iso}$. It is clear that node A should be in at least one of the other $N-1$ nodes' communication range in order to get the broadcasting packet, assuming that the other $N-1$ nodes are reachable by the broadcast. Thus, the conditional probability of $P_{non-iso}$ can be expressed as:

$$P_{non-iso} = 1 - P_1(0)P_2(0), \tag{7}$$

If we assume statistical independence for all the nodes, the probability that the other $N-1$ nodes are reachable the the broadcast can be calculated as:

$$P_{non-iso}^{N-1} = \left[1 - P_1(0)P_2(0)\right]^{(N-1)}, \tag{8}$$

Thus, the broadcast reachability is upper bounded by:

$$\begin{aligned} P_{non-iso}^{N} &= \left[1 - P_1(0)P_2(0)\right]^{N} \\ &= \left[1 - P_1(0)P_2(0)\right]^{(n1+n2)}. \end{aligned} \tag{9}$$

Monte-Carlo simulations are performed to validate the analysis. We simulate two-dimensional heterogeneous networks with a deployment area of size 1000×1000. Adjacency matrix is constructed represent the digraph [14]. Depth-first-search algorithm [14] is used to check the reachability by selecting a random Type I node as the starting node. Plotted figures are average from 100 simulation runs. Fig. 5 shows analytical and simulation results on the broadcast reachability for the heterogenous case where $r_{c1} = 2r_{c2}$, $n_1 = 200$, and $n_2 = 300$, and for the homogenous case where $r_{c1} = r_{c2}$. Radius r_{c2} varies from 40 to 100. Fig. 6 shows results when n_2, r_{c2} and r_{c1} are fixed at 300, 70 and 140 respectively, but n_1 varies from 10 to 300.

In the homogenous case when $r_{c1} = r_{c2}$, the broadcast reachability is equivalent to the connectivity since there are no asymmetric links. Our analysis is different from Bettstetter's work on the connectivity of heterogeneous networks [11], since only connectivity analysis considers bi-directional. The analytical results of the connectivity in heterogenous case [11] are also shown in Fig. 5 and Fig. 6. We can see that our analysis on the upper bound of broadcast reachability provides a close bound to the actual simulation results. The results also illustrate that heterogeneity does affect the broadcast reachability much more dramatically than it affects connectivity. Fig. 5 indicates that, in a heterogeneous WSN, broadcast reachability increases at a much faster rate than connectivity as the transmission radius of both types of nodes grows. Fig. 6 indicates that even a small number of Type I nodes in a large network can greatly improve broadcast reachability while connectivity only improves gradually as nodes with large transmission range proliferate.

IV. Related Work

Recent works on the connectivity and coverage of WSNs have addressed some of these issues [7], [8], [9], [10], [11], [12]. The coverage problem in sensor networks was first investigated as one of the network QoS metrics by Meguerdichian *et al.* [7]. In paper [8], the authors have established the optimal polynomial time worst and average case algorithm for network coverage calculation using graph theoretic and computational geometry. In paper [9], the authors have modeled the coverage problem in sensor networks as a decision problem, and proposed a method to determine whether every point in the service area of the sensor network is covered by at least k-sensors,

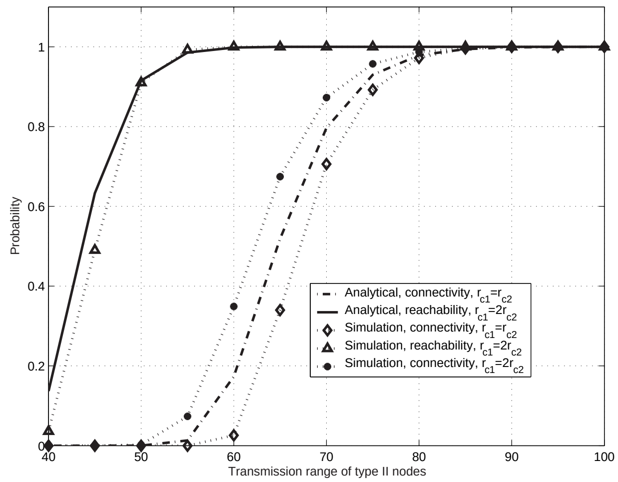

Fig. 5. Effects of transmission range on the reachability in heterogenous network

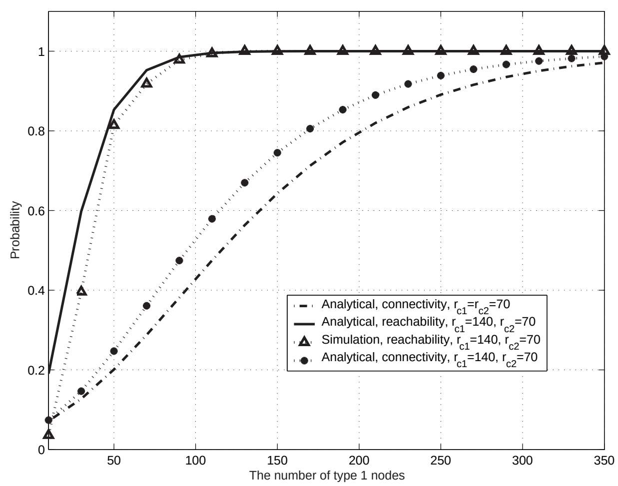

Fig. 6. Effects of type I nodes on the reachability in heterogenous network

under the assumption of homogenous node configuration. In paper [12], the authors have presented probabilistic analysis for the sensor coverage problem, where coverage is defined as the same as in our paper, and their analysis is constricted to homogeneous sensor networks. As to the broadcast reachability analysis, to our best of knowledge, few papers have paid attention to this area. In paper [15], the authors have presented a distributed topology control algorithm to calculate the per-node minimum transmission power, so that reachability between any two nodes is guaranteed to be the same as in the initial topology, and the reachability is defined as multi-hop connectivity from any source to any destination in the directed graph, which is different from the focus of our research.

V. Conclusion

There exist fundamental limits on the operation of WSN, including low date rate, sheer network size, limited computing power, communication range and battery capacity. We believe that the analytical characterization of WSN is important since it provides real insights on the design of WSN. Probabilistic analysis on the effects of heterogeneity on coverage and reachability has been presented in this paper. It substantiates our intuition that the introduction of nodes with better capability, namely longer sensing range and transmission range, can dramatically increase the network coverage and broadcast reachability, though the effects on connectivity are only modest. This work can also provide useful guide on choosing the optimal number of different types of nodes, as well as sensing and transmission ranges of large-scale heterogenous WSN design.

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