CS-6021-001: Mathematical Logic — Nearly final draft

Class Meeting: ... **Time:** MWF 2:30-3:25 ... & ... **Room:** 840 Old Chemistry Bldg. (a.k.a. Alchemy Bldg.)

Professor: John Schlipf
> **Email:** `John` dot `Schlipf` at `UC` dot `Edu`
> **Office:** 810 A Old Chemistry .... **Telephone:** 556-hyphen-2569 .... **Home Telephone:** 281-hyphen-9476[1]
> **Office Hours:** **TBD** *and by appointment.*
> > *If you are having trouble with the material, come to me for help as soon as possible.*

> **My Web Page:** `www.eecs.ceas.uc.edu/˜schlipf`
> **Course Web Page:** `www.eecs.ceas.uc.edu/˜schlipf/MathLogic`

Catalog Course Description: *First order logic. Proof systems, Compactness, completeness, and an introduction to model-theoretic techniques. Gödel's incompleteness theorem.* 3 undergraduate or graduate credits.

Textbook: *A Mathematical Introduction to Logic*, by H. Enderton (2nd ed.)[2], plus distributed notes.

> **Some Other References:** (Warning: all use somewhat different formal systems)
> - *Language, Proof, & Logic*, by J. Barwise & J. Etchemendy
> - *Logic for Applications*, by A. Nerode & R. Shore
> - *Logic in Computer Science*, by M. Huth & M. Ryan
> - *Computability and Logic*, by G. Boolos

Prerequisite: A $\geq$ 5000-level theoretical mathematics or CS course or a *majors-level logic* course.

Topics:
1. Propositional Logic.
2. First Order (Predicate) Logic: Syntax & Semantics.
3. Formal Proof System; Soundness, 6ompleteness and Compactness.
4. Normal Forms.
5. Formal Proof System; Soundness, Completeness and Compactness.
6. Introduction to Model-Theoretic Techniques
7. Models of First Order Theories
8. Introduction to Number Theory
9. Introduction to Computability
10. Arithmetization of syntax
11. Gödel's First Incompleteness Theorem

> That *averages* 1-2 weeks per topic — but follow along in class to see where we are for any day. If you cannot attend, contact a classmate to see what we have done that day. And reserve the right to change the list of topics if I see necessry.

> I'll announce additional issues by email — to whatever email address UC links to your course enrollment. So be sure to check that email account *daily*.

Goals (and a bit of background)**:** This material arises from the work of the ancient Greeks — their method of demonstrating the correctness of arguments. In the last 150 years it has become a tool for formalizing the foundations of mathematics. Computer science was founded, in large part, by mathematical logicians, such as Alan Turing and John von Neumann.

> I feel that one must understand the process of "doing" logic in order to apply it. **So:**

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[1]Normally, use email to contact me when I am not in my office. Please call me at home only for emergencies, and then only after 9:00 AM and before 9:00 PM. Most help wth reading the book or doing the homework is possible only in my office.

[2]I found a `.pdf` copy of the 1st edition on the web. I *think* the text is largely the same, *but* the problems have been changed.

- I shall put heavy emphasis upon actually writing correct proofs about the logic; expect that to be a significant challenge for some of you. (Of course, this means I shall also go through material more slowly than one otherwise would.)

- Another concern is developing your ability to read logic materials. So, since Enderton's text can be somewhat dense, I may assign some of you to present some of the material to the class.

- Although some of you may be more interested in issues such as "formal methods" of program verification, I shall limit attention to more "classical" topics, since these are the paragigms for later work.

Grading & Attendance: I *hope* to grade almost entirely upon (a) your presentation of work in class and (b) your contributions during others' presentations. If I do not see enough of some students' work, I may assign homework to turn in, give tests, or both. (If I do give tests, I shall give a final exam on the day UC specifies for final exams for this class time.)

Thinking through problems will develop your intellectual sophistication. So, for this class, please *do not* look elsewhere for sample problem answers; the goal is to stretch you mind with each problem. [3] (You are welcome to come to me for help as needed. But, normally, I'll try to give you only enough of a hint to get you past your current difficulty, so that you can continue intellectual practice.)

When I assign homework, unless I say otherwise, turn in to me, at the *start of the class period*, your list of"*claims*" — the numbers of the homework problems you completed.[4] Then I shall call students to the board to present their solutions of some of the problems to their classmates on the board.[5] Typically, I grade each solution you give, on correctness an clarity; then I get an overall score by multiplying your average grade times the number of problems you have "claimed."

You have the responsibility to attend class, so that you are intellectually involved in the class and so that you are there to present their solutions when I call on you. [6]

You also have the responsibility to help identify and correct their classmates' errors. This will normally feel very uncomfortable at first — both having your classmates find errors in your work and having to correct classmates' work. However, we all make mistakes, and this is an important learning experience. I shall normally make corrections only if class members miss them. Obviously, this means that you will need to keep up on the work and attend almost all classes.

Plagiarism: See my web page and the University rules cited there.

For students with Disabilities: Contact the Disability Services office *as soon as possible*; they will instruct me on how to compensate for your disability. If you need any accommodations on tests, since I'm a nearly paradigmatic absent-minded professor, *please remind me 2-7 days before the test* so that I can plan for the accommodations. Also see my web page, in particular if you feel the above offices have not dealt fairly with your disability.

Please also see the notes on my web page about mental health problems or crises.

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[3]And, if you do just look up solutions, besides committing plagiarism, (i) you will be less intellectually prepared to address more sophisticated problems later on, and (ii) you will be less likely to present solutions well when you come to the board.

[4]You may continue claiming each problem until someone puts a solution on the board, I ask for written solutions, I pass out a sample solution, or I declare it "closed."

[5]You may use your written solutions as reminders in your presentatons, but you *may not* simply copy off them.

[6]If you are absent when I call on you, I will give you 0 points on that problem — but I shall forgive each student the first 1-3 such 0's. If you have have an illness or other conflict that makes you miss > 2 classes or miss a test, bring in the usual documentation to confirm the problem, *e.g.*, a physician's excuse, on professional stationary, saying you had an illness severe enough to miss > 2 classes, a newspaper obituary identifying you among the next-of-kin, or, *in advance*, a note from somebody else at UC telling of your need to attend some other function.