Alpha Beta Pruning for Expected Minimax

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Recall the the Expected Minimax results from the Minimax strategy when chance nodes are considered.

As we have seen, the Minimax strategy, extended to this case calls for the definition of the of the Minimax value for a a node n in three different situations:

1. n is a terminal node
2. n is a MAX node
3. n is a MIN node
4. n is a $CHANCE$ node

For the first three cases the **minimax value** is calculated in the usual manner; for the third case, its definition must be extended:

Recall that a chance node n really means that there is a probability distribution for the actions taken by the player whose turn is next: Max or Min.

More precisely,

$$n \equiv \{(s_1^n, p_1^n), \ldots, (s_m^n, p_m^n n)\}$$

where s_j, $j = 1, \ldots, m^n$ denote the *successors* for the node n, and for each j, $p_j^n \equiv P(s_j^n),\ 0 \leq p_j^n \leq 1;\ and\ \sum_{j=1}^{m} p_j^n = 1$.

If we denote by $Successors(n) = \{s; s\ is\ a\ successor\ of n\}$, the chance node induces a probability distribution on the minimax values of its elements.

In this case the **minimax value** for n is then defined as the **expected value** of the values of its successors nodes. The **minimax value** is referred now to as **expectedminimax value**, which is defined as:

$$ExpectedMiniMax(n) = \begin{cases} Utility(n) & \text{if } n \text{ is a terminal node} \\ Max_{s \in Successors(n)} ExpectedMiniMax(s) & \text{if } n \text{ is a } MAX \text{ node} \\ Min_{s \in Successors(n)} ExpectedMiniMax(s) & \text{if } n \text{ is a } MIN \text{ node} \\ ExpectedValue(ExpectedMiniMax(s)) & \text{if } n \text{ is a CHANCE node} \end{cases}$$

Example Let us consider the following imaginary game:

MAX actions: $\{M_1, M_2, M_3\}$ Each of these leads to chance node;

$CHANCE$: $\{C_{M1}, C_{M2}, C_{M3}\}$; Let us assume the following probability distributions for each of these:

$$C_{M1} : \{(m_{11}, 1/2), (m_{12}, 1/2)\}$$

$$C_{M2} : \{(m_{21}, 1/4), (m_{22}, 3/4)\}$$

$$C_{M3} : \{(m_{31}, 2/3), (m_{32}, 1/3)\}$$

where m_{ij} denotes the MIN node corresponding to the action M_i of the MAX player and jth "branch" of the corresponding $CHANCE$ node.

Table 1: Utility: terminal nodes

node	utility value
M_{111}	2
M_{112}	6
M_{113}	1
M_{121}	7
M_{122}	4
M_{123}	2
M_{211}	8
M_{212}	9
M_{213}	-3
M_{221}	7
M_{222}	2
M_{223}	3
M_{311}	1
M_{312}	5
M_{313}	3
M_{321}	-2
M_{322}	6
M_{323}	3

MIN : Each MIN node is now followed by **ONE** (to make things easier) $CHANCE$ node. For example, assuming that each chance node generates three equally likely outcomes, we would have

$$C_{m_{ij}} : \{(M_{ij1} : 1/3), (M_{ij2} : 1/3), (M_{ij3} : 1/3)\}$$

where $M_{ijk},\ i = 1, \ldots, 3;\ j = 1, \ldots, 2;\ k = 1, \ldots, 3$, denote the MAX nodes generated from the last $CHANCE$ node.

Assume that $M_{ijk},\ i = 1, \ldots, 3;\ j = 1, \ldots, 2;\ k = 1, \ldots, 3$ are all terminal nodes with utility value shown in Table **??**.

Back up the utility up one level in the tree to obtain the result shown in Table **??**.

