Math Logic, Week#XIV (*i.e.*, Week#E_{16})

Moon's Day

Temporal Logic & Model Checking Cont.

We have seen 2 (of many) temporal logics:

CTL: Every temporal operator quantifies separately over paths.

The operators we've seen — operating on formulas ϕ, ψ — are

$$EX\phi, AX\phi, EF\phi, AF\phi, EG\phi, AG\phi, E[\phi U\psi], A[\phi U\psi].$$

LTL: The operators we've seen: $X\phi, F\phi, G\phi, A[\phi U\psi]$.

The formula does not explicitly reference the path.

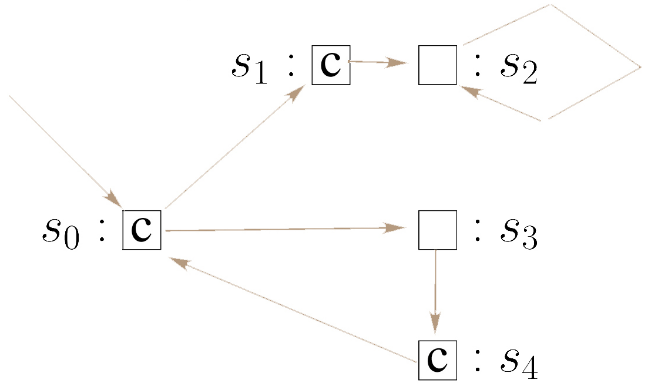
Rather, we evaluate the formula along each path.

The formula is true at a state s if it's true at *every path* starting at s.

(So it's possible that $s \not\Vdash \phi$ and also $s \not\Vdash \neg\phi$.)

1. I indicated an algorithm for determining satisfaction (forcing individual states) in CTL
   — and I assigned you to write out the full algorithm.
2. We did not cover an algorithm for LTL, which is more complicated.
3. LTL and CTL are not comparable: Some properties are expressible in both, some in LTL but not CTL, and some in CTL but not LTL.

4. **Differences in a simple "translation":**



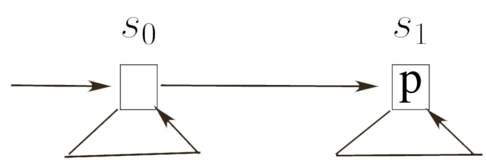
In LTL: ¿Does $s_0 \Vdash F(c \wedge Xc)$?

in CTL: ¿Does $s_0 \Vdash AF(c \wedge AXc)$?

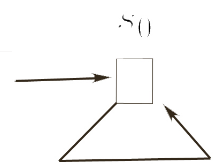
5. **Theorem:** *No LTL formula is logically equivaent to $AG(EFp)$.*

Proof: Suppose there were such an LTL formula φ. Consider the 2 transition systems below:

System S_1:



System S_2:



In CTL: In S_1, $s_0 \Vdash AG(EFp)$. **(¿Why?)**

In S_2, $s_0 \not\Vdash AG(EFp)$. **(¿Why?)**

In LTL: We assumed that φ is logically equivalent to $AG(EFp)_{in\ CTL}$.

So in S_1, $s_0 \Vdash \varphi$ — φ is true on every path starting at s_0.

And every path in S_2 is isomorphic to one in system S_1. So, φ must be true in every such path.

So, in S_2, $s_0 \Vdash \varphi$ — differing from the CTL result.

6. **Theorem** *No CTL formula is logically equiv. to $GFr_1 \rightarrow GFp_1$.*

¿What does that formula "say"? (It's a common example of "fairness.")

Proof: Left for some other course.