This leads us to $\alpha - \beta$ **pruning for this. The key idea is to identify bounds for the chance nodes.**

Table 2: Utility:backed up **one** level

node	utility value	node	Expected minimax for chance nodes $C_{m_{ij}}$
M_{111}	2		
M_{112}	6	$C_{m_{11}}$	$1/3(2+6+1) = 3$
M_{113}	1		
M_{121}	7		
M_{122}	4	$C_{m_{12}}$	$1/3(7+4+1) = 4$
M_{123}	1		
M_{211}	8		
M_{212}	9	$C_{m_{21}}$	$1/3(8+9-2) = 5$
M_{213}	-2		
M_{221}	7		
M_{222}	2	$C_{m_{22}}$	$1/3(7+2+3) = 4$
M_{223}	3		
M_{311}	1	$C_{m_{31}}$	$1/3(1+5+3) = 3$
M_{312}	5		
M_{313}	3		
M_{321}	-3	$C_{m_{32}}$	$1/3(-3+6+3) = 2$
M_{322}	6		
M_{323}	3		

Table 3: Utility:backed up **two** levels

node	utility value	node	Expected minimax for chance nodes $C_{m_{ij}}$	node	Expected minimax m_{ij}
M_{111}	2				
M_{112}	6	$C_{m_{11}}$	$1/3(2+6+1) = 3$	m_{11}	3
M_{113}	1				
M_{121}	7				
M_{122}	4	$C_{m_{12}}$	$1/3(7+4+1) = 4$	m_{12}	4
M_{123}	1				
M_{211}	8				
M_{212}	9	$C_{m_{21}}$	$1/3(8+9-2) = 5$	m_{21}	5
M_{213}	-2				
M_{221}	7				
M_{222}	2	$C_{m_{22}}$	$1/3(7+2+3) = 4$	m_{22}	4
M_{223}	3				
M_{311}	1	$C_{m_{31}}$	$1/3(1+5+3) = 3$	m_{31}	3
M_{312}	5				
M_{313}	3				
M_{321}	-3	$C_{m_{32}}$	$1/3(-3+6+3) = 2$	m_{32}	2
M_{322}	6				
M_{323}	3				

Table 4: Utility:backed up **three** levels

node	utility value	node	Expected minimax for chance nodes $C_{m_{ij}}$	node	Expected minimax m_{ij}	node	Expected minimax C_{Mi}
M_{111}	2						
M_{112}	6	$C_{m_{11}}$	$1/3(2+6+1)=3$	m_{11}	3	C_{M1}	3.5
M_{113}	1						
M_{121}	7						
M_{122}	4	$C_{m_{12}}$	$1/3(7+4+1)=4$	m_{12}	4		
M_{123}	1						
M_{211}	8						
M_{212}	9	$C_{m_{21}}$	$1/3(8+9-2)=5$	m_{21}	5	C_{M2}	17/4
M_{213}	-2						
M_{221}	7						
M_{222}	2	$C_{m_{22}}$	$1/3(7+2+3)=4$	m_{22}	4		
M_{223}	3						
M_{311}	1	$C_{m_{31}}$	$1/3(1+5+3)=3$	m_{31}	3	C_{M3}	11/3
M_{312}	5						
M_{313}	3						
M_{321}	-3	$C_{m_{32}}$	$1/3(-3+6+3)=2$	m_{32}	2		
M_{322}	6						
M_{323}	3						

Table 5: Utility:backed up **four** levels

node	utility value	node	Expected for chance nodes $C_{m_{ij}}$ minimax	node	Expected m_{ij} minimax	node	Expected C_{Mi} minimax	node	Expected MAX minimax
M_{111}	2								
M_{112}	6	$C_{m_{11}}$	$1/3(2+6+1)=3$	m_{11}	3	C_{M1}	3.5		
M_{113}	1								
M_{121}	7								
M_{122}	4	$C_{m_{12}}$	$1/3(7+4+1)=4$	m_{12}	4			MAX	17/4
M_{123}	1								
M_{211}	8								
M_{212}	9	$C_{m_{21}}$	$1/3(8+9-2)=5$	m_{21}	5	C_{M2}	17/4		
M_{213}	-2								
M_{221}	7								
M_{222}	2	$C_{m_{22}}$	$1/3(7+2+3)=4$	m_{22}	4				
M_{223}	3								
M_{311}	1	$C_{m_{31}}$	$1/3(1+5+3)=3$	m_{31}	3	C_{M3}	11/3		
M_{312}	5								
M_{313}	3								
M_{321}	-3	$C_{m_{32}}$	$1/3(-3+6+3)=2$	m_{32}	2				
M_{322}	6								
M_{323}	3